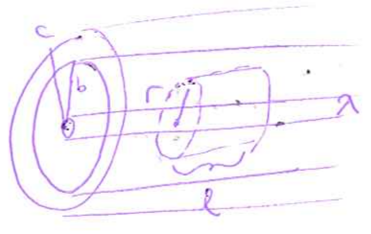
PS·2 Ch 22 solutions

22.39 Coaxial cable

a)

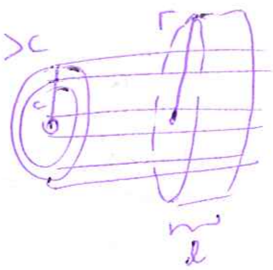


$a<r<b$; apply Gauss law to gaussian cylinder.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda \cdot l}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$, radially outward. (since positive charge enclosed)

b) $r>c$



$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r},$$

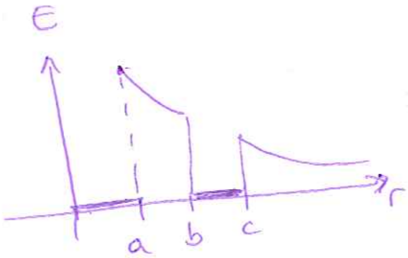
radially outward.

c) $r<a$; inside conductor $E=0$

$a<r<b$; $E = \frac{\lambda}{2\pi\epsilon_0 r}$

$b<r<c$; $E=0$ inside conductor

$r>c$; $E = \frac{\lambda}{2\pi\epsilon_0 r}$



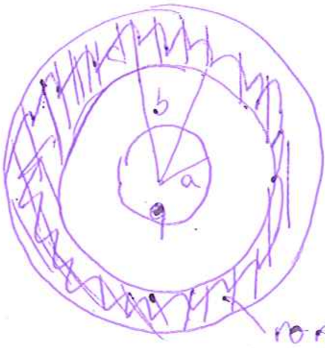
d) charge per unit length on inner surface of outer cylinder:

take a gaussian cylinder larger than inner surface; since inside conductor $E=0$, from gauss' law $Q_{encl}=0$. ⇒ inner conductor carries λl charge, so inner surface of outer cylinder should have $-\lambda l$ charge.

Since net charge on outer cylinder is zero, outer surface line charge density is $+\lambda$.

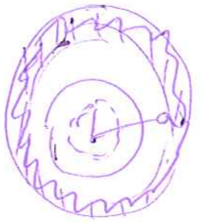
22.44 Sphere in a sphere

conducting solid charge q. | hollow conducting

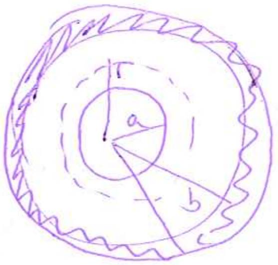


no net charge

a) $r<a$; Gaussian sphere is inside the conductor $E=0$.



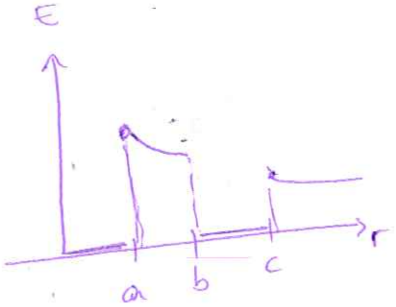
$a<r<b$;



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E\,4\pi r^2 = q/\epsilon_0 \rightarrow E = \frac{q}{4\pi\epsilon_0 r^2}$$

b)



$b<r<c$;

inside spherical conducting shell $\rightarrow E=0$

$r>c$; gaussian surface encloses everything.

$$E\cdot 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0} = q/\epsilon_0 \rightarrow E = \frac{q}{4\pi\epsilon_0 r^2}$$

c) charge on inner surface of shell.

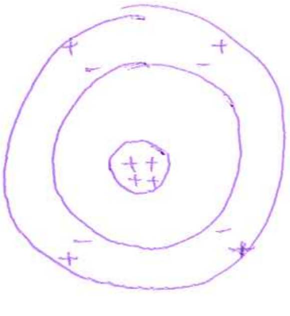
$E=0$; from Gauss' law

$Q_{encl}=0$

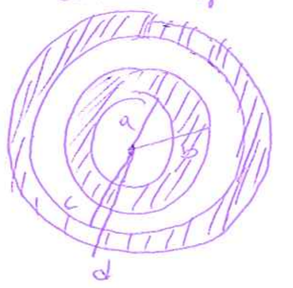
↳ charge on inner shell surface must be $-q$.

d) Since no net charge on the outer spherical shell; charge on outer surface must be $+q$.

e)



22.47 Concentric spherical shells



~~inner~~ shell $+2q$
outer shell $+4q$.

a) $r<a$; $E=0$ no enclosed charge.

