PHYS 102: General PhysicsII KOÇ UNIVERSITY Spring Semester 2016

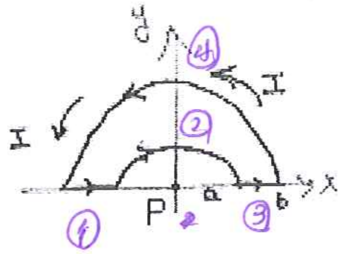
College of Sciences

Section 4 Quiz 7 31 March 2016

**Closed book. No calculators are to be used for this quiz.**
**Quiz duration: 10 minutes**

**Name:** **Student ID:** **Signature:**

Using the Biot-Savart law find the magnetic field at point P due to the current loop shown in the figure.




For parts 1 and 3, because $\vec{dl} \parallel \hat{r}$, then $\vec{B}_1 = \vec{B}_3 = \frac{\mu_0}{4\pi}\int \frac{I\vec{dl}\times\hat{r}}{r^2} = 0$

(because $\vec{dl}\times\hat{r} = 0$)

Therefore, $\vec{B}_{total} = \vec{B}_2 + \vec{B}_4 = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl_2}\times\hat{r}_2}{r^2} + \underbrace{\frac{\mu_0 I}{4\pi}\int \frac{\vec{dl_4}\times\hat{r}_4}{r^2}}$

direction is into the page. direction is out of the page

and, $\vec{dl_2}\perp\hat{r}_2 \Rightarrow |\vec{dl_2}\times\hat{r}_2| = |\vec{dl_2}||\hat{r}_2| \sin 90 = dl_2$

$\vec{dl_4}\perp\hat{r}_{24} \Rightarrow |\vec{dl_4}\times\hat{r}_2| = |\vec{dl_4}||\hat{r}_4| \sin 90 = dl_4$

$\Rightarrow B_{total} = + \frac{\mu_0 I}{4\pi}\int \frac{dl_2}{a^2} - \frac{\mu_0 I}{4\pi}\int\frac{dl_4}{b^2} = \frac{\mu_0 I}{4\pi}\frac{1}{a^2}\overbrace{\int dl_2}^{\pi b} - \frac{\mu_0 I}{4\pi}\frac{1}{b^2}\overbrace{\int dl_4}^{\pi a}$

Plus sign stands for the positive direction (into the page).

Minus sign stands for the negative or opposite direction (out of the page)

$$\Rightarrow \boxed{B_{total} = \frac{\mu_0 I}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right] \otimes \text{ into the page}}$$

PHYS 102:General PhysicsII    KOÇ UNIVERSITY    Spring Semester 2016

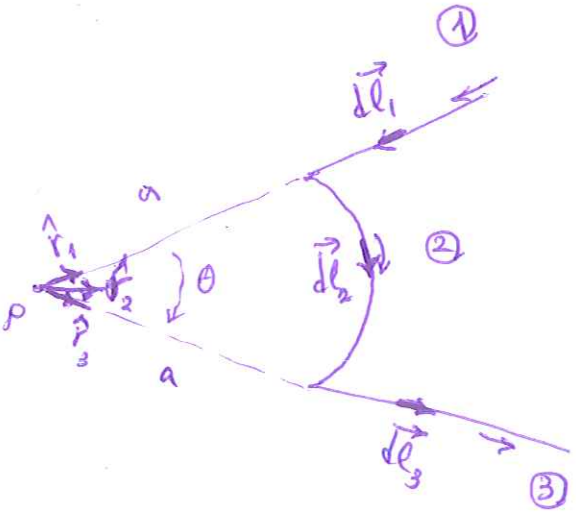
College of Sciences

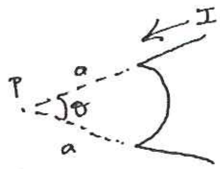
Section 2    Quiz 7    31 March 2016

**Closed book. No calculators are to be used for this quiz.**
**Quiz duration: 10 minutes**

**Name:**    **Student ID:**    **Signature:**

Calculate the magnetic field at point P of the current-carrying wire segment shown in the figure using the Biot-Savart law. The wire consists of two straight portions and a circular arc of radius a, which subtends an angle θ.




