

Section 1

Quiz 2

05 March 2015

Closed book. No calculators are to be used for this quiz.

Quiz duration: 15 minutes

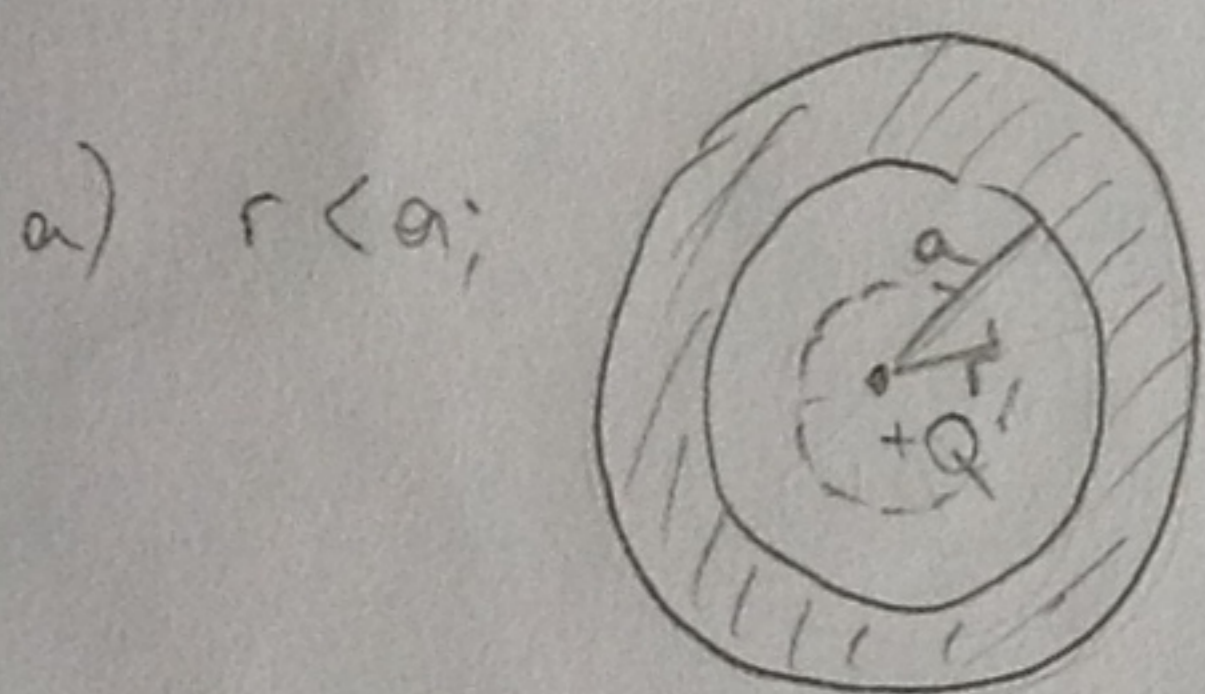
Name:

Student ID:

Signature:

A conducting spherical shell with inner radius a and outer radius b contains a total charge $2Q$. A positive point charge Q is located at the center of the spherical shell.

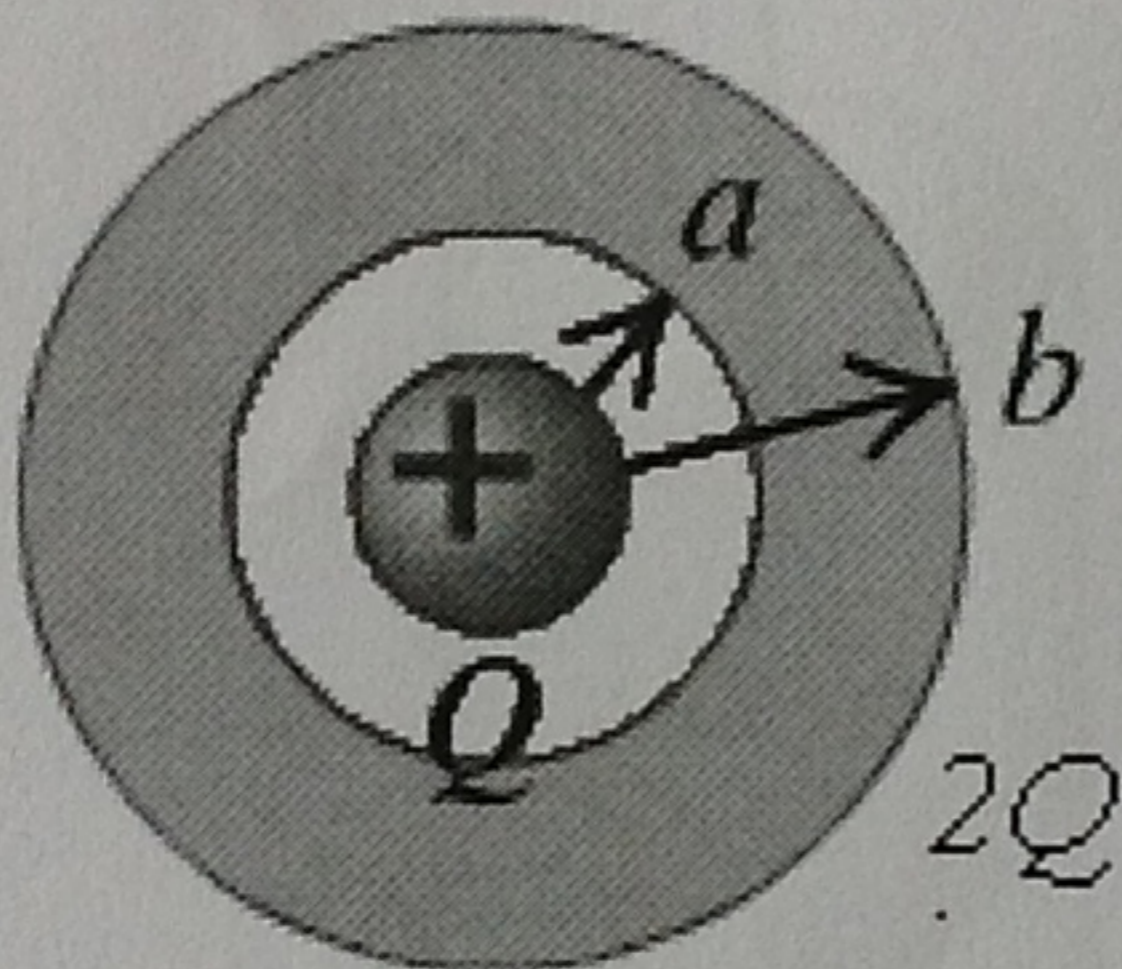
- (a) Derive the expression for the electric field magnitude as a function of the distance r from the center for the regions $r < a$, $a < r < b$, and $r > b$.
- (b) Graph the electric field magnitude as a function of r .
- (c) What is the charge on the inner surface and on the outer surface of the conducting spherical shell?



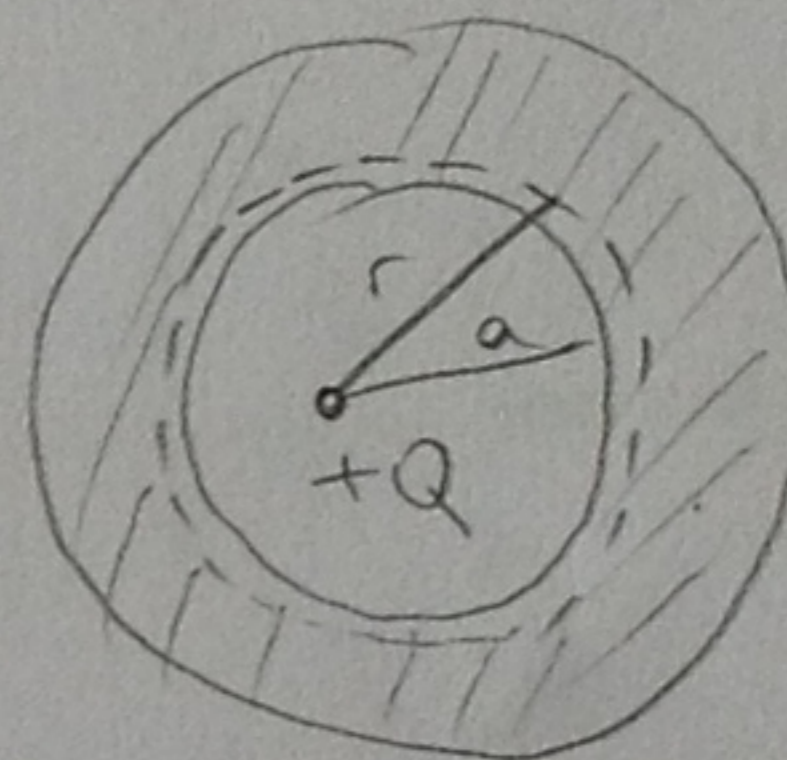
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

(radially outward)



* to find the charge
c) on the inner surface;



we select a gaussian surface slightly larger than inner surface.

