Space Interferometry Mission Instrument Model and Astrometric Performance Validation

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The Space Interferometry Mission (SIM), will perform very accurate astrometric measurements to measure the positions of stars using a 10 m baseline optical interferometer. The lack of signal from the science targets precludes using the star as a feedback signal to control the space interferometer delay line. In order to solve this problem SIM uses pathlength feed forward (PFF) control of the science interferometer. In the case of controlling the science interferometer optical path, the information to position the science delay line comes from a combination of internal metrology, external metrology, and guide interferometer measurements. The accuracy of the internal and external metrology measurements and the guide interferometer measurements are important for the quality of the feed forward signal and also for the ultimate astrometric performance of the instrument.

An instrument model of SIM has been built to evaluate optical performance and to emulate various observational scenarios. The effect of averaging methods to reduce metrology cyclic error and the viability of on-orbit calibration maneuvers are studied. The model consists of a real-time dynamics formulation of the spacecraft and a real-time attitude control system. Simulation results investigate the sensitivity of the feed forward signal to the various error sources and time-varying terms.

1. INTRODUCTION

The Space Interferometry Mission (SIM) scheduled for launch in 2009, is one of the premier missions in the Origins Program, NASA’s endeavor to understand the origins of the galaxies, of planetary systems around distant stars, and perhaps the origins of life itself (see Fig. 1) [1]. The SIM instrument uses three Michelson white light interferometers to make the delay measurements for the fundamental observation of the instrument. Two of the interferometers acquire fringes on bright guide stars in order to make highly precise measurements of changes in the spacecraft attitude, while the third makes the delay measurement on the science target. Fig. 2 shows how a single Michelson interferometer makes an astrometric measurement. The interferometer collects the starlight from two physically separated collectors, and via a series of mirrors, brings the light to a beam combiner where they are interfered. The vector quantity determined by the line joining the vertices of the two optical fiducials on each of the collectors defines the baseline vector. In the current Shared Baseline configuration, the two guide interferometers share the same baseline vector. For a broadband source like a star, interference fringes can only occur when the optical path from the star through the left interferometer arm is equal to the optical path from the star through the right interferometer arm. In order to achieve this, a Michelson interferometer uses a delay line to introduce additional path into one of the interferometer arms. The white light fringe can be formed and detected by actively controlling the position of the delay line. SIM will measure the path-length delay between the two arms of the interferometer. The instantaneous delay is given by the interferometer astrometric equation

\[ d_s = \langle s_s, b_s \rangle + k_s \]  (1)

where \( \langle , \rangle \) denotes dot product which provides the angle \( \theta \) (see Fig. 2) between the baseline vector and the star position. The optical pathlength delay \( d_s \) is measured by the interferometer using internal metrology and the white light fringe, \( s_s \) is the unit vector to the observed star, \( b_s \) is the baseline vector, and \( k_s \) is the so-called constant term that represents the instrumental offset between the true and the measured delay. Note that all the quantities on the right are unknown, and thus must be estimated. The star position \( s_s \) and baseline vector \( b_s \) are derived from delay measurements of multiple stars. SIM measures several stars in its 15° × 15° field of regard at a given baseline orientation \( b_s \), forming a so-called tile. A tile is defined as the set of stars observed with the SIM spacecraft orientation \( b_s \). Guide stars are typically bright stars...
(7th and 8th magnitude) to simplify the acquisition and fringe tracking and provide information about high frequency disturbances in the interferometer necessary to acquire and measure fringes on dim science targets. The baseline is then reoriented to observe a new tile having several stars in common with the previous tile. This process is repeated until a large section of sky is covered. With sufficient sky coverage, one can determine the baseline vector in each tile and all observed star positions to the required accuracy (few microarcseconds) [2]. SIM offers the promise of 4 microarcseconds accuracy for 20,000 stars at magnitudes down to 20 over the entire sky [3].

The lack of signal from the dim science targets prevents using the star as a feedback signal to control the science interferometer delay line. In order to solve this problem SIM uses pathlength feed forward (PFF) control of the science interferometer. In the case of controlling the science interferometer optical path, the information to position the science delay line comes from the internal metrology, external metrology, and the guide interferometer measurements. The combination of the external metrology measurements and guide interferometer measurements (fringe signal and the internal metrology signals) provide enough observations to compute the science interferometer baseline vector in inertial space. Details of this algorithm are described in Section IID. Any error in the PFF signal will result in a degradation of the interference signal. The PFF signal is exact when the parameters that define the mapping are without error. However, a priori error in the guide star positions, the relative positions of the fiducials in the metrology local frame, and the interferometer constant terms will produce a residual error between the science baseline vector in inertial space and its estimate as derived by the feed forward algorithm.

