

# A Rigorous and Efficient Analysis of 3D Printed Circuits: Vertical Conductors Arbitrarily Distributed in Multilayer Environment

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**Abstract**—The combination of the method of moments and the discrete complex image method is extended to multilayered printed circuits with vertical conductors crossing more than one layer and/or located in different layers. Some realistic printed circuits with multilayered vertical strips are analyzed, and results are compared either to those presented in the literature or to those obtained from the commercial software *em* by SONNET Software, North Syracuse, NY. In addition, it is demonstrated that this extension has facilitated the use of a recently developed algorithm, which was specifically developed for the efficient handling of multiple vertical conductors, for the analysis of structures with multiple vertical strips running in multilayer environment.

**Index Terms**—Multilayered 3D Printed Circuits, Method of Moments, Discrete-Complex-Image Method.

## I. INTRODUCTION

Demand on miniaturization and complex functional operations of high-frequency systems leads engineers to design circuits in multilayered environment, which requires vertical conductors to facilitate connectivity between the layers. In addition, a recent trend on packaging, known as system-on-package approach, proposes integrating RF modules with embedded passive components and monolithic microwave integrated circuits in a multilayer environment [1]. Such trends can only be accepted on a large scale, if a computer-aided-design tool is developed to accurately simulate 3D structures in multilayer environment [1], and to simultaneously simulate digital, analog and optical circuits coexisting in a package [2]. As the latter is more involved and may require efficient and accurate simulation tools for individual type of circuitry, the former has to be improved for computational efficiency and accuracy, which is the main goal of the study presented in this paper.

For the analysis of 3D conducting objects in layered media, multilayer or single layer, method-of-moments (MoM) based algorithms for the solution of mixed-potential integral equations have commonly been used, due to its well-known advantages, [3]–[11]. Earlier treatments of 3D conducting objects in layered media provided the foundations of mixed-potential integral equations [3], and demonstrated the implementation of the MoM via numerically integrating the Sommerfeld-type integrals involved in the formulation [4]. Then, it was proposed in [5] that the MoM matrix entries associated with

the vertical conductors can be evaluated partially analytically with the use of the explicit expressions of spectral-domain Green's functions in  $z$  and  $z'$ . In addition, Sommerfeld-type integrals were numerically evaluated and numerical tables for different source-to-observation distances were formed to accelerate the computation of the MoM matrix entries. The same approach, i.e., analytic integration of the part of the MoM matrix entries involving  $z$  and  $z'$  integrals, was also used in [6], [7], where they demonstrated the derivation of spectral-domain Green's functions as explicit functions of  $z$  and  $z'$ , even for different observation and source layers, analytically in [6] and numerically in [7]. As another approach, the discrete-complex-image method (DCIM) in conjunction with the MoM was also employed for the analysis of printed structures with vertical metallizations [8]–[11]. It was also demonstrated that the extension of the method developed in [10] results in an efficient simulation tool for the analysis of multilayered printed structures with multiple vertical conductors, provided all vertical conductors are positioned in only one layer [12]. Since the method proposed in [10] is widely accepted as being limited for printed structures with vertical conductors positioned only in a single layer in a multilayer environment [6], [11], as the main contribution, it is demonstrated that this approach, with some modification in the implementation of the MoM, can handle more general printed structures, such as those with vertical conductors located in different layers and/or extending more than one layer. In addition, this development has facilitated the implementation of the algorithm proposed in [12] for multiple vertical conductors distributed arbitrarily in multilayered planar media.

Since the main algorithm has already been detailed for geometries with vertical conductors in a single layer, additional work required for its extension to more general geometries is discussed in Section II. Then, it is followed by some representative examples in Section III to demonstrate the accuracy, as compared to *em* by SONNET Software, North Syracuse, NY, or to the results available in literature. Finally, in Section IV, conclusions are provided.