We know that $E=0$ inside conductor, Gauss' law says that $Q_{enc} = 0$

Hence inner surface charge is $-Q$.

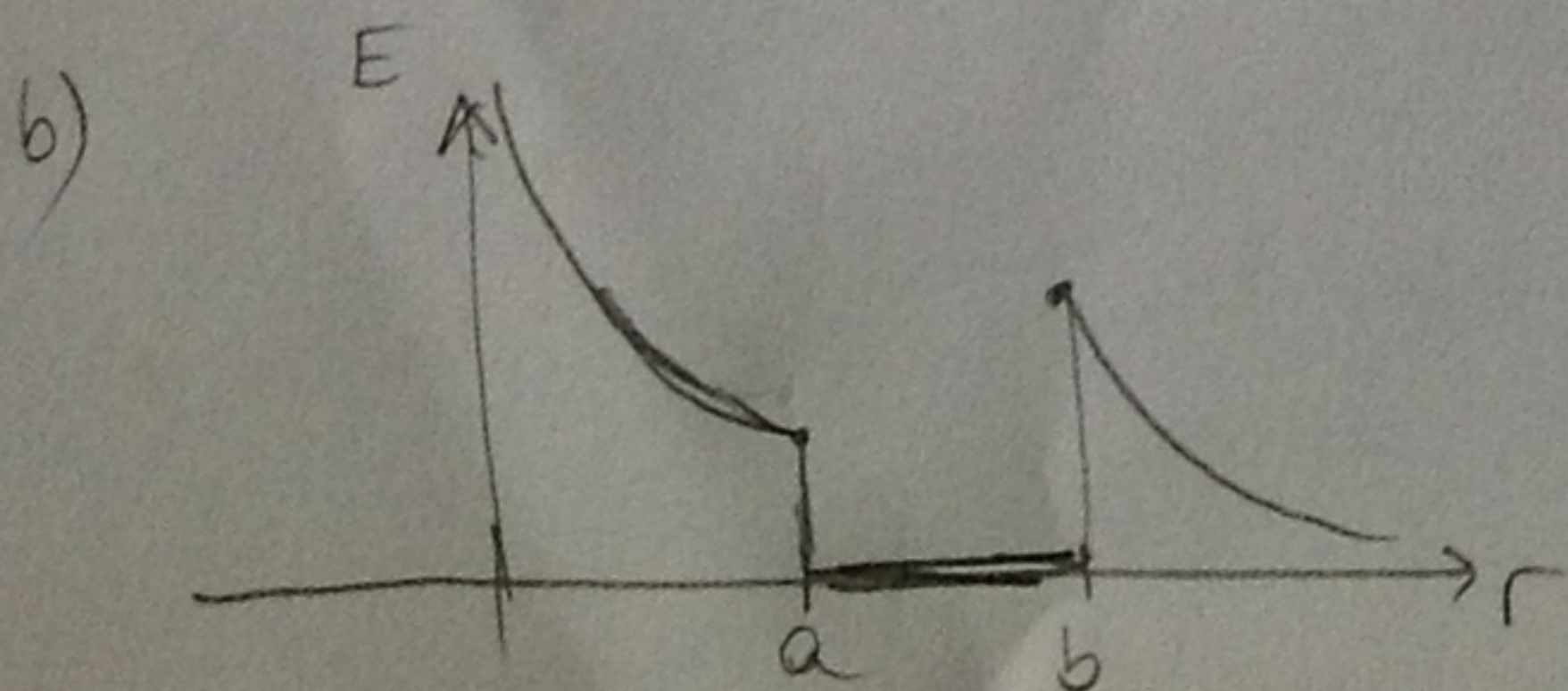
* The total charge on the shell is given $2Q$. Since inner surface charge $-Q$, outer surface charge becomes $3Q$.

$a < r < b$;
since the selected gaussian surface inside the conductor, $E=0$.

$r > b$; total enclosed charge $Q+2Q=3Q$

$$E 4\pi r^2 = \frac{3Q}{\epsilon_0} \rightarrow E = \frac{3Q}{4\pi\epsilon_0 r^2}$$

(radially outward)



Section 2

Quiz 2

05 March 2015

Closed book. No calculators are to be used for this quiz.

Quiz duration: 15 minutes

Name:

Student ID:

Signature:

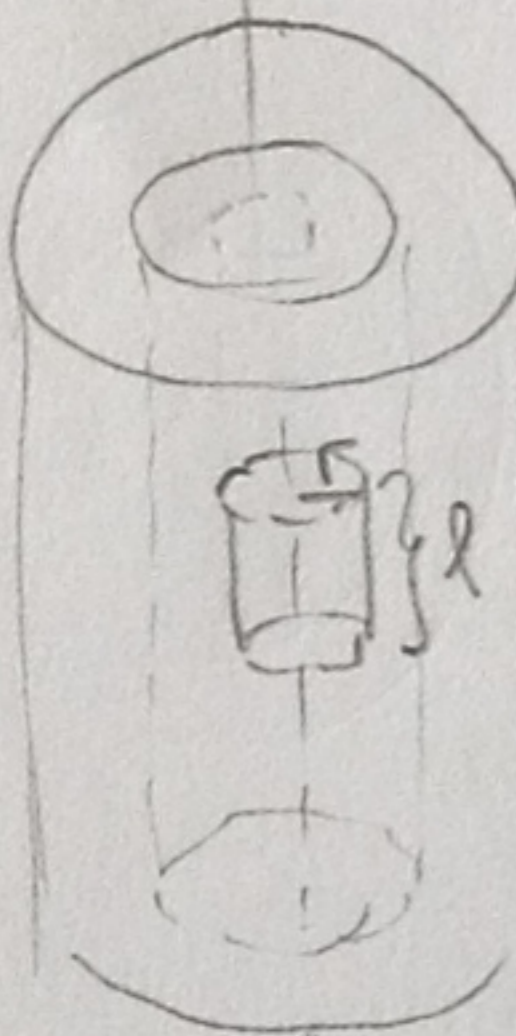
A very long conducting tube (hollow cylinder) has inner radius a and outer radius b . It carries charge per unit length $+\alpha$, where α is the positive constant with units of C/m. A line of charge lies along the axis of the tube. The line of charge has charge per unit length $+\alpha$.

(a) Calculate the electric field in terms of α and the distance r from the axis of the tube for $r < a$, $a < r < b$ and $r > b$.