The development of an “astrometric performance model” for the SIM spacecraft and the instrument is described. This model is designed to investigate the sensitivity of the feed forward signal to the various error sources and time-varying terms. It also enables early identification of the performance and design problems that would degrade the fringe visibility. Predicting the performance of the SIM instrument, through simulation and modeling early in the design process can significantly impact the mission’s success. This model is used to support the astrometric error budget by verifying the error allocations and also to identify the design flaws early in the formulation phase of the project.

The validation and verification strategy for SIM is quite complex and involves a suite of subsystem model and testbeds. Another simulation model to simulate the entire five year mission has been developed to validate and support the astrometric error budget in a global sense [4]. This model runs very fast compared with the astrometric performance model and builds model fidelity in other areas but it is not does not capture the spacecraft dynamics and does not include the attitude control system (ACS).

The astrometric performance model consists of a real-time dynamics formulation of the spacecraft and a...
real-time attitude control system (see Sections IIA and IIB for details). The PFF algorithms are implemented in the MATLAB Simulink environment, and include the internal and external metrology measurements and guide interferometer measurements. Details of these algorithms are described in Sections IIC and IID. The simulation environment enables many testing scenarios and end-to-end performance evaluations that would not be viable on the ground. We present results on two spacecraft observational scenarios that have been proposed for instrument calibration and reduction of cyclic metrology errors. Section III summarizes the on-orbit calibration scheme and demonstrates how the model is exercised to study the viability of the on-orbit calibration. Section IV studies the averaging methods to reduce the metrology cyclic error and discusses the validity of the approach.

II. SIMULATION MODEL

A. Spacecraft Dynamic Model

The shared baseline configuration of the SIM dynamic model is generated by the Dynamics Algorithms for Real-Time Simulation Software (DARTS/DSHELL) package developed at Jet Propulsion Laboratory (JPL) [5, 6]. It is a multimission spacecraft simulation environment for real-time in-the-loop simulations for testing and verification of flight software and hardware. The DSHELL environment integrates the DARTS SpaceCraft (S/C) dynamics simulator and a library of hardware models (actuators, sensors, and motors) into an integrated simulation environment that can be easily configured and interfaced with flight software and hardware for various real-time and non-real-time S/C simulation needs. DARTS dynamics computing engine implements a fast efficient “spatial operator algebra” formulation for multibody dynamics simulation [7]. The spatial operator algebra is a mathematical approach for modeling the dynamical behavior of complex, articulated collections of bodies interacting with each other in free-space or in contact with the environment. In addition to large angle articulation of bodies, DARTS can also handle the coupling between the rigid body and the flexible dynamics of the system.

In the astrometric performance model, the SIM spacecraft is modeled as a rigidbody with actuator and sensors nodes (see Fig. 3). The randomly generated displacements are then superimposed on to the rigid body rotations of the spacecraft to emulate the flexible dynamics of the structure. The random model is only a placeholder until detailed finite element models of the instrument and supporting structure become available. The actuator and sensor nodes allow the user to track the real-time positions/rates of the specific nodes located on the spacecraft. Sensor nodes at the fiducial positions are used to measure the relative distance measurements between the collectors. Another sensor node on the spacecraft collects the attitude and rate information so that the position of the spacecraft can be corrected through the ACS system. Torques generated by the ACS system are applied at the actuator nodes on the spacecraft. The dynamics model module interfaces with the ACS, external metrology and PFF algorithms in the MATLAB Simulink environment (see Fig. 4). In this study, the astrometric performance metric of the SIM instrument is the science delay error, \( \varepsilon(t) \). It is calculated by subtracting the estimated science delay from the true science delay.

B. Attitude Control System

Once acquired, the stars must be tracked continuously by the angle pointing control system to 30 milliarcsecond (mas) accuracy. In order to achieve this level of pointing control error for the interferometers, the rigid body motion of the spacecraft itself must be controlled to an accuracy of about \( \sim 2 \) arcsec for SIM using the attitude control subsystem of the SIM spacecraft. The ACS is also used for performing large angle slewing maneuvers (for example, on-orbit calibration). The ACS compensator uses a proportional-derivative (PD) controller with a 0.01 Hz bandwidth. The spacecraft’s three axis attitude is estimated by a Kalman-Bucy filter that uses the sensor measurements from the DARTS/DSHELL model. Three reaction wheel assemblies (RWAs) are used to control the spacecraft and the imperfections of the wheels such as dynamic imbalance and static imbalance are modeled as disturbances going into the system. The other imperfections included in the ACS model are viscous friction forces, quantization effects, and various sensor noises [8]. The RWA controller design captures “imperfections” associated with the attitude sensors, RWA motor dynamics and also the controller limitations (e.g., command resolution and timing...
delay). The main components of the ACS design are shown in Fig. 5.