$a<r<b$; inside conductor $E=0$

$b<r<c$; $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

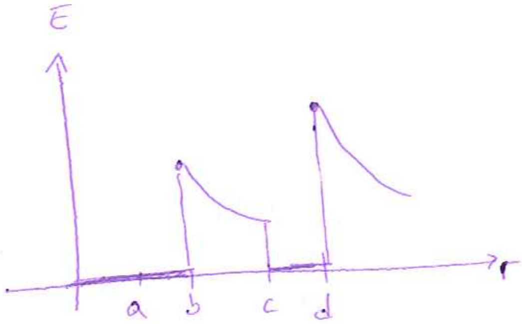
$E\,4\pi r^2 = +2q/\epsilon_0$

$E = \frac{2q}{4\pi\epsilon_0 r^2}$, radially outward

$c<r<d$; inside conductor $E=0$

$r>d$; $E\cdot 4\pi r^2 = \frac{2q+4q}{\epsilon_0}$

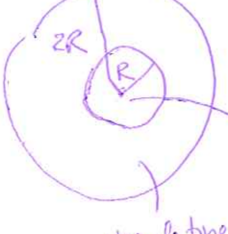
$E = \frac{6q}{4\pi\epsilon_0 r^2}$, radially outward



b)

	inner shell	outer shell
inner surface	inside conductor $E=0$, $Q_{encl}=0$ no charge	$E=0$ inside conductor ↳ $Q_{encl}=0$ ↳ $-2q$
outer surface	net charge on shell $2q$. ↳ $+2q$	net charge on shell $4q$ ↳ $+6q$

22.50



a) to have net charge of system $=0$; the shell must carry $-Q$ because inner sphere has $+Q$.

$-Q = \rho V_{shell}$

$-Q = \rho\left(\frac{4}{3}\pi(2R)^3 - \frac{4}{3}\pi R^3\right)$

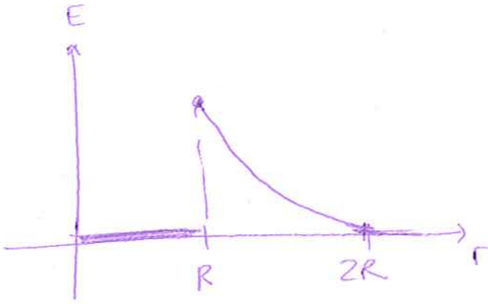
$\rho = -\frac{3Q}{28\pi R^3}$

b) $r<R$; inside conductor $E=0$.

$r>2R$; $E=0$ since net charge is zero.

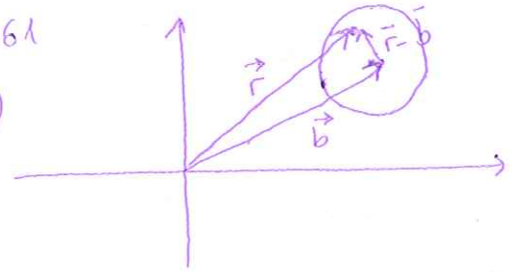
$R<r<2R$; Gauss' law → $E\,4\pi r^2 = \frac{Q_{encl}}{\epsilon_0} = \underbrace{\frac{Q}{\epsilon_0}}_{\text{from inner sphere}} + \underbrace{\frac{4\pi\rho(r^3-R^3)}{3\epsilon_0}}_{\text{from spherical shell}}$

$E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\rho}{3\epsilon_0 r^2}(r^3-R^3)$ (substitute ρ ~~found~~ in part (a)) c)



22.61

a)



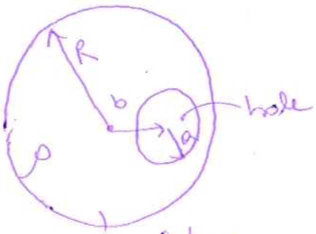
insulating sphere not centered at origin, but at $\vec{b}$.
of radius a,
charge density ρ.

If we would have a sphere centered at origin, electric field inside the sphere is given by $E = \frac{Qr'}{4\pi\epsilon_0 R^3}$, where $\vec{r'}$ is the vector from the center of sphere to point where ~~E~~ is calculated.

$\rho = \frac{Q}{V} = \frac{3Q}{4\pi R^3} \implies \vec{E} = \frac{\rho \vec{r'}}{3\epsilon_0}$

In our situation, $\vec{r'} = \vec{r} - \vec{b} \implies \boxed{\vec{E} = \frac{\rho(\vec{r}-\vec{b})}{3\epsilon_0}}$

b)



insulating sphere of radius R with hole

$\vec{E}$ inside hole?

Charge distribution can be represented by uniform sphere of charge density ρ and uniform sphered centered at $\vec{r}=\vec{b}$ with charge density $-\rho$.
(So hole with "0" charge)

$\vec{E}_{uniform} = \frac{\rho\vec{r}}{3\epsilon_0} \qquad \vec{E}_{hole} = \frac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}$

$\implies \vec{E}_{total} = \vec{E}_{uniform} + \vec{E}_{hole} = \frac{\rho\vec{r}}{3\epsilon_0} + \left(\frac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}\right) = \frac{\rho\vec{b}}{3\epsilon_0}$

$\vec{E}$ is independent of ~~r~~ $\vec{r}$, so $\vec{E}$ is uniform inside hole.