Hassan Babaei

PHYS 102: General Physics 2 **KOÇ UNIVERSITY** **Spring Semester 2015**

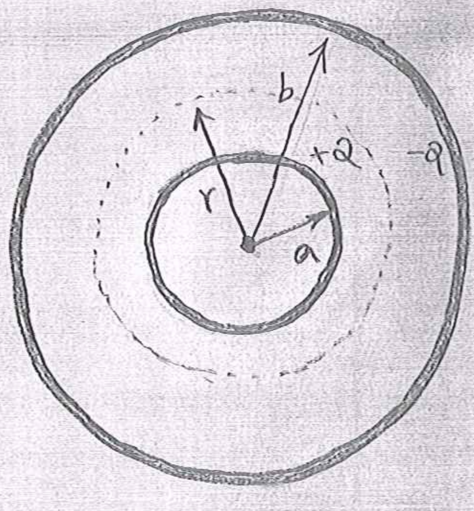
College of Arts and Sciences

Section 2 **Quiz 4** **19 March 2015**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Find the capacitance of a capacitor consisting of two concentric conducting shells of inner radius a and outer radius b.



From Gauss law $\oint_S \vec{E} \cdot \vec{da} = \frac{Q_{net}}{\varepsilon_0}$

Since Gauss surface is spherically symmetric, electric field has a definite value on the radius r from center. And the direction is radially outward. $\vec{E} = |E| \hat{r}$

$$|E| \hat{r} \cdot (4\pi r^2) \hat{r} = \frac{Q}{\varepsilon_0} \Rightarrow |E| = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r}} \rightarrow \text{radially outward.}$$

$$V_{ab} = V(a) - V(b) = -\int_b^a \vec{E} \cdot \vec{dr} = -\frac{Q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0} \left(-\frac{1}{r}\right)_b^a$$

$$\rightarrow V_{ab} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right) \quad \left(\text{since } a < b \rightarrow \frac{1}{a} > \frac{1}{b} \quad V_{ab} > 0\right)$$

$$C = \frac{Q}{V_{ab}} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

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Section 3 **Quiz 4** **19 March 2015**

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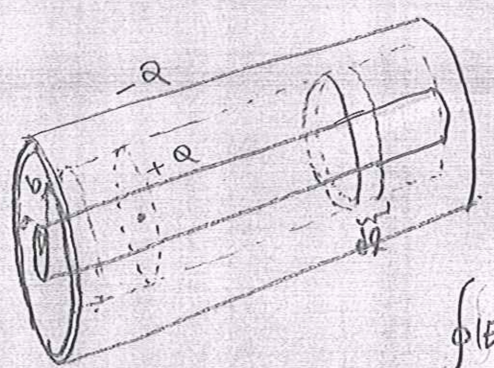
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Find the capacitance of a capacitor consisting of two concentric conducting cylinders of inner radius a, outer radius b, and length L.



From Gauss law $\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{net}}{\varepsilon_0}$

Since Gauss surface is cylindrically symmetric, electric field has a definite value on the surface. And its direction is radially outward.

$$\vec{E} = |E|\,\hat{r}$$

$$\oint_S |E|\,\hat{r}\cdot(2\pi r)\,d\ell\,\hat{r} = \frac{Q_{net}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$|E|(2\pi r \ell) = \frac{\lambda L}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}}$$

$$V_{ab} = V(a) - V(b) = -\int_b^a \vec{E}\cdot d\vec{r} = -\frac{\lambda}{2\pi\varepsilon_0}\int_b^a \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\,\mathrm{Ln}\,a/b$$

$$\Longrightarrow \boxed{V_{ab} = V(a) - V(b) = \frac{\lambda}{2\pi\varepsilon_0}\,\mathrm{Ln}\,\frac{b}{a}}$$

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0}\,\mathrm{Ln}\,\frac{b}{a}} = \frac{2\pi\varepsilon_0 L}{\mathrm{Ln}(b/a)}$$

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Section 1 Quiz 4 19 March 2015

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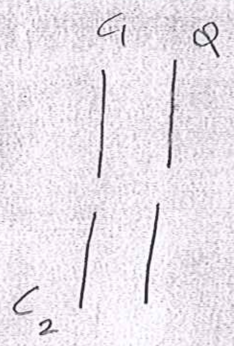
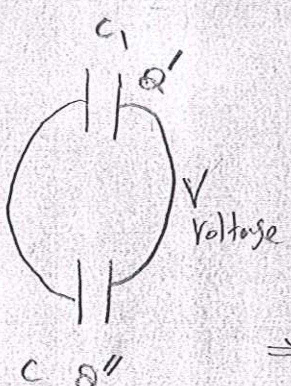
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A capacitor of capacitance C_1 of charge Q is connected to an uncharged capacitor of capacitance C_2 in parallel. Calculate the energy lost when the connection made.

a) 

When we connect them, total charge stays the same. therefore some of charge of capacitor C_1 goes to C_2. However, total charge is Q.

b) 

$Q' + Q'' = Q$, Next, we see that V is the same. Also total C increased from C_1 to $C_1 + C_2$.