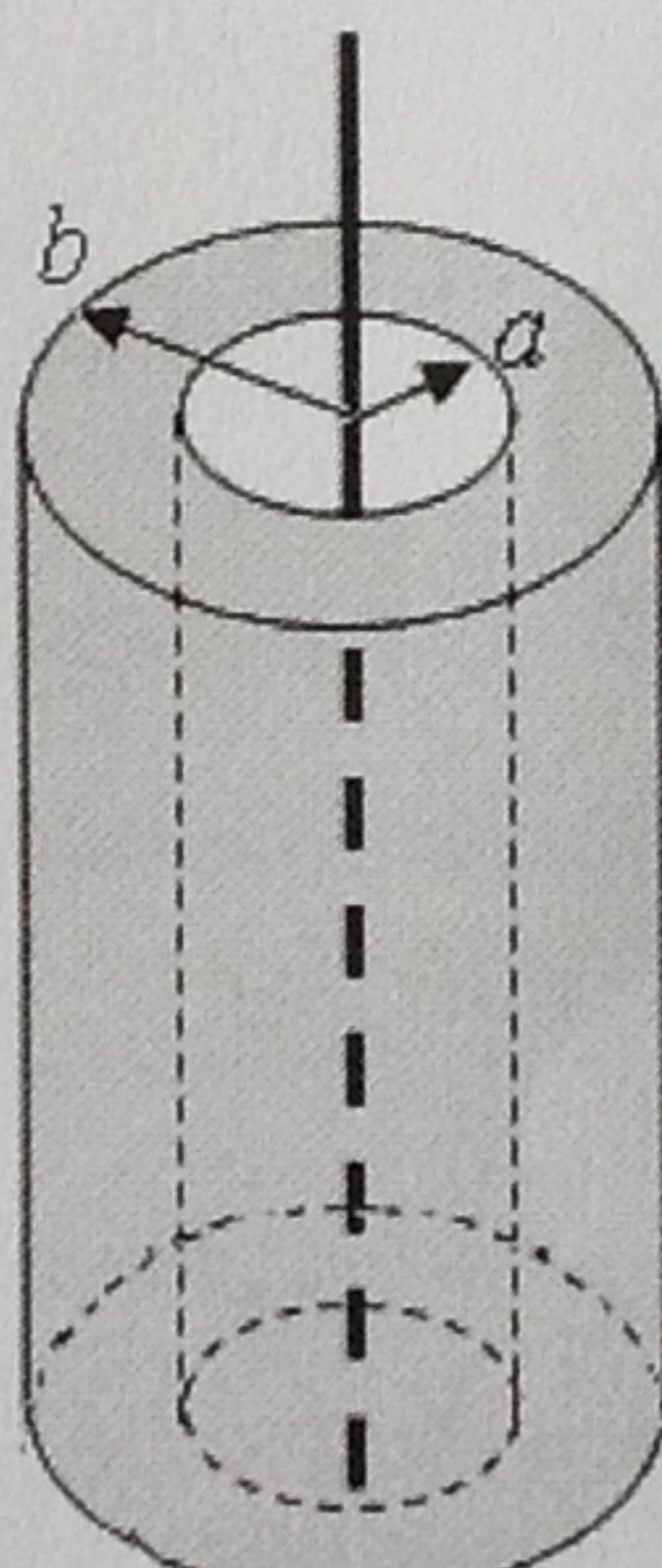
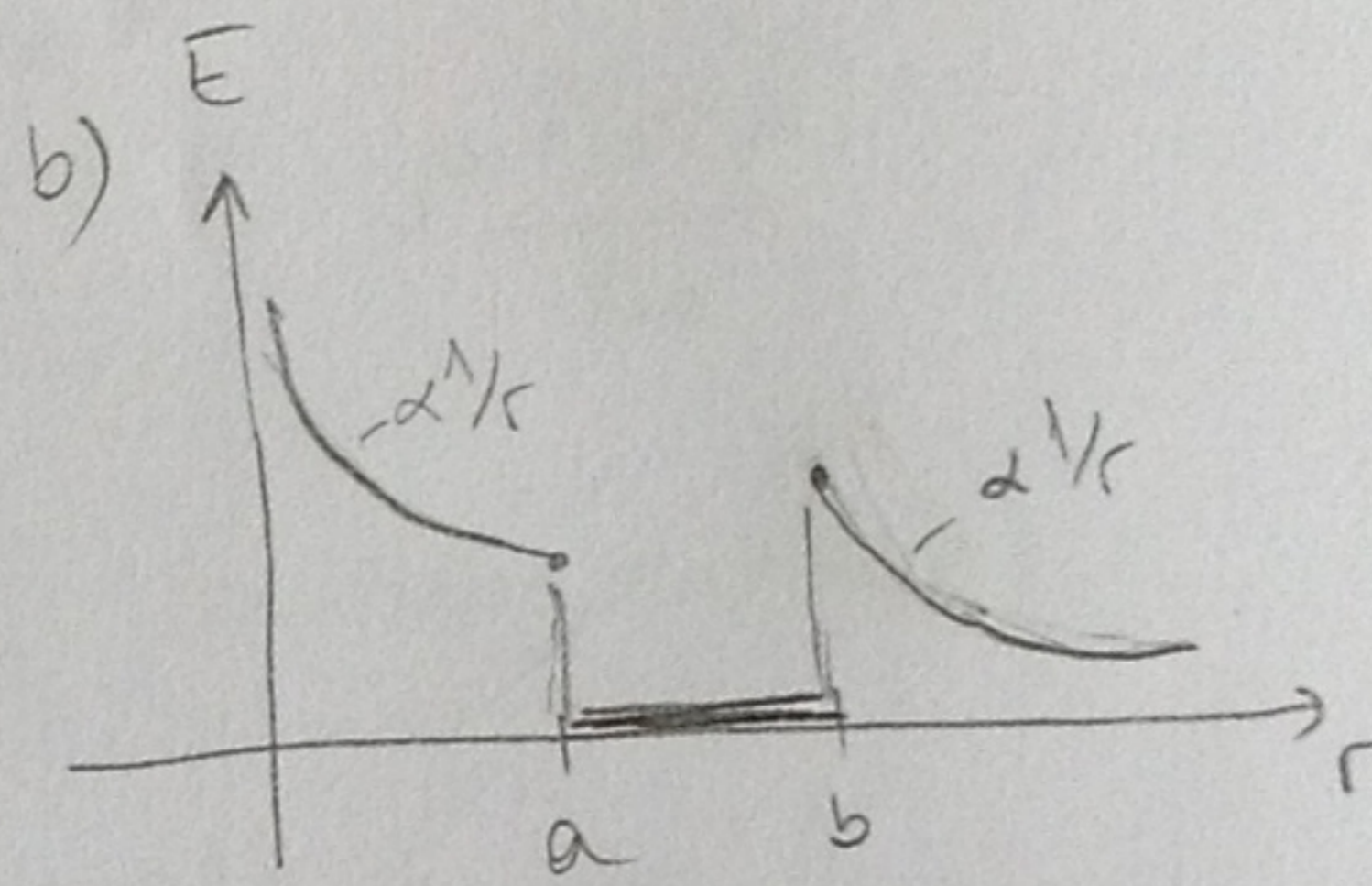
(b) Graph the electric field magnitude as a function of r in all the regions?

(c) What is the charge per unit length on (i) the inner surface of the tube and (ii) the outer surface of the tube? (Tube is the conducting hollow cylinder.)

a) $r < a$;



We select a gaussian cylinder of radius r , length l .

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl.}}{\epsilon_0}$

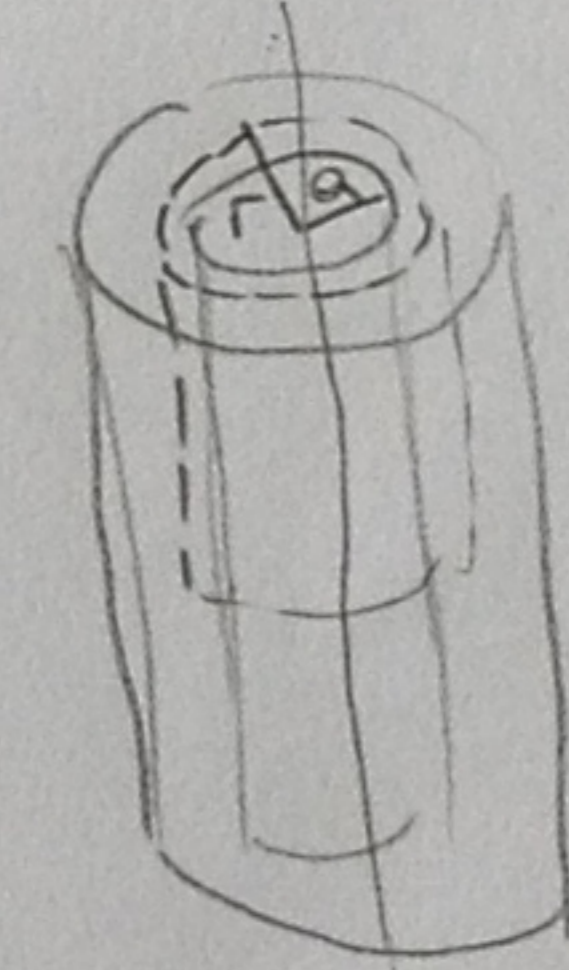
$E \cdot 2\pi r l = \frac{(\alpha) \cdot l}{\epsilon_0} \rightarrow E = \frac{\alpha}{2\pi \epsilon_0 r}$
(radially outward)

b) $a < r < b$;

since this region is inside the conductor,

$E = 0$.

c) $r > b$;



Select gaussian cylinder larger than inner cylinder of radius a .

Since it is inside the conductor, $E = 0$.

$\hookrightarrow Q_{encl} = 0$

\hookrightarrow line charge density on inner surface of the tube $-\alpha$.

The net line charge density of tube is given as $+\alpha$, since inner surface carries $-\alpha$, outer surface of the tube carries $+2\alpha$.

$E \cdot 2\pi r l = \frac{Q_{encl}}{\epsilon_0} = \frac{\alpha l + \alpha l}{\epsilon_0}$

$E = \frac{2\alpha}{2\pi \epsilon_0 r}$

Section 3

Quiz 2

05 March 2015

Closed book. No calculators are to be used for this quiz.