$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}_1 \times \hat{r}_1}{r^2} = 0, \text{ because } \vec{dl}_1 \parallel \hat{r}_1$$

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}_3 \times \hat{r}_3}{r^2} = 0, \text{ because } \vec{dl}_2 \parallel \hat{r}_2$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}_2 \times \hat{r}_2}{r^2} \xrightarrow[r=a]{\theta=90} \frac{\mu_0 I}{4\pi a^2}\underbrace{\int dl}_{a\theta} \quad \otimes$$

$$\Rightarrow \quad \vec{B}_2 = \frac{\mu_0 I}{4\pi a^2}\, a\theta = \frac{\mu_0 I \theta}{4\pi a}$$

$$\vec{B}_{total} = \vec{B}_2 = \frac{\mu_0 I \theta}{4\pi a}$$ the direction is into the page. $\otimes$

PHYS 102:General PhysicsII KOÇ UNIVERSITY Spring Semester 2016

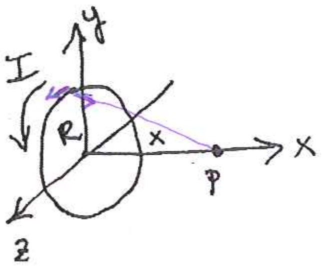
College of Sciences

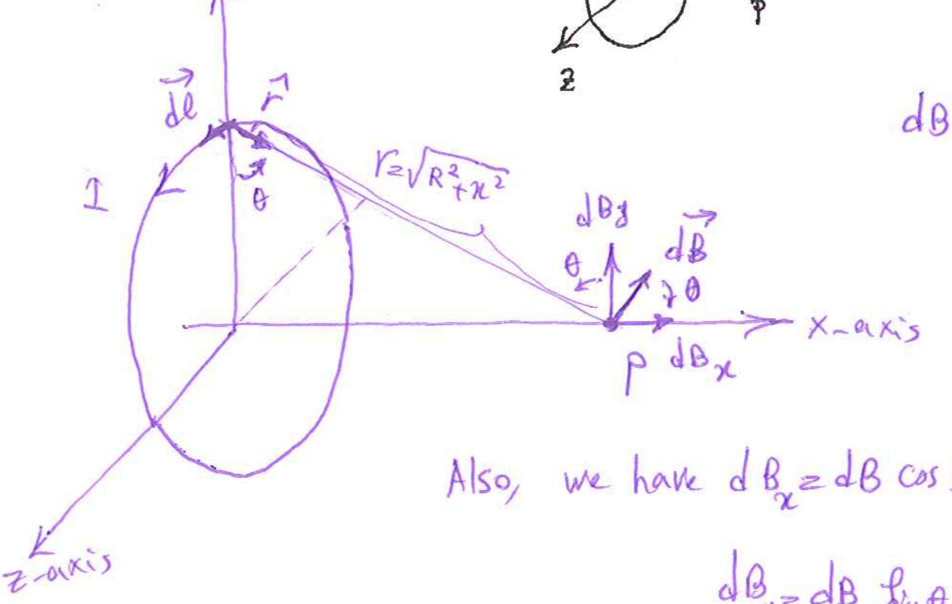
**Section 3** **Quiz 7** **31 March 2016**

**Closed book. No calculators are to be used for this quiz.**
**Quiz duration: 10 minutes**

**Name:** **Student ID:** **Signature:**

Consider a circular wire of loop of radius R located in the yz plane and carrying a steady current I as shown in the figure. Calculate the magnetic field at an axial point P at a distance x from the center of the loop using the Biot-Savart law.




note that,

$$(\vec{dl} \perp \hat{r}) \Rightarrow |\vec{dl} \times \hat{r}| = dl$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{|\vec{dl} \times \hat{r}|}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+R^2)}$$

Also, we have $dB_x = dB \cos\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+R^2)} \times \frac{R}{\sqrt{x^2+R^2}}$

$$dB_y = dB \sin\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+R^2)} \times \frac{x}{\sqrt{x^2+R^2}}$$

Notice that for each element $\vec{dl}$ There is another one in the opposite side of the loop, which cancel the y-component of magnetic field. Therefore, the total magnetic field $\vec{B}$ at P has only an x-component.

$$\Rightarrow \quad B_{total} = B_x = \frac{\mu_0 I}{4\pi} \int \frac{R\,dl}{(x^2+R^2)^{3/2}} \xlongequal[\text{have definite Values}]{\text{as } x \text{ and } R} \frac{\mu_0 I R}{4\pi (x^2+R^2)^{3/2}} \overbrace{\int dl}^{2\pi R}$$

$(x^2+R^2)$ is constant for all points on loop

$$\Rightarrow \boxed{B_{total} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}} \hat{i}}$$