C. External Metrology

The purpose of the external metrology system is to determine the positions of the fiducials of the optical truss and thereby track the flexible-body motions of the guide and science interferometer baseline vectors. The SIM design uses three separate baselines to measure the two guide and science stars. Consequently, SIM must measure the relative orientation between the three baselines in addition to the individual baseline lengths. Metrology beams are launched to measure the relative distance between the collector fiducials (see Fig. 6). These 1D external metrology measurements are inverted into position vectors in the spacecraft local frame using the procedure described in Section IID. Absolute metrology is necessary to give a priori knowledge of the geometry of the initial fiducial positions. This knowledge is essential for proper conversion of the relative metrology measurements into fiducial position changes.

D. Estimating Baseline Vector

As described in the earlier sections, the process of reconstructing the science baseline vector when implemented onboard in real-time is termed PFF and is a critical component to the operation of the interferometer. There are essentially two components to the problem of determining the science baseline, \( \mathbf{b}_s \). The first part inverts the 1-D external metrology measurements into position vectors computed in the spacecraft local frame for all of the fiducials. The second part uses the guide interferometer measurements to determine the transformation between the local frame and the inertial frame.

We now get into the details of how \( \mathbf{b}_s \) is obtained from the measurements and a priori data [9]. For this purpose, we treat SIM as a set of fiducials, \( \mathbf{x}_1, \ldots, \mathbf{x}_N \) \((N = 5)\). Our interests center around the evolution of the fiducials \( \mathbf{x}_i(t) \) over a time period \( t_0 \leq t \leq T \), where \( t = t_0 \) denotes the beginning of an observation of a tile and \( t = T \) is the time of completion. The problem is solved using the on-board optical sensing systems that include the external metrology system and the guide star interferometers. The observed variables associated with the external metrology system are the relative distance measurements between the \( i \)th and \( j \)th fiducials calculated by function \( F(x) \)

\[
F_{ij}(x) = |x_i(t) - x_j(t)| - |x_i(t_0) - x_j(t_0)| \quad (2)
\]

where \( x_i \) is the position of the \( i \)th fiducial. Then the estimate of the fiducial positions \( \hat{x}_i \) are determined by solving a nonlinear least square problem. Details of the estimation scheme are available in [9].
Once a solution has been obtained, the science baseline, \( b_{i}^{\text{loc}} \) and guide baseline vector, can be determined in the local frame. Now let \( U \) define the transformation between the local and inertial coordinate frames so that \( b_{i} = Ub_{i}^{\text{loc}} \). The problem of determining \( b_{i} \) in inertial coordinates is solved once we obtain \( U \).

The equations for obtaining \( U \) are provided by the guide interferometer measurements. SIM uses a pair of guide interferometers to produce two independent delay measurements per observation. The guide measurements may be written as

\[
d_{g,i} = \left( s_{g,i}, Ub_{g}^{\text{loc}} \right)
\]  

with \( b_{g}^{\text{loc}}, i = 1,2 \) denoting the guide interferometer baseline vectors determined via the inversion of the 1-D external metrology measurements. \( U \) can be parameterized in a series expansion form such that

\[
U = I + S + \frac{S^{2}}{2!} + \frac{S^{3}}{3!} + \cdots
\]

where \( S \) is a skew symmetric matrix and

\[
S(\omega) = \begin{pmatrix}
0 & -\omega_{3} & \omega_{2} \\
\omega_{3} & 0 & -\omega_{1} \\
-\omega_{2} & \omega_{1} & 0
\end{pmatrix}
\]

and \( \omega \) is the rotation vector

\[
\omega = \begin{pmatrix}
\omega_{1} \\
\omega_{2} \\
\omega_{3}
\end{pmatrix}.
\]

Without loss of generality, we may assume that \( U \approx I \) because of on-board attitude knowledge. The quadratic approximation to (3)–(4) using (5)–(6) is

\[
y = T\omega + G(\omega)
