

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

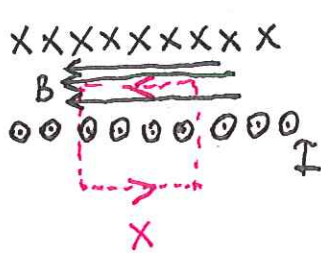
Signature:

A long, straight solenoid has  $N$  turns, uniform cross-sectional area  $A$ , and length  $l$ . Show that the inductance of this solenoid is given by  $L = \mu_0 AN^2 / l$ . Assume that the magnetic field is uniform inside the solenoid and zero outside.

Inductance is defined as:

$$L = \frac{N\Phi_B}{I}$$

And flux is defined as:  $\Phi_B = BA$ . Using Ampere Law we can calculate the magnetic field of the solenoid:



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow Bx = \mu_0 n x I$$

$$\Rightarrow B = \mu_0 \frac{N}{l} I$$

Using this magnetic field in the definition of the inductance we have:

$$L = \frac{N\Phi_B}{I} = \frac{NBA}{I} = \frac{NA}{I} \cdot \mu_0 \frac{N}{l} I \Rightarrow$$

$$\Rightarrow \boxed{L = \frac{\mu_0 AN^2}{l}}$$

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A long, straight solenoid has  $N$  turns, uniform cross-sectional area  $A$ , length  $l$ , and carries a current  $I_0$ . Find the total energy contained in the coil's magnetic field assuming the field is uniform.

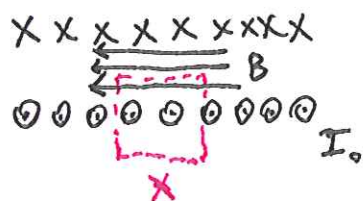
Energy stored in the coil is given as:

$$U = \frac{1}{2} L I_0^2$$

Inductance is defined as:  $L = \frac{N \Phi_B}{I_0}$ . Where flux is given with  $\Phi_B = BA$ . Using these:

$$U = \frac{1}{2} L I_0^2 = \frac{N \Phi_B}{2 I_0} \cdot I_0^2 = \frac{N B A I_0}{2}$$

Using Ampere's law we can calculate the magnetic field:



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc.} \Rightarrow Bx = \mu_0 \frac{N}{l} x I_0$$

$$\Rightarrow B = \frac{\mu_0 N I_0}{l}$$

If we substitute  $B$  to the expression of  $U$  we get:

$$U = \frac{N A B I_0}{2} = \boxed{\frac{\mu_0 N^2 I_0^2 A}{2l}}$$

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An LC circuit containing an inductor  $L_0$  and a capacitor  $C_0$  oscillates with a maximum current of  $I_0$ . Calculate the maximum charge on the capacitor.

Total charge in the capacitor can be written as:

$$q(t) = Q \cos(\omega t + \phi)$$

where  $Q$  is maximum charge, and  $\omega = \frac{1}{\sqrt{LC}}$  for LC circuit. Then current is given as:

$$i(t) = \frac{dq}{dt} = -Q\omega \sin(\omega t + \phi)$$

From this equation we see that maximum current is given as:

$$I_0 = Q\omega \Rightarrow Q = \frac{I_0}{\omega} \Rightarrow$$

$$\boxed{Q = I_0 \sqrt{LC}}$$

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An LC circuit containing an inductor  $L_0$  and a capacitor  $C_0$  oscillates with a maximum current of  $I_0$ . Assuming the capacitor has its maximum charge at time  $t = 0$ , calculate the energy stored in the inductor after  $t$  seconds.

For inductor energy stored is given as:

$$U = \frac{1}{2} L i^2$$

The current for any LC circuit in general can be written as:

$$i(t) = I_0 \cos(\omega t + \phi) ; \quad \omega = \frac{1}{\sqrt{L_0 C_0}}$$

Since  $i(0) = I_0$  we have:

$$\cos(\phi) = 1 \Rightarrow \phi = 0$$

Thus:

$$i(t) = I_0 \cos(\omega t)$$

Therefore:

$$U = \frac{1}{2} L_0 (i(t))^2 = \boxed{\frac{1}{2} L_0 I_0^2 \cos^2\left(\frac{t}{\sqrt{L_0 C_0}}\right)}$$



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An ideal battery with voltage  $V_0$ , a resistor with resistance  $R_0$ , and an ideal inductor with inductance  $L_0$  are all connected in series with an open switch. The switch is suddenly closed. How long after closing the switch will the current through the inductor reach one-half of its maximum value?

Current of R-L circuit with emf is given as:

$$i = \frac{V_0}{R_0} (1 - e^{-(R_0/L_0)t})$$

At  $t=0$ ,  $i = \frac{V_0}{R_0}$ . Assume at time  $t_1$ , the current drop one-half of its maximum value:

$$i_1 = \frac{V_0}{2R_0} = \frac{V_0}{R_0} (1 - e^{-(R_0/L_0)t_1}) \Rightarrow$$

$$\Rightarrow \frac{1}{2} = (1 - e^{-(R_0/L_0)t_1}) \Rightarrow 1 = 2 - 2e^{-(R_0/L_0)t_1}$$

$$\Rightarrow e^{-(R_0/L_0)t_1} = \frac{1}{2} \Rightarrow -\frac{R_0}{L_0} t_1 = \ln(1/2) \Rightarrow$$

$$\Rightarrow t_1 = -\frac{L_0}{R_0} \ln(1/2) \Rightarrow$$

$$t_1 = \frac{L_0}{R_0} \ln(2)$$