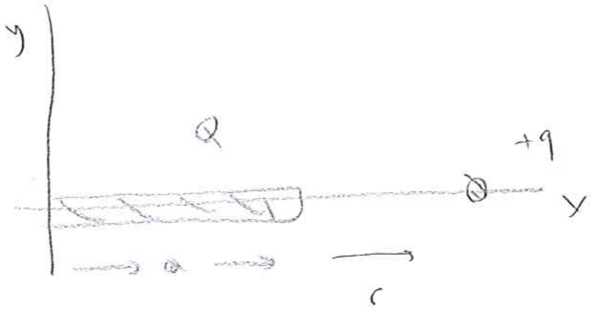
21.89



$dQ = (Q/a)\,dx$

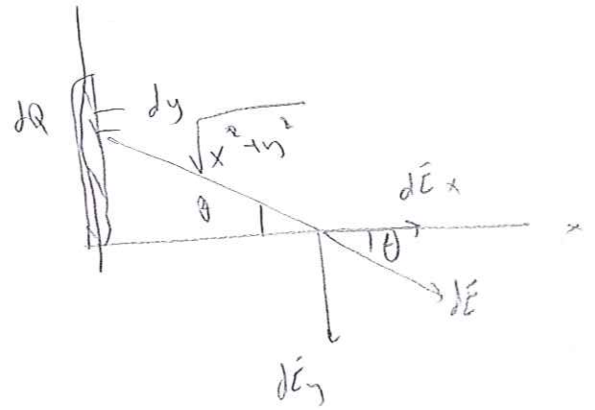
a) $dE_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{(a+r-x)^2} \rightarrow E_x = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_0^a \dfrac{Q\,dx}{a(a+r-x)^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a}\left(\dfrac{1}{r}-\dfrac{1}{a+r}\right)$

$E_y = 0$

b) $\vec{F} = q\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a}\left(\dfrac{1}{r}-\dfrac{1}{a+r}\right)\hat{\imath}$

c) $x \gg a \rightarrow \vec{F} = \dfrac{kqQ}{ax}\left((1-a/x)^{-1}-1\right) = \dfrac{kqQ}{ax}\left(1+a/x+\dots-1\right) = \dfrac{kqQ}{x^2}$

21.97



$$d\vec{E}_x = d\vec{E}\cos\theta = \frac{Q\,x}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right)$$

$$d\vec{E}_y = -d\vec{E}\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{y\,dy}{(x^2+y^2)^{3/2}}\right)$$

$$E_x = \int d\vec{E}_x = +\frac{Q\,x}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Q\,x}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]\Bigg|_0^a$$

$$= \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dE_y = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{y\,dy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]\Bigg|_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

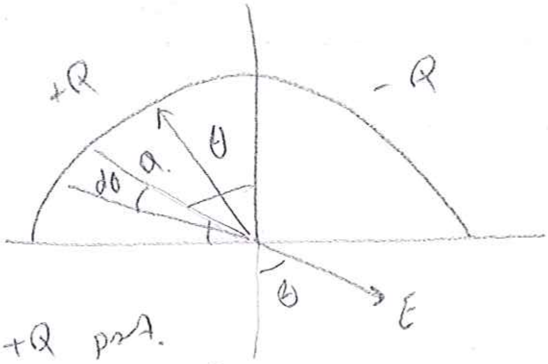
b) $\vec{F} = q\vec{E} \rightarrow F_x = \dfrac{-qQ}{4\pi\epsilon_0 x}\dfrac{1}{\sqrt{x^2+a^2}}$; $F_y = \dfrac{qQ}{4\pi\epsilon_0 a}\left(\dfrac{1}{x} - \dfrac{1}{\sqrt{x^2+a^2}}\right)$

c) For $x \gg a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x} - \dfrac{a^2}{2x^3}$

plug into F_y expression $\rightarrow F_y \approx \dfrac{qQa}{8\pi\epsilon_0 x^3}$

$$F_x = -\frac{qQ}{4\pi\epsilon_0 x^2}$$

21.38



consider +Q part.

$$\lambda = \frac{Q}{\frac{2\pi a}{4}} = \frac{2Q}{\pi a}$$

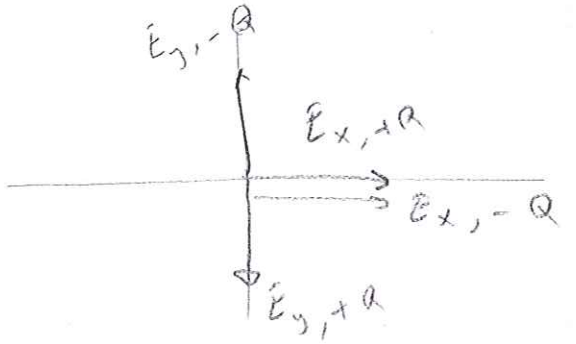
$$dE_y = dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda a\,d\theta}{a^2}\cos\theta \rightarrow E_y = \frac{\lambda}{4\pi\epsilon_0 a}\int_0^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$dE_x = dE\sin\theta$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0 a}\int_0^{\pi/2}\sin\theta\,d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

-Q part also creates the same fields



$\rightarrow$ E_y component cancel

$$E_{y,net} = 0$$

$$E_{x,net} = 2\times\frac{Q}{2\pi^2\epsilon_0 a^2} = \frac{Q}{\pi^2\epsilon_0 a^2}$$

21.99

Each line produces an electric field at P due to a finite line. These fields add by vector addition.

Each field has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2+a^2}}$$

$$E_{net} = 2E_1 \cos 45^\circ = 2 \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2+a^2}} \cos 45^\circ$$

a) $$E_{net} = 2 \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2+a^2}} \cos 45^\circ$$

$$= \frac{2(9 \cdot 10^9 \, N m^2/C^2)(2.5 \times 10^{-6} C)\cos 45}{(0.6\, m)\sqrt{(0.6\, m)^2 + (0.6\, m)^2}} = 6.25 \times 10^4 \, N/C$$

b) $F = eE = (1.6 \times 10^{-19} C) \times (6.25 \times 10^4 \, N/C) = 1 \cdot 10^{-14}$, opposite to the direction of electric field