PHYS 102: General PhysicsII KOÇ UNIVERSITY Spring Semester 2016

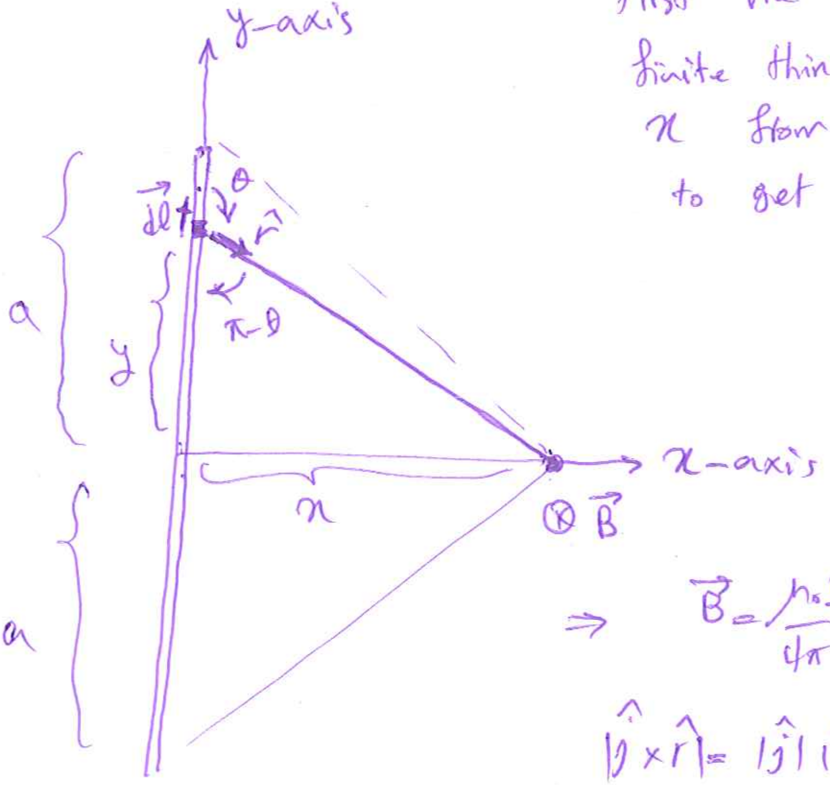
College of Sciences

Section 1 Quiz 7 31 March 2016

**Closed book. No calculators are to be used for this quiz.**
**Quiz duration: 10 minutes**

**Name:** **Student ID:** **Signature:**

Using the Bio-Savart law, find the magnetic field of a thin, infinitely long, straight current-carrying wire.




First we obtain the magnetic field of a finite thin wire with length $2a$ in a distance $x$ from its center. Then take $a \to \pm\infty$ to get the magnetic field of an infinite wire.

$$\vec B = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}\times\hat r}{r^2}, \quad \vec{dl} = dy\,\hat j, \quad \hat r = \frac{\vec r}{r}$$

where $r = \sqrt{x^2+y^2}$

$$\Rightarrow \vec B = \frac{\mu_0 I}{4\pi}\int \frac{dy\,\hat j\times\hat r}{(x^2+y^2)}$$ in the direction into the page $\otimes$

$$|\hat j\times\hat r| = |\hat j|\,|\hat r|\sin\theta = 1\times 1\sin\theta = \sin(\pi-\theta)$$

and $\sin(\pi-\theta) = \frac{x}{\sqrt{x^2+y^2}}$

$$\Rightarrow \vec B = \frac{\mu_0 I}{4\pi}\int_{-a}^{+a}\frac{x\,dy}{(x^2+y^2)^{3/2}} \xrightarrow[dy = x(1+tg^2\varphi)]{\text{take } y = x\,tg\varphi} \frac{\mu_0 I}{4\pi}\int\frac{x^2(1+tg^2\varphi)}{x^3(1+tg^2\varphi)^{3/2}}\,dy$$

$$\Rightarrow \vec B = \frac{\mu_0 I}{4\pi x}\int\frac{d\varphi}{\sqrt{1+tg^2\varphi}} = \frac{\mu_0 I}{4\pi x}\int\cos\varphi\,d\varphi = \frac{\mu_0 I}{4\pi x}\sin\varphi = \frac{\mu_0 I}{4\pi x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{-a}^{+a}$$

$$\Rightarrow \vec B = \frac{\mu_0 I}{4\pi}\frac{2a}{\sqrt{x^2+a^2}} \otimes$$ (the magnetic field of a finite wire)

Now, let us take $a \to \pm\infty$, we get,

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi} \frac{2\cancel{a}}{x\cancel{a}\sqrt{\frac{x^2}{a^2}+1}} = \frac{\mu_0 I}{2\pi x} \times \frac{1}{\sqrt{1+\frac{x^2}{a^2}}} = \frac{\mu_0 I}{2\pi x} \underbrace{\left(1+\frac{x^2}{a^2}\right)^{-\frac{1}{2}}}$$

$$(1+x)^n \xlongequal{\text{if } |x|<1} 1 + nx + \cdots$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi x}\left(1 - \frac{1}{2}\frac{x^2}{a^2} + \cdots\right) \xrightarrow[\text{only first term has the contribution.}]{\text{as } a \to +\infty} \frac{\mu_0 I}{2\pi x}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi x} \quad \otimes}$$