Quiz duration: 15 minutes

Name:

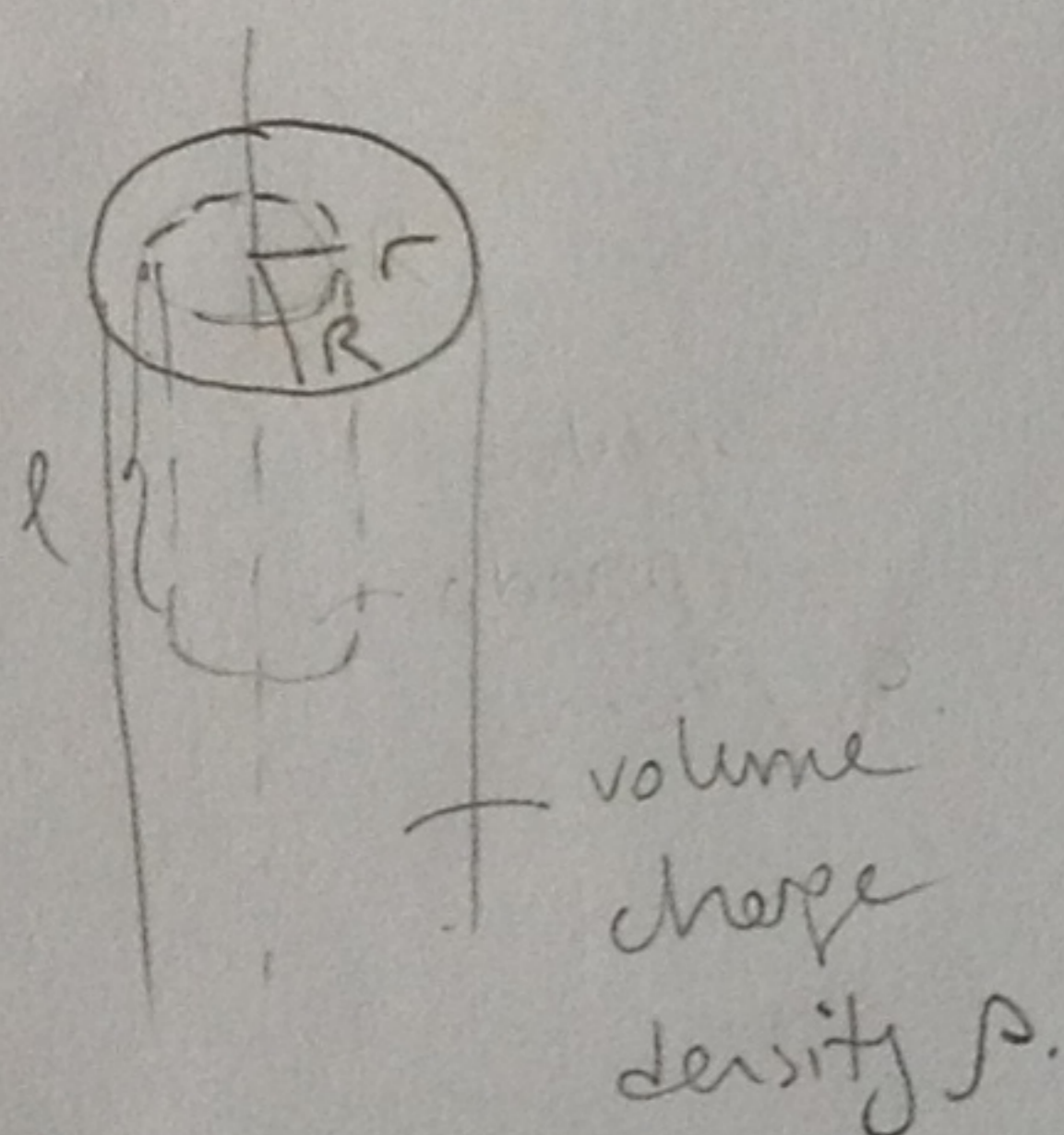
Student ID:

Signature:

A very long, solid cylinder with radius R has positive charge uniformly distributed throughout it, with charge per unit volume ρ .

(a) Calculate the electric field in terms of the charge density ρ and the distance r from the axis of the cylinder for $r < R$ and $r > R$.

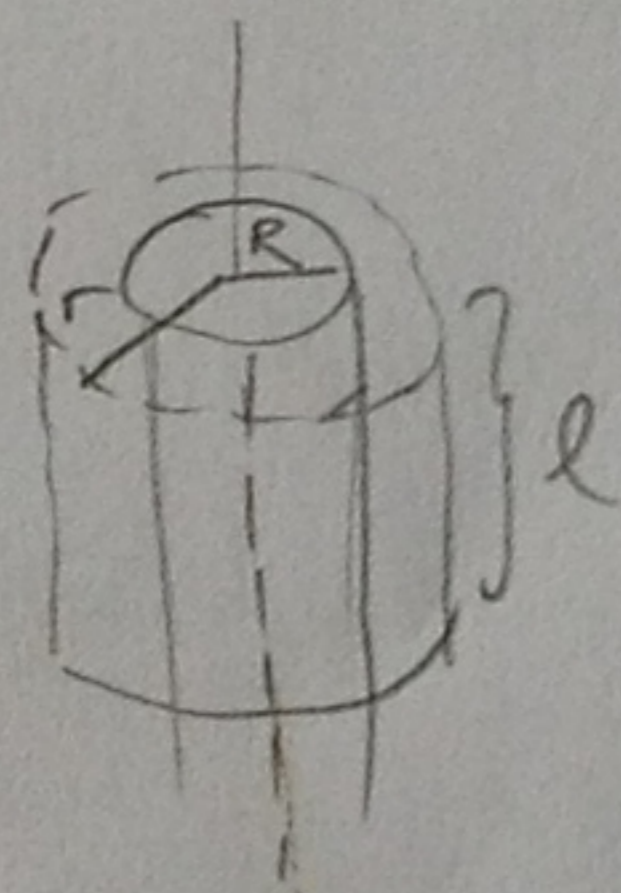
(b) Graph the electric-field magnitude as a function of r from $r = 0$ to $r = 3R$.



a) $r < R$; $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

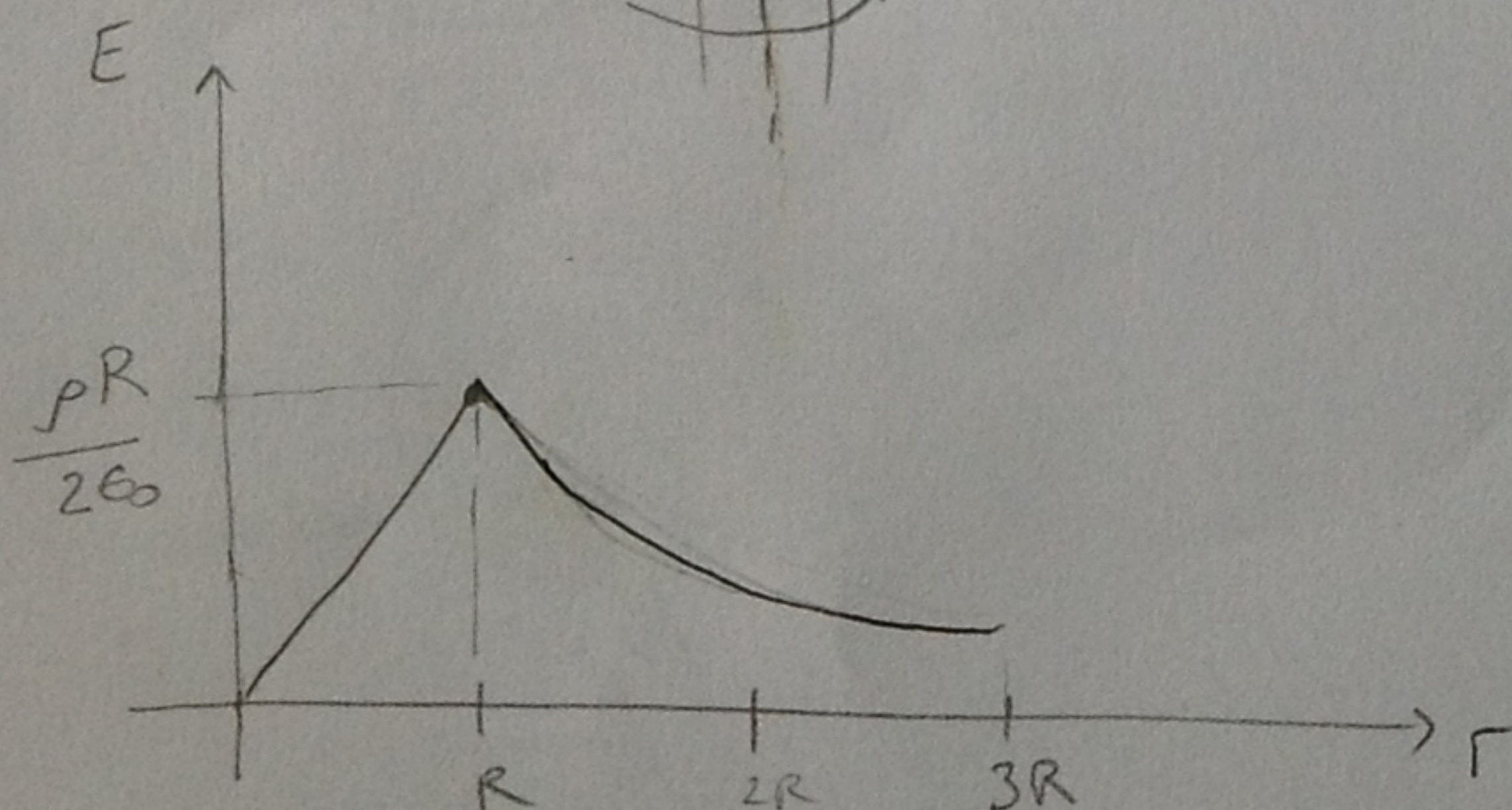
$$E \cdot 2\pi r l = \frac{\rho V}{\epsilon_0} = \frac{\rho \pi r^2 l}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0} \quad (\text{radially outward})$$



$r > R$; $E \cdot 2\pi r l = \frac{\rho \pi R^2 l}{\epsilon_0}$

$$E = \frac{\rho R^2}{2\epsilon_0 r} \quad (\text{radially outward})$$



Section 4

Quiz 2

05 March 2015

Closed book. No calculators are to be used for this quiz.

Quiz duration: 15 minutes

Name:

Student ID:

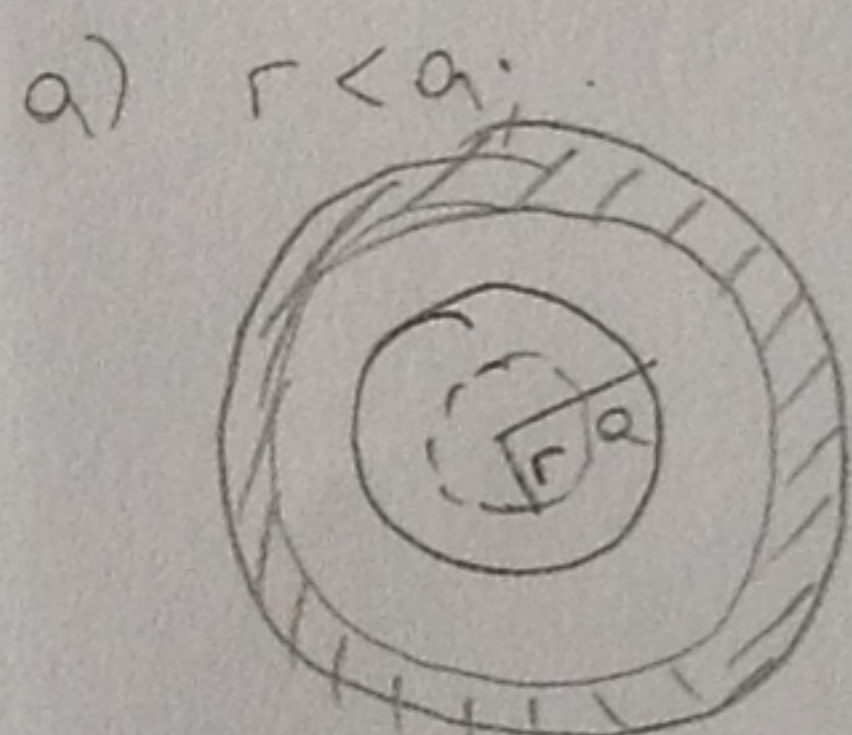
Signature:

A solid conducting sphere carrying charge q has radius a . It is inside a concentric hollow conducting sphere with inner radius b and outer radius c . The hollow sphere has no net charge.

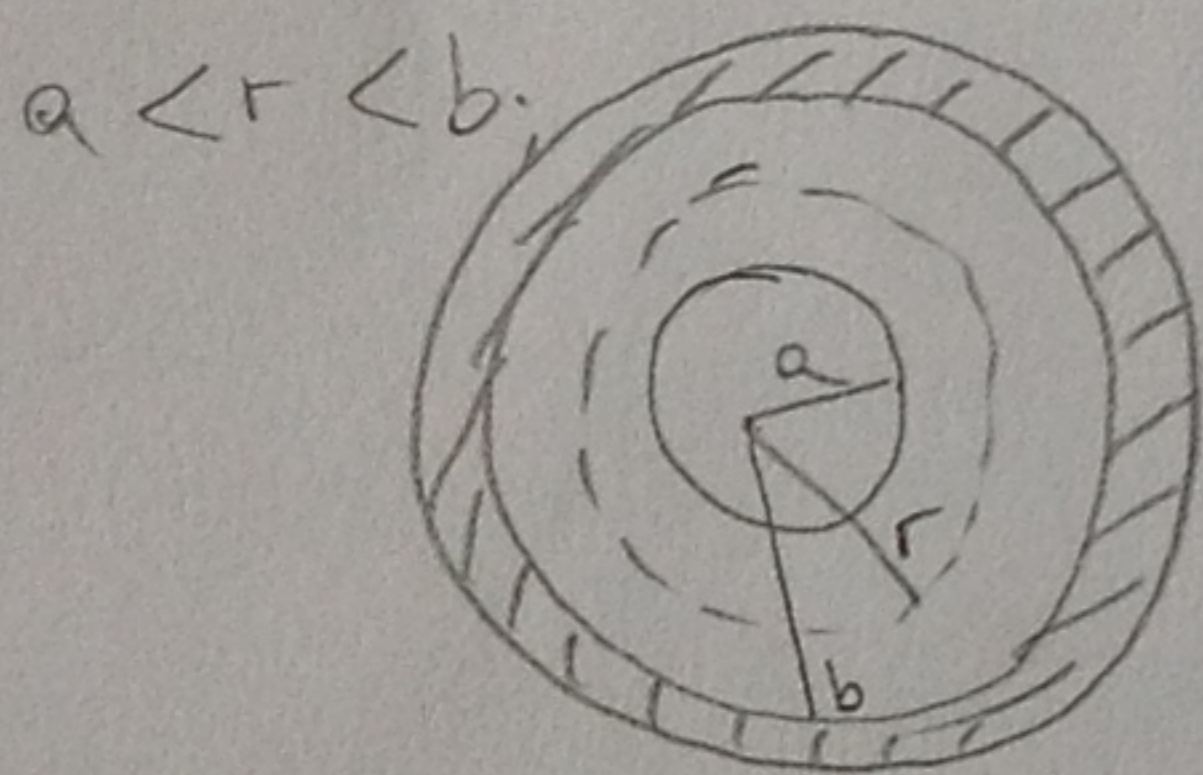
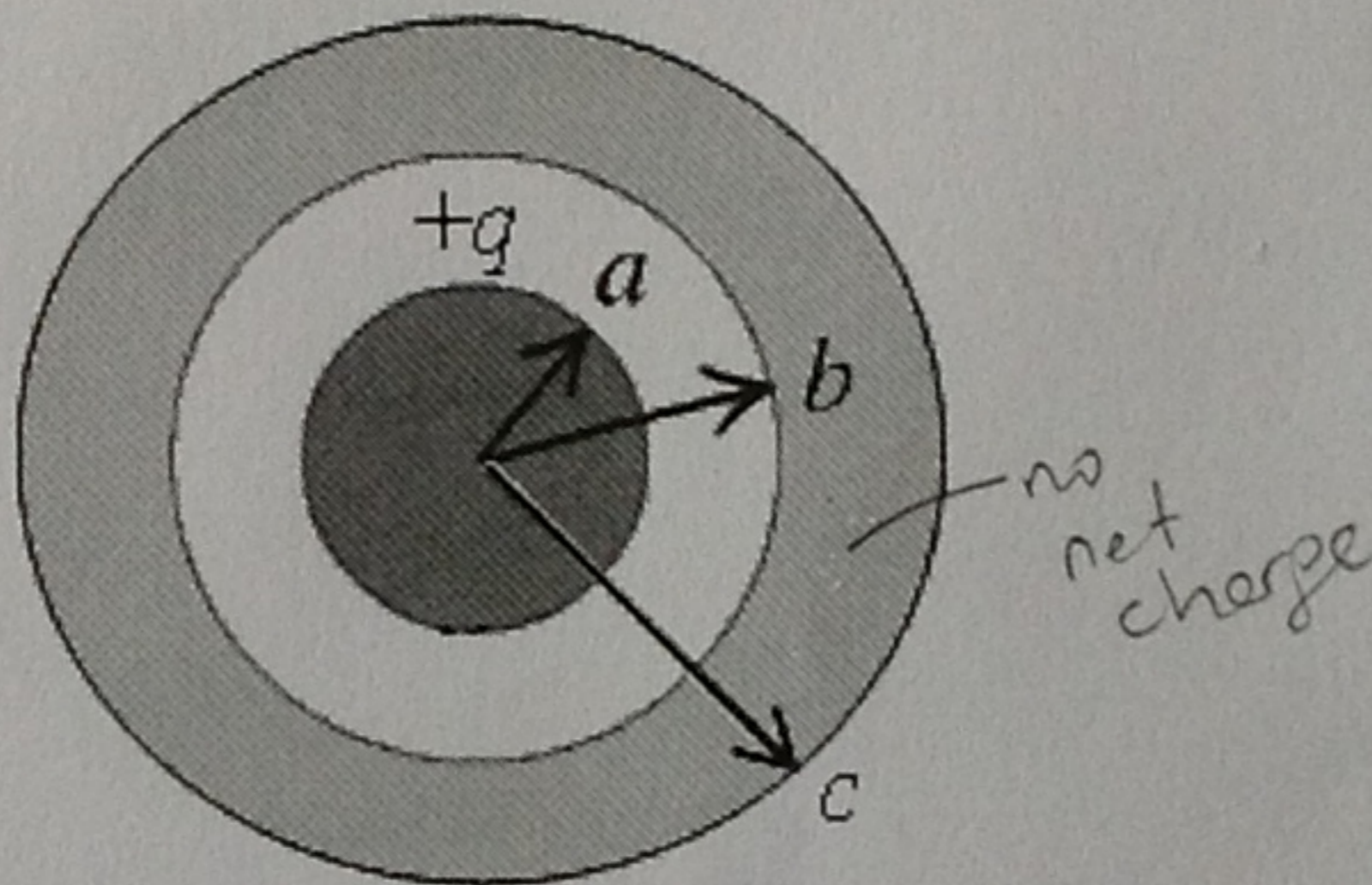
(a) Derive expressions for the electric field magnitude in terms of the distance r from the center for the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$.

(b) Graph the magnitude of the electric field as a function of r from $r = 0$ to $r = 2c$.

(c) What is the charge on the inner surface and on the outer surface of the hollow sphere?



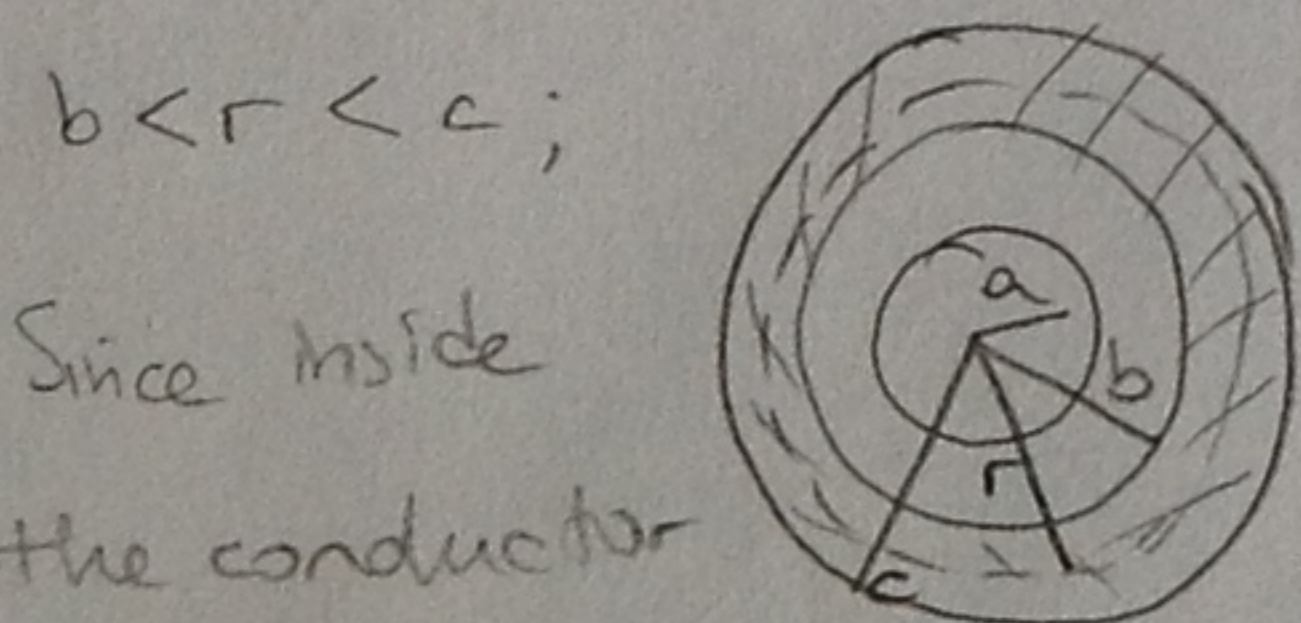
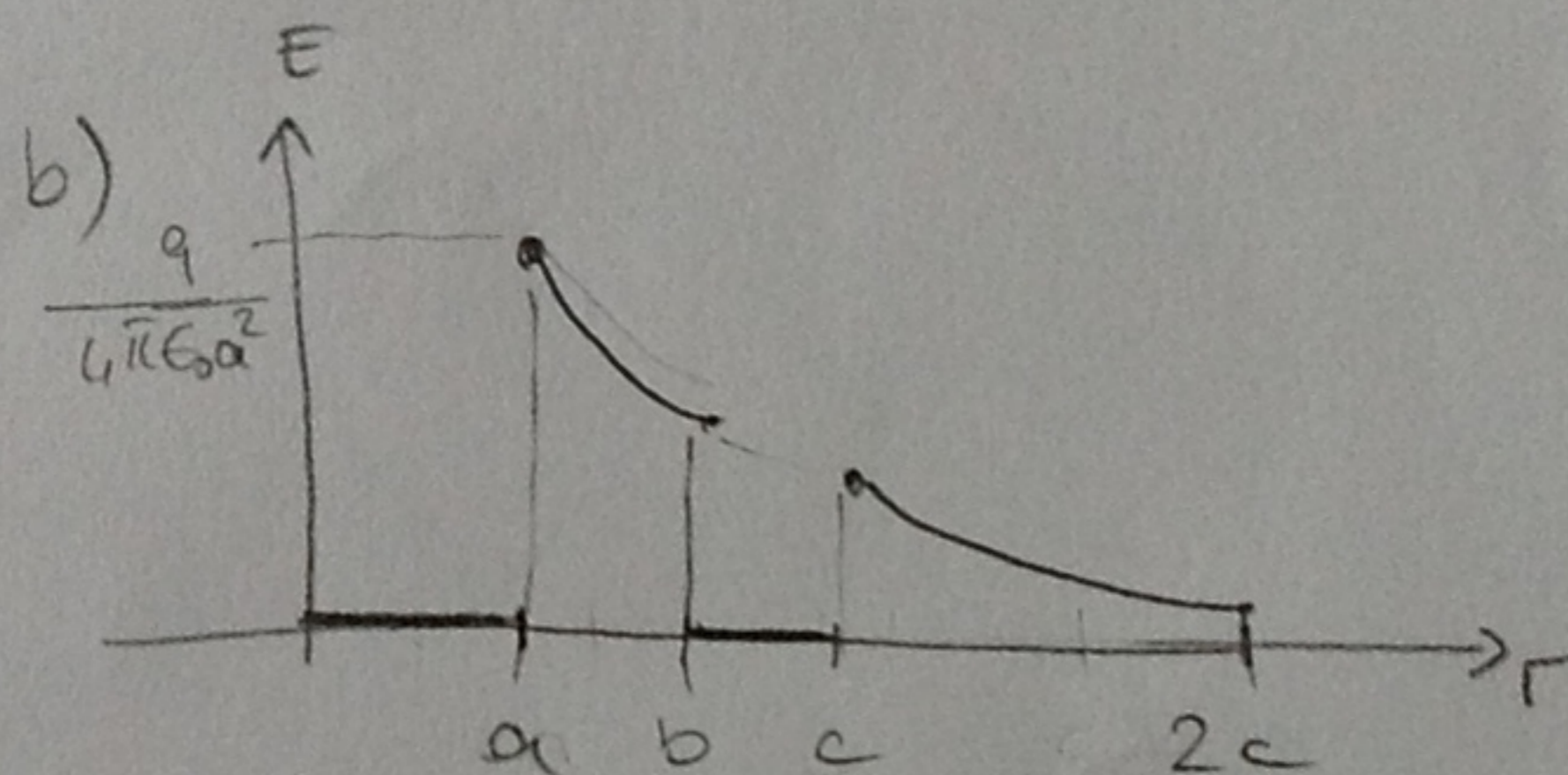
Since it is inside the conductor, $E = 0$.



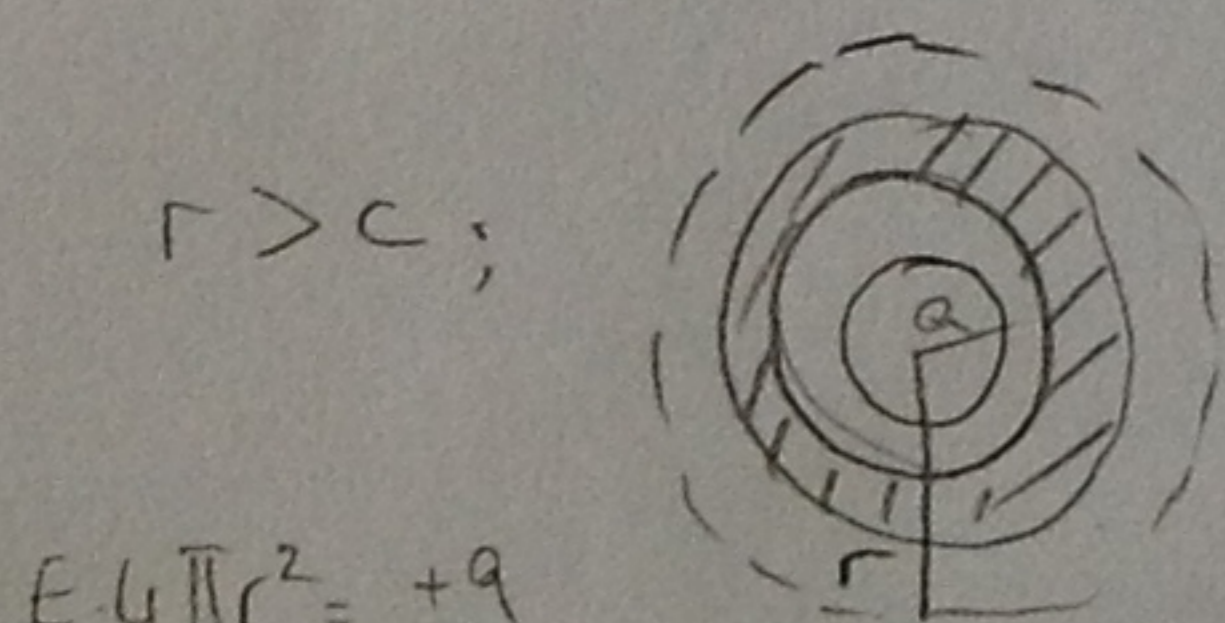
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{+q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$



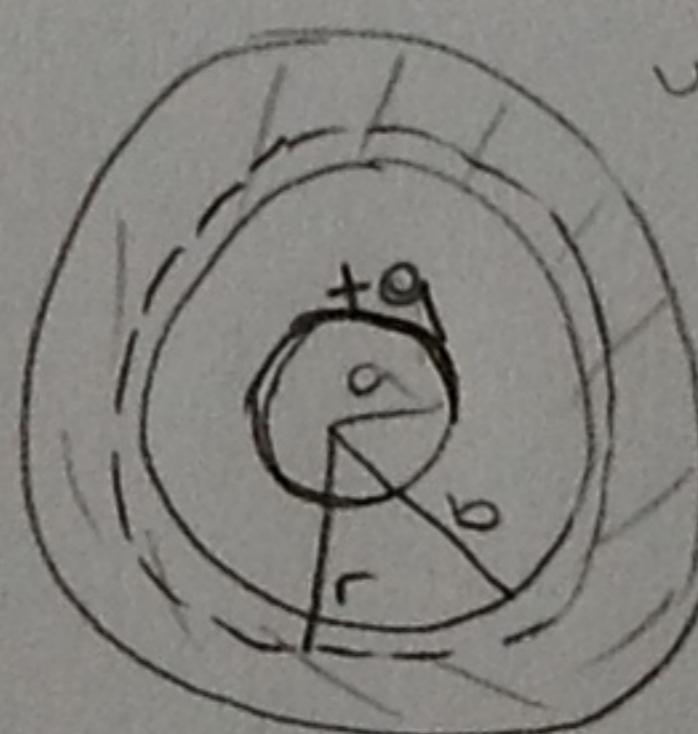
Since inside the conductor $E = 0$.



$$E 4\pi r^2 = \frac{+q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

c) charge on inner surface of hollow sphere;



we select a gaussian sphere of radius slightly larger than b . Since we are inside the conductor $E = 0$.
 $\rightarrow Q_{enc} = 0$.

Hence charge on inner surface is $-q$.

Since no net charge on hollow sphere, the outer surface must carry $+q$.

Section 5

Quiz 2

05 March 2015

Closed book. No calculators are to be used for this quiz.
 Quiz duration: 15 minutes

Name:

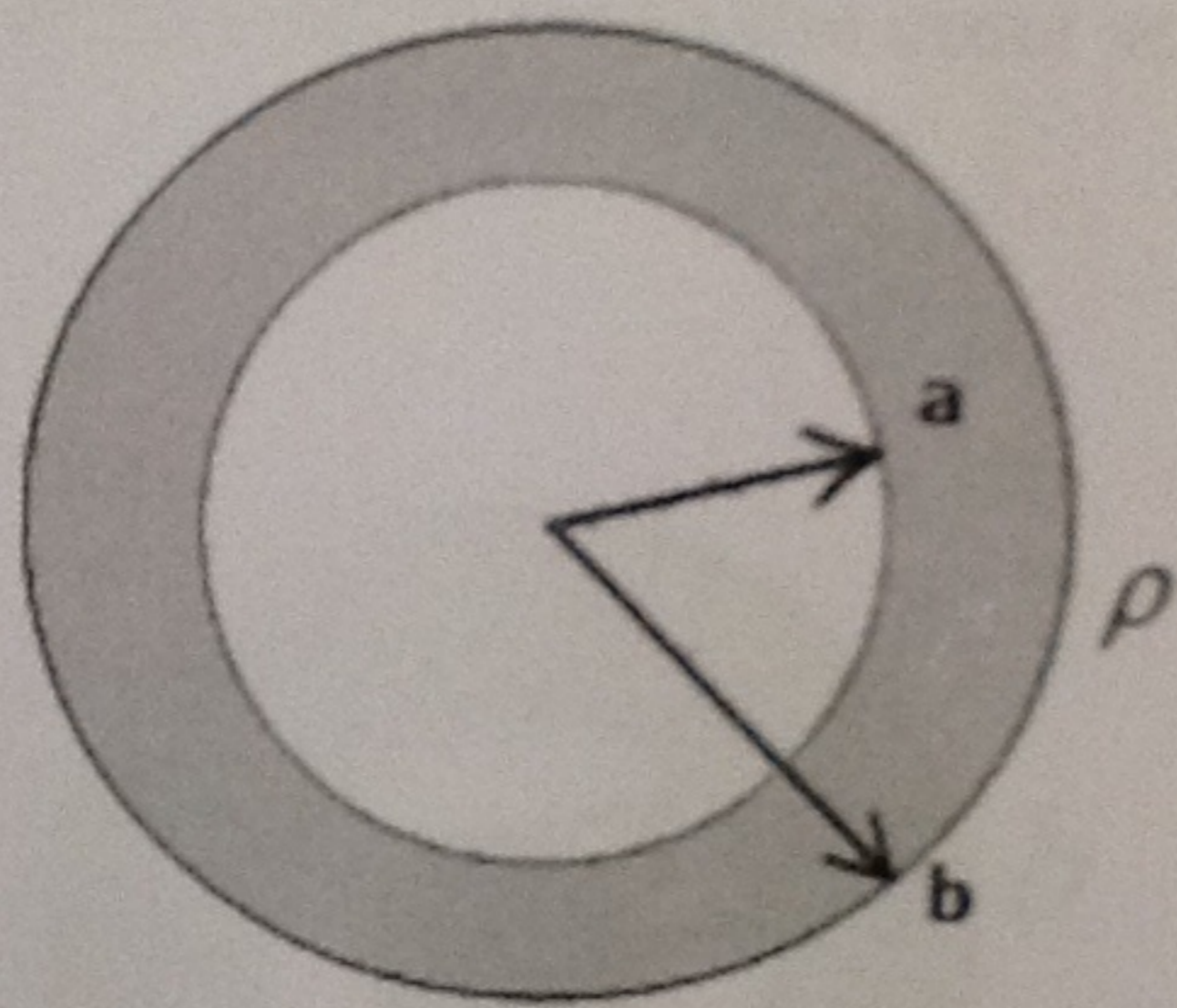
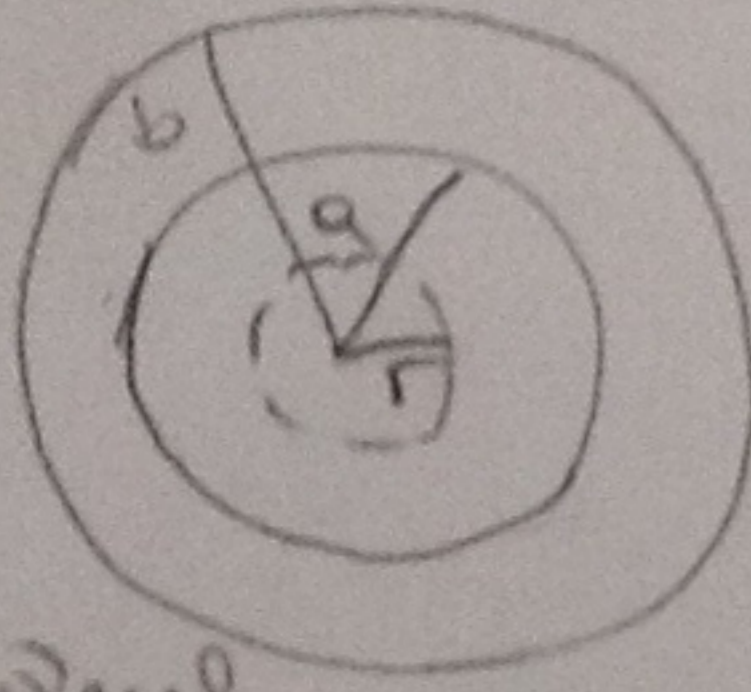
Student ID:

Signature:

An insulating spherical shell with inner radius a and outer radius b has positive charge uniformly distributed throughout, with charge per unit volume ρ .