$\Rightarrow$ energy lost: $\Delta U = U_f - U_i = \frac{1}{2}\frac{Q^2}{C_1} - \frac{1}{2}\frac{Q^2}{C_1+C_2}$

$\rightarrow \Delta U = \frac{Q^2}{2}\left(\frac{1}{C_1} - \frac{1}{C_1+C_2}\right) = \frac{Q^2}{2} \times \frac{C_2}{C_1(C_2+C_1)}$

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Section 4 **Quiz 4** **19 March 2015**

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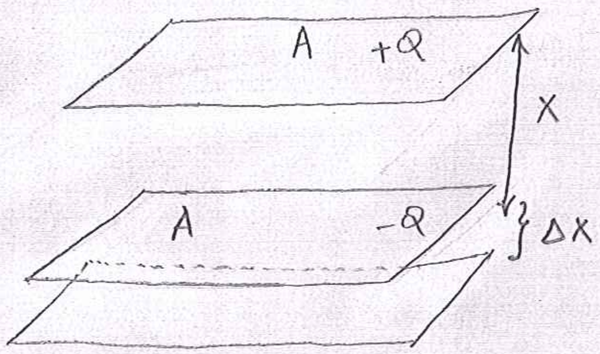
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A parallel plate capacitor of area A has charge Q and separation x. (i) Find the force F between the plates. (ii) Find the work ΔW done by F to separate the plates by an additional Δx. (iii) Compare ΔW by the change in the energy stored in the capacitor ΔU.



i) $C_0 = \frac{\varepsilon_0 A}{X}$, $Q = C_0 V$, $E = \frac{V}{X}$

$F = \frac{1}{2} Q E = \frac{1}{2} \frac{C_0 V^2}{X}$ ← the force acting on one plate by another

ii) Work done to separate plates by an additional ΔX $\Delta W = F \Delta X = \frac{1}{2} C_0 V^2 \frac{\Delta X}{X}$

iii) $C_0 = \frac{\varepsilon_0 A}{X}$ is the capacitance before moving plates. After moving for an additional ΔX $C = \frac{\varepsilon_0 A}{X + \Delta X} = \frac{\varepsilon_0 A}{X(1 + \frac{\Delta X}{X})} \xrightarrow[\frac{\Delta X}{X} \ll 1]{\text{Since } \Delta X \ll X} \underbrace{\frac{\varepsilon_0 A}{X}}_{C_0}\left(1 - \frac{\Delta X}{X}\right)$

$\Rightarrow C = C_0\left(1 - \frac{\Delta X}{X}\right) \Rightarrow \Delta C = -C_0 \frac{\Delta X}{X}$

Since voltage is constant while moving plates apart,

$U = \frac{1}{2} C V^2 \xrightarrow[V = \text{constant}]{\text{implies}} \Delta U = \frac{1}{2} \Delta C V^2 = -\frac{1}{2} C_0 V^2 \frac{\Delta X}{X}$

(energy stored in capacitor)

$\boxed{\Delta U = -\Delta W}$

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Section 5 **Quiz 4** **19 March 2015**

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Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A parallel plate capacitor of area A and plate separation L is filled with a removable dielectric slab with dielectric constant K. The capacitor is always connected to a battery keeping it at voltage V. Find (i) the charge on the plates with and without the slab, (ii) the electric field between the plates with and without the slab, and (iii) the induced charge Q_i on the dielectric.

Since V is constant.

i) - Without slab $C_0 = \frac{\varepsilon_0 A}{L} \Rightarrow Q_0 = C_0 V = \frac{\varepsilon_0 A V}{L}$

- With slab $C = \frac{k\varepsilon_0 A}{L} \Rightarrow Q = CV = \frac{k\varepsilon_0 A V}{L} = kQ_0$

ii) Since V is constant, $E = \frac{V}{d}$ is Constant. Therefor in both cases electric field is $E = \frac{V}{d}$.

iii) Note that, first by adding slab inside plates, some additional charge place in both plates which takes Q_0 to Q in a fixed Voltage. Henceforth,

$$Q_{additional} = Q - Q_0 = \frac{\varepsilon_0 A V}{L}(k-1) = Q_0(k-1).$$

Now, as we see the actual charge placed on plates is Q, where σ is surface density responsible for Q. And σ_i is surface density for induced charges in dielectric. From book, we know, $\sigma_i = \sigma(1 - \frac{1}{k})$

$$\Rightarrow Q_{induced} = Q_i = Q(1 - \tfrac{1}{k}) = kQ_0(1 - \tfrac{1}{k}) = Q_0(k-1)$$

Therefore, by adding slab electric field doesn't change, because the induced charge cancels the additional charge.