

Section 1

Quiz 1

11 February 2016

Closed book. No calculators are to be used for this quiz.

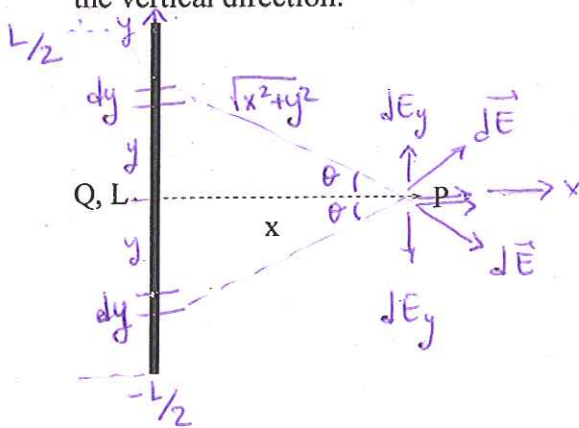
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Find the electric field at point P due to the straight wire, where the total charge Q is uniformly distributed over the length L . The point P is symmetrically located (i.e., at the midpoint) in the vertical direction.



y -components of $d\vec{E}$ cancel each other.

$$\lambda = \frac{Q}{L}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dy}{x^2 + y^2}$$

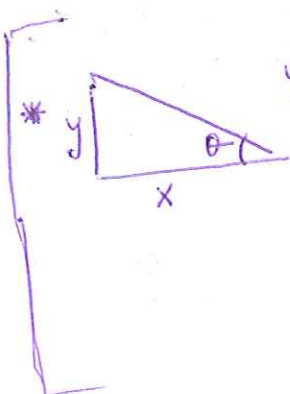
$$dE_x = dE \cos\theta \quad \cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda x dy}{(x^2 + y^2)^{3/2}}$$

$$E_x = \int dE_x = \frac{\lambda}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{x dy}{(x^2 + y^2)^{3/2}}$$

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{\sqrt{x^2 + y^2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{x} \frac{(L/2)}{\sqrt{x^2 + (L/2)^2}} \quad (\text{+x direction})$$



$$y = x \tan\theta$$

$$dy = x \sec^2\theta d\theta$$

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \int \frac{x \sec^2\theta d\theta}{(x^2)^{3/2} (1 + \tan^2\theta)^{3/2}} = \int \frac{x \sec^2\theta d\theta}{x^3 \sec^3\theta}$$