## II. DISCUSSION

Note that the underlying method used in this work, the spatial-domain MoM in conjunction with the DCIM, is the same as the one proposed in [10]. Therefore, the formulation of this combination is not detailed here any further, and only its extension to printed circuits with multilayered vertical conductors will be briefly discussed. However, for the sake of completeness, two major issues on the use of DCIM, for getting the closed-form expressions of Green's functions and/or the auxiliary functions, in the applications of MoM need to be stated with some clarifications. Since every image, i.e., exponential function, used in the approximation of Green's functions has to be accounted for in the calculation of the MoM matrix entries, their number definitely plays some role in the overall efficiency of the method: larger the number of images less efficient the algorithm would get. But, the experience so far has shown that the number of exponentials necessary for good approximation does not reach to a level that

Manuscript received November 1, 2006; revised April 21, 2007.

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would render the overall algorithm inefficient as compared to other traditional methods. This is partly because the number of images required is not generally that high [13], and partly because some tools for efficient calculations of the MoM matrix entries have been developed for Green's functions cast in exponential forms [14], [15]. In addition, the algorithm proposed in [12] for multiple vertical conductors has been facilitated by the use of the DCIM, and therefore, large number of exponentials that may be required in some applications can be counterbalanced by the efficiency gained in the calculations of the MoM matrix entries by their use. Deterioration of the accuracy of the approximation of Green's functions by the DCIM for large source-to-observation distances, usually much larger than the free-space wavelength, has been an issue for a long time, which was recently discussed in details and attributed to non-spherical wave nature of the functions to be approximated, [16]. Throughout this work, vertical strips are employed as vertical conductors, with no loss of generality, and that no slanted connections between the layers are considered as it may require different class of Green's functions [3]. It is also assumed that all layers and ground plane extend to infinity in the transverse directions and the conductors are lossless and infinitesimally thin. A time convention of  $e^{j\omega t}$  has been adopted and suppressed in this work.

It is well-known that the spectral-domain Green's functions in planar layered media can be obtained analytically in closed-forms. However, in order to utilize the full advantageous of the closed-form Green's functions when used in conjunction with the MoM, it is important to cast them as explicit functions of  $z$  and  $z'$ , even when the observation and source points are in different layers. Although Green's functions with explicit  $z$  and  $z'$  parameters were detailed in [17], [18] using the transmission line approach, a few steps of the derivation using the wave approach are provided following the procedure provided in [19], [20], in order to be able to demonstrate the functional dependence of the individual terms in Green's functions.

Consider a general planar stratified medium where a point source is located in layer- $i$  and observation points can be in any layer, including the source layer as well. To clarify the notation, note that  $\sim$  over fields denotes the spectral-domain representation of the corresponding field, while  $\sim$  over reflection and transmission coefficients denotes the generalized versions of these parameters that account for the multiple reflections and transmissions. Assuming the observation points are in layer- $j$ , which could be located either below or above the source layer, the spectral-domain field expressions (for TE and TM waves) in the observation layer can be written in the form of

$$\tilde{F} = A_j \left[ e^{-jk_{z_j}(z-z')} + \tilde{R}^{j,j+1} e^{jk_{z_j}(z-z')} e^{-jk_{z_j}2(z_{ju}-z')} \right] \quad (1)$$

for  $j > i$ ,

$$\tilde{F} = A_j \left[ e^{jk_{z_j}(z-z')} + \tilde{R}^{j,j-1} e^{-jk_{z_j}(z-z')} e^{-jk_{z_j}2(z_{jl}+z')} \right] \quad (2)$$

for  $j < i$ . Generalized reflection coefficients,  $\tilde{R}^{j,j\mp 1}$ , are defined at  $z = z_{ju}$  (upper interface of layer- $j$ ) and  $z = -z_{jl}$  (lower interface of layer- $j$ ) for  $j > i$  and  $j < i$ ,

respectively, and  $A_j$  is the amplitude of the wave in layer- $j$  to be determined. Starting from the amplitudes of the fields in the source layer,  $A_i^+$  and  $A_i^-$  for up- and down-going waves, one can determine the amplitudes of the fields in any layer iteratively. As a result, the amplitude transfer function from the source layer (layer- $i$ ) to any other layer (layer- $j$ ) can be easily obtained as