- (a) Derive expressions for the electric field magnitude in terms of the distance r from the center for the regions $r < a$, $a < r < b$, and $r > b$.
- (b) Graph the magnitude of the electric field as a function of r .

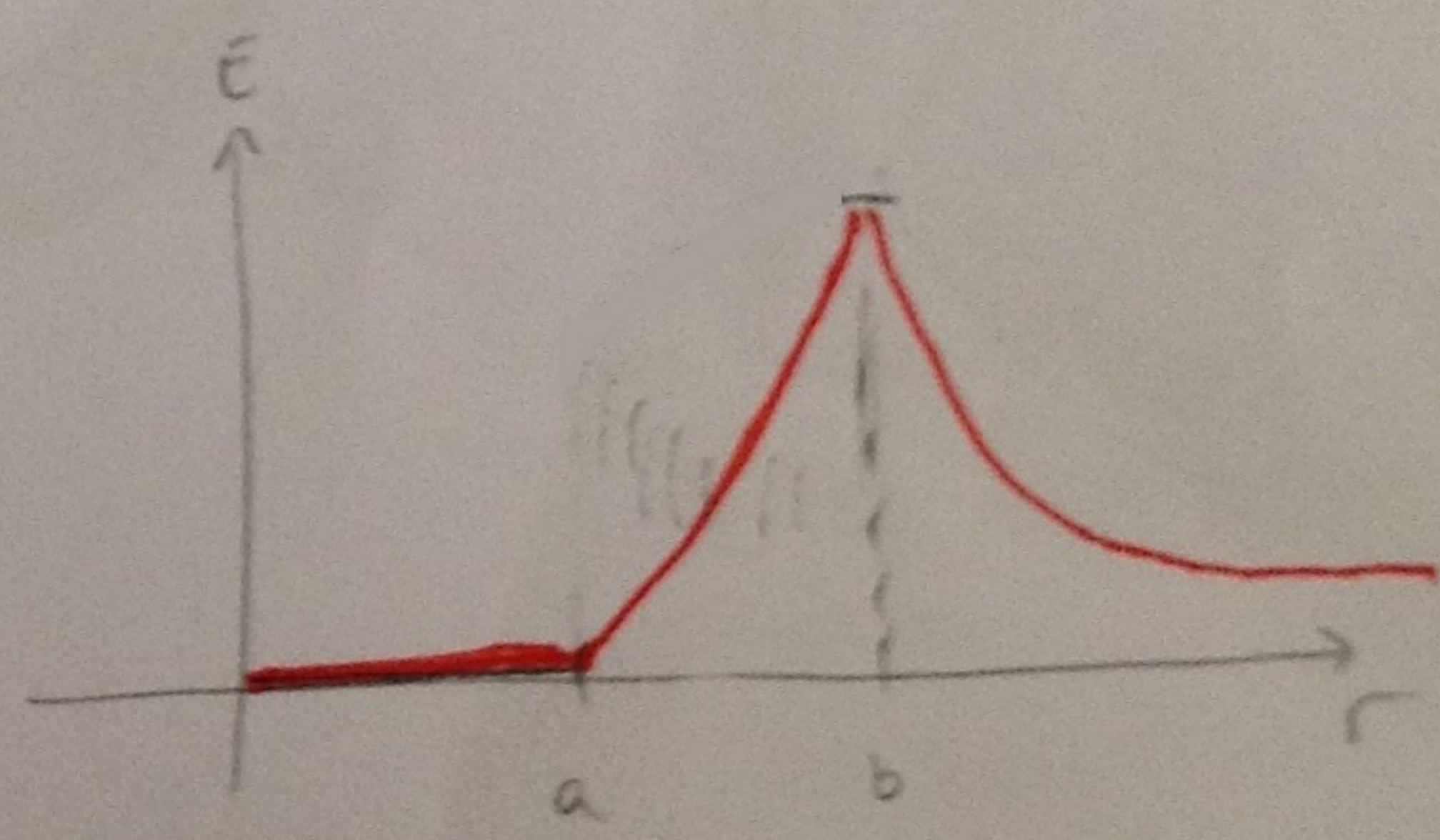
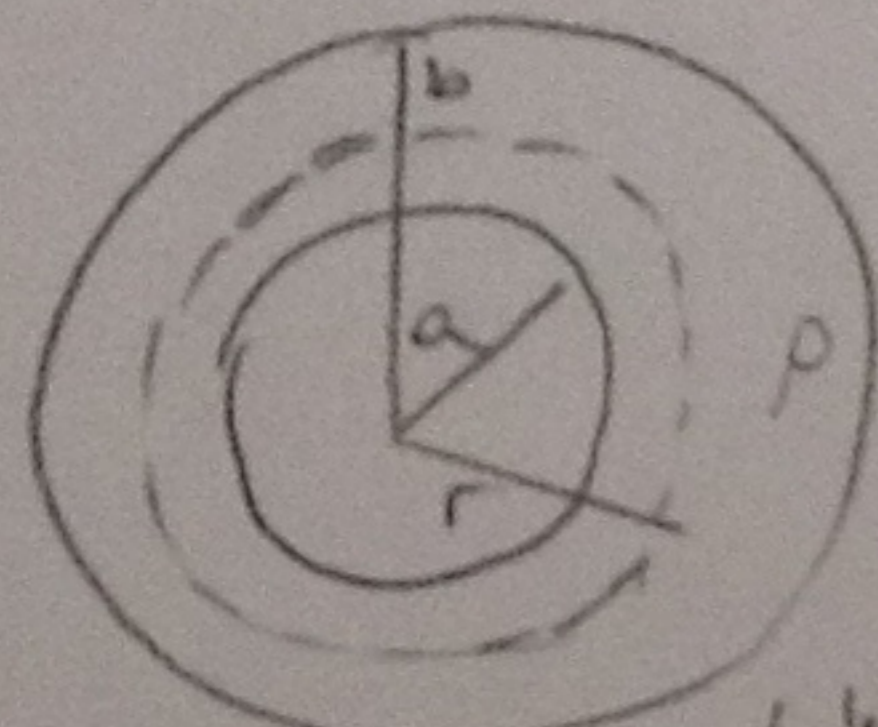
a) $r < a$,



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = 0 \rightarrow E = 0$$

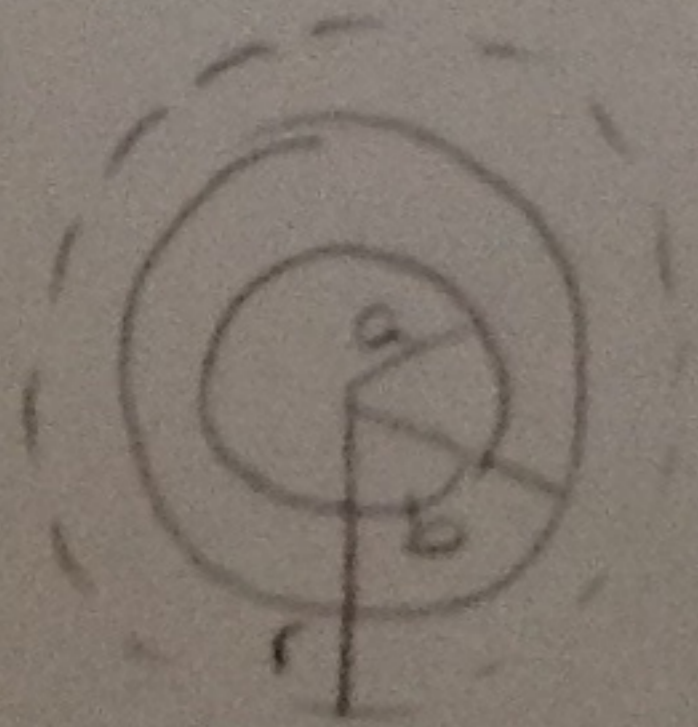
$a < r < b$;



$$E \cdot 4\pi r^2 = \frac{\rho V}{\epsilon_0} = \frac{\rho (\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3)}{\epsilon_0}$$

$$\vec{E} = \frac{\rho (r^3 - a^3)}{3\epsilon_0 r^2} \hat{r} = \frac{\rho}{3\epsilon_0} \left(r - \frac{a^3}{r^2} \right) \hat{r}$$

$r > b$;



$$E \cdot 4\pi r^2 = \frac{\rho (\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3)}{\epsilon_0}$$

$$\vec{E} = \frac{\rho (b^3 - a^3)}{3\epsilon_0 r^2} \hat{r} = \frac{\rho}{3\epsilon_0} (b^3 - a^3) \frac{1}{r^2} \hat{r}$$