$$= \frac{1}{x^2} \int \cos\theta d\theta = \frac{\sin\theta}{x^2}$$

$$= \frac{y}{x^2 \sqrt{x^2 + y^2}}$$

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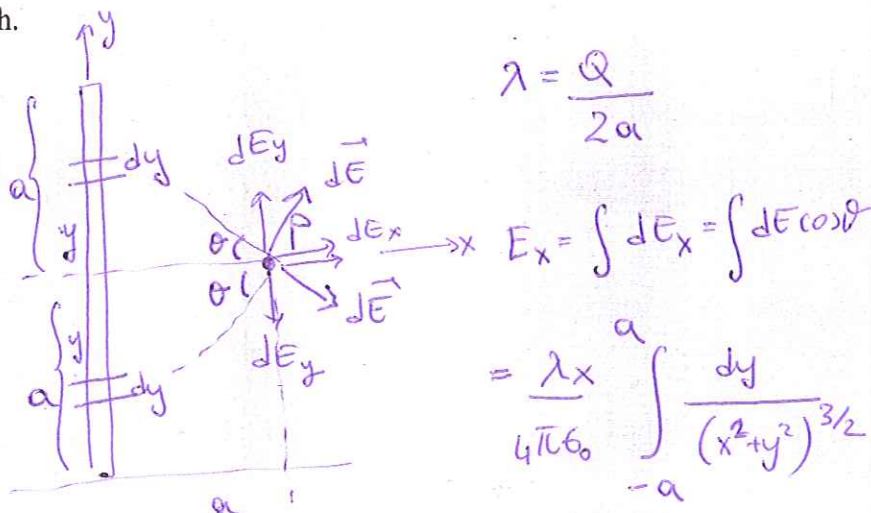
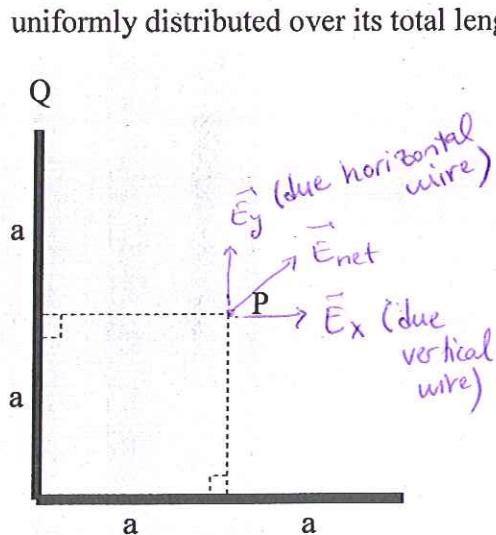
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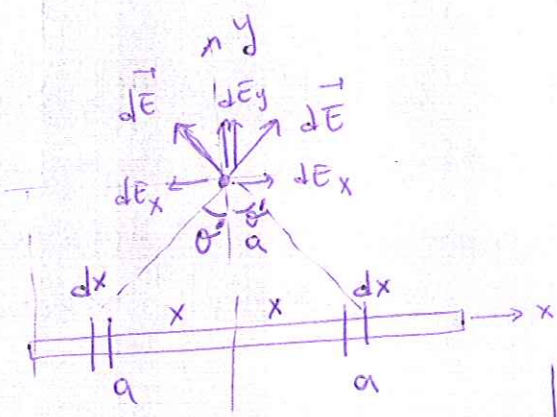
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Find the electric field at point P due to the L-shaped straight wire, where the total charge Q is uniformly distributed over its total length.



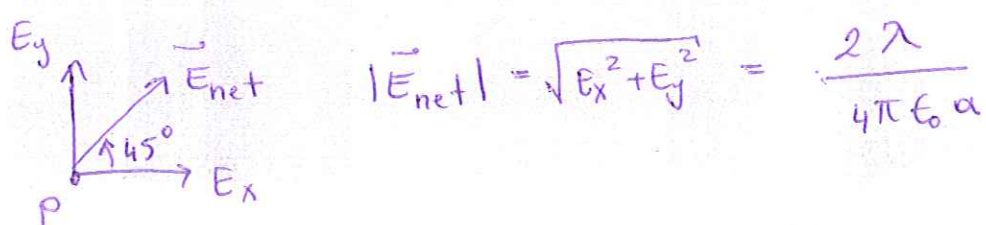
$$E_x = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{x} \frac{a}{\sqrt{x^2+a^2}}$$



$$E_y = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{y} \frac{a}{\sqrt{y^2+a^2}}$$

$$x=a \rightarrow E_x = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}\lambda}{a}$$

$$y=a \rightarrow E_y = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}\lambda}{a}$$



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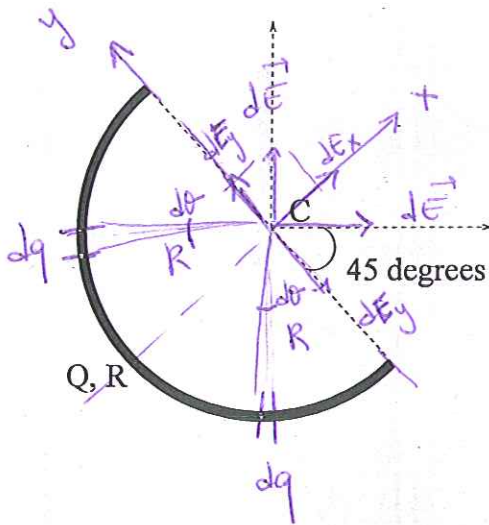
Quiz duration: 10 minutes

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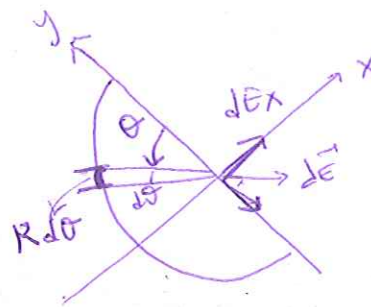
Student ID:

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Find the electric field at point C due to the semi-ring-shaped wire, where the total charge Q is uniformly distributed over its length. Here, R is the radius of the ring.



dE_y 's cancel each other.
 \vec{E}_{net} is along +x axis.



$$\lambda = \frac{Q}{\pi R}$$

$$dE_x = dE \sin\theta$$

$$= \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2} \sin\theta = \frac{1}{4\pi\epsilon_0} \frac{\lambda(R d\theta)}{R^2} \sin\theta = \frac{\lambda}{4\pi\epsilon_0 R} \sin\theta d\theta$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0 R} \int_0^\pi \sin\theta d\theta = \frac{\lambda}{4\pi\epsilon_0 R} (-\cos\theta) \Big|_0^\pi$$

$$= \frac{\lambda}{4\pi\epsilon_0 R} (1 - (-1)) = \frac{2\lambda}{4\pi\epsilon_0 R}$$

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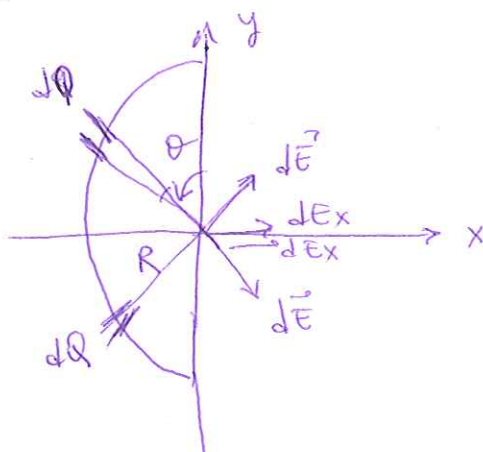
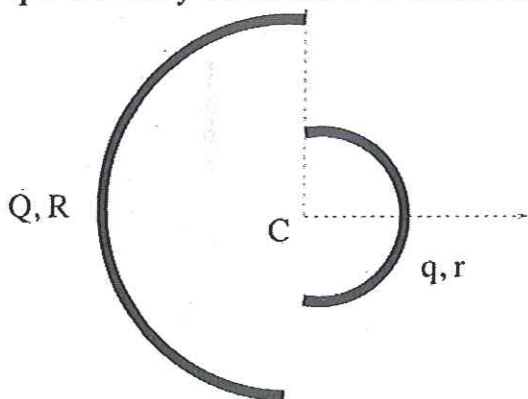
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Find the electric field at point C due to a combination of co-central semi-ring-shaped wires, where the charge Q is uniformly distributed over the bigger ring with radius R and the charge q is uniformly distributed over the smaller ring with radius r .



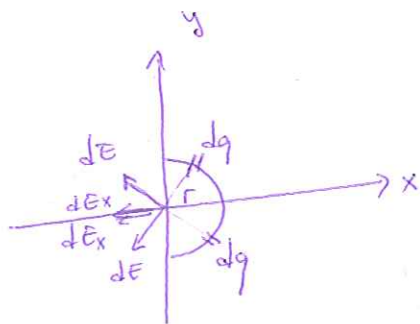
$$\vec{E}_{Q,R} = \frac{2\lambda}{4\pi\epsilon_0 R} \hat{i} \quad \left(\lambda = \frac{Q}{\pi R}\right)$$

$$dE_{x(Q,R)} = \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\theta}{R^2} \sin\theta$$

$$E_{x(Q,R)} = \frac{\lambda}{4\pi\epsilon_0 R} \int_0^\pi \sin\theta d\theta$$

$$= \frac{2\lambda}{4\pi\epsilon_0 R} \quad (+x \text{ direction})$$

Similarly:



$$\vec{E}_{q,r} = -\frac{2\lambda'}{4\pi\epsilon_0 r} \hat{i} \quad \left(\lambda' = \frac{q}{\pi r}\right)$$

$\Rightarrow \vec{E}_{net}$ is the superposition of $\vec{E}_{Q,R}$ and $\vec{E}_{q,r}$

$$\vec{E}_{net} = \vec{E}_{Q,R} + \vec{E}_{q,r} = \left(\frac{2Q}{4\pi^2\epsilon_0 R^2} - \frac{2q}{4\pi^2\epsilon_0 r^2} \right) \hat{i}$$