$$A_j e^{-jk_{z_j}(z_{(j-1)u}-z')} = A_i^+ e^{-jk_{z_i}(d_i-z')} \times \underbrace{\left( \prod_{n=i}^{j-2} \tilde{T}^{n,n+1} e^{-jk_{z_{n+1}}d_{n+1}} \right)}_{=1 \text{ for } j-2 > i} \tilde{T}^{j-1,j} \quad (3)$$

for  $j > i$ , and

$$A_j e^{jk_{z_j}(z_{(j+1)l}-z')} = A_i^- e^{-jk_{z_i}z'} \times \underbrace{\left( \prod_{n=i}^{j+2} \tilde{T}^{n,n-1} e^{-jk_{z_{n-1}}d_{n-1}} \right)}_{=1 \text{ for } j+2 > i} \tilde{T}^{j+1,j} \quad (4)$$

for  $j < i$ . Note that the terms denoted by  $\tilde{T}^{i,i+1}$  and  $\tilde{T}^{i,i-1}$  describe the generalized transmissions from the top of layer- $i$  to the bottom of layer- $(i+1)$ , and from the bottom of layer- $i$  to the top of layer- $(i-1)$ , respectively. Once the amplitude of the field in the observation layer is obtained, the spectral-domain Green's function can be written as explicit functions of  $z$  and  $z'$ , by substituting (3) or (4) into (1) or (2), respectively. Hence, the spectral-domain Green's functions will be composed of four terms

$$\tilde{F} = \sum_{p=0}^1 \sum_{q=0}^1 c_{pq} e^{-j[(-1)^p k_{z_j} z + (-1)^q k_{z_i} z']} \quad (5)$$

where  $c_{pq}$ 's are not functions of  $z$  and  $z'$ . Once the spectral-domain Green's functions are expressed as the explicit functions of  $z$  and  $z'$ , then the MoM matrix entries can be obtained fully analytically, if it is desired, using the same approach proposed in [10]. Although the approach is the same, the implementation of the algorithm, especially the calculation of the MoM matrix entries corresponding to the basis functions crossing the boundaries, needs to be modified.

Throughout this work, basis functions used to approximate the induced current density along the horizontal and vertical conductors of the geometry are chosen to be rooftop and half-rooftop functions, respectively. In addition, at the intersections of the vertical and horizontal conductors, saw-tooth attachment functions are introduced on the horizontal cells of the junctions and, for computational purpose, used as the part of the corresponding vertical half-rooftop basis functions, to satisfy the charge conservation and current continuity at the junctions. As a vertical conductor runs through a layer, it may connect horizontal conductors at both interfaces of the layer (including a ground plane at one interface), or just pass through the interface with no horizontal connections. For the former use, two attachment functions with opposite slopes are used, one at each end of the vertical conductor, except at the ground connection. However, for the latter use, no need for the attachment function, two half-rooftop functions join and make up a single rooftop function spanning the interface.

### III. NUMERICAL EXAMPLES

Since the extension of the method developed in [10] to more general structures has been briefly described, it needs now to be validated on a relatively simple but intuitive example, like two microstrip lines on different layers in a multilayer environment, as shown in Fig. 1. This is a 3-Port microstrip geometry in a five-layer medium backed by a ground plane, whose details are provided in Fig. 1; two horizontal conductors are connected with a vertical strip and two vertical strips are placed between each of the horizontal conductors and the ground plane. Since the current distributions on the

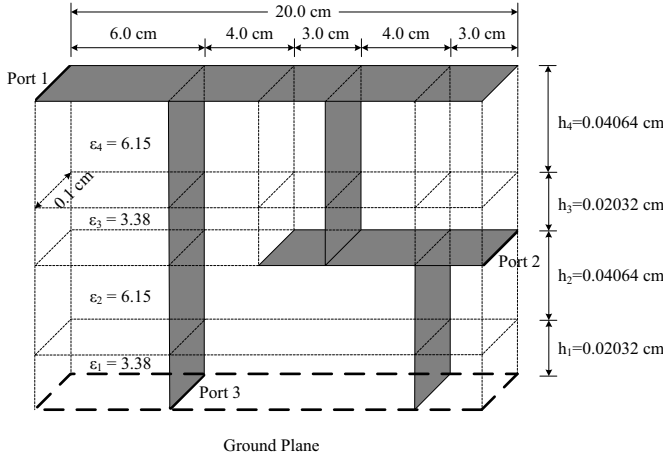


Fig. 1. A 3-port printed circuit with vertical strips in a multilayer environment.

microstrip lines can be predicted intuitively, using transmission line analogy, they are obtained for two different sets of port terminations: Port-1 is excited (with a unity-amplitude current source) while Port-2 and Port-3 are terminated in open- and short-circuit, respectively; Port-3 is excited while both Port-2 and Port-3 are left open-circuited. For the sake of brevity, only the current distribution on the bottom microstrip line for the former case is provided in Fig. 2 along with the results obtained from a commercial software, *em* by SONNET Software, North Syracuse, NY. It is observed that the results are in good agreement and the slight differences between the two results can be attributed to the difference between the environments that numerical techniques assume: *em* by SONNET Software, North Syracuse, NY, solves the geometry in shielded environment while the method proposed in this paper solves it in an open environment.

Another example is a square patch antenna ( $L=4.02$  cm) with multiple shorting strips over a two-layer substrate; 1st layer over PEC:  $\epsilon_{r1}=4.77$ ,  $h_1=0.068$  cm; 2nd layer:  $\epsilon_{r2}=2.33$ ,  $h_2=0.132$  cm. The antenna is fed by a microstrip line, width=0.268 cm, at the mid point of its one edge, and shorting strips (width=0.268 cm each) are positioned 1.876 cm away from its feeding edge. According to the cavity model, the patch would be expected to have resonances at the frequencies of 2.3 GHz [ $=c/(2 \times 4.02 \times \sqrt{\epsilon_{eff}})$ ] and 2.5 GHz [ $=c/(2 \times 1.876 \times \sqrt{\epsilon_{eff}})$ ] with no shorting strip, and with full short-circuiting at a distance of 1.876 cm from the feeding edge, respectively, where  $\epsilon_{eff}$  is calculated heuristically from [21]. The magni-

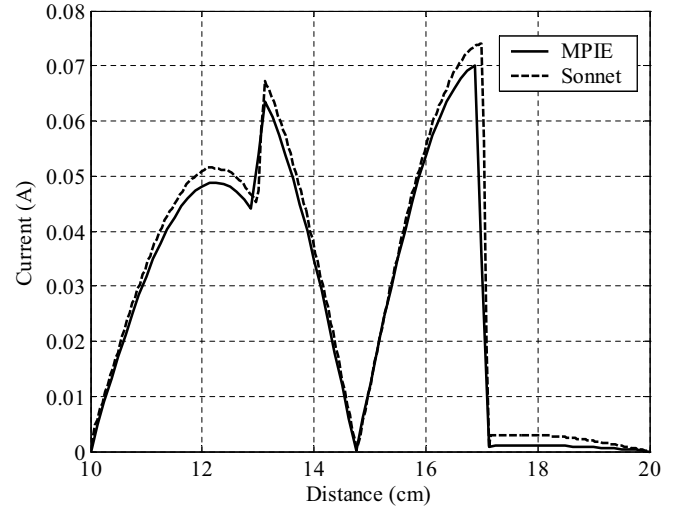


Fig. 2. Current distribution on the bottom microstrip line when Port-1 is excited, Port-2 and Port-3 are terminated in open- and short-circuit, respectively, in Fig. 1 at  $f = 1.5$  GHz.

tudes of  $S_{11}$  are obtained and the resonant frequencies of the patch with no vertical strip and with fifteen vertical strips are observed to be about 2.22 GHz, and 2.53 GHz, which are in good agreement with the above predicted resonant frequencies.

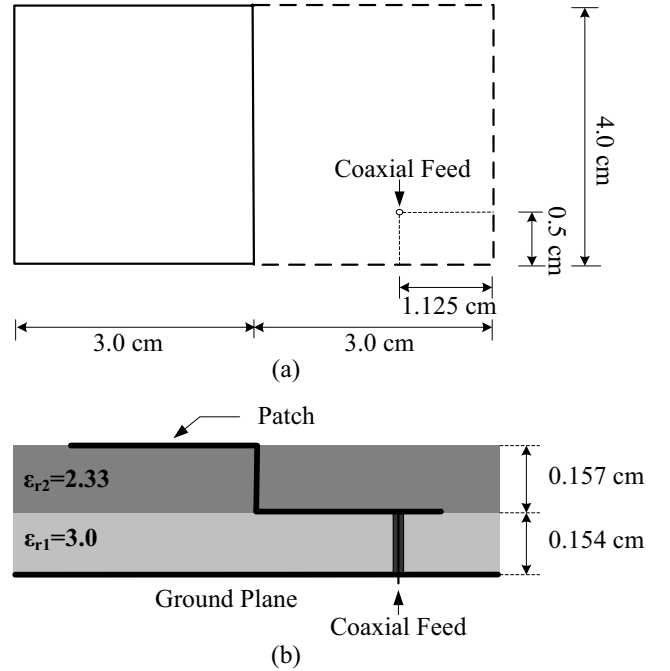


Fig. 3. Two layered patch antenna (a) Top view (b) Side view.

As a final example, a two-layered rectangular microstrip antenna, whose one half is printed on the top interface and the other is on the lower interface, connected via non-touching strips, is studied. This structure is the modified version of the geometry studied in [22] using a whole wide metallic connection between the two halves of the patch antenna, as shown in Fig. 3. The input impedance of this probe-fed

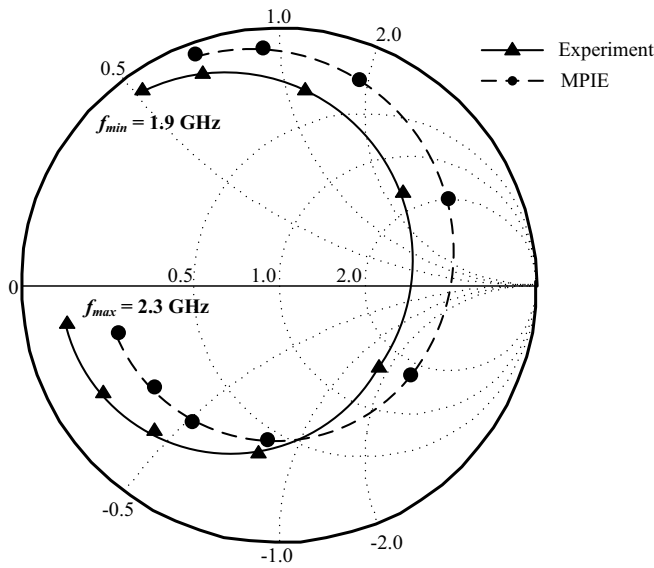


Fig. 4. Input impedance of the antenna shown in Fig. 3.

antenna is obtained over a band of frequency (1.9-2.3 GHz) using the method presented in this paper, and the results are compared to the experimental results provided in [22], as shown in Fig. 4. Although the results are in good agreement, particularly concerning the resonant frequencies, the slight difference between the simulation and the experimental results can be attributed to different kind of vertical connections (non-touching strips vs. a whole conducting strip) between the two layers of the antenna.

As a final remark, it should be noted that multiple vertical strips in these examples have been implemented by the algorithm developed recently [12] with similar improvement in computational efficiency.

#### IV. CONCLUSIONS

In this study, a well-known method for its numerical efficiency and accuracy, namely the combination of MoM and the discrete complex image method, has been extended to analyze an important class of printed geometries, multilayered printed structures with truly multilayered vertical conductors. The method is applied to some realistic geometries, and the results are compared to those presented in literature and to those obtained from the commercial software *em* by SONNET Software, North Syracuse, NY. As a final statement, this combination has now matured to the level that can efficiently and accurately analyze almost any kind of printed structures, and has become a good candidate for a CAD simulation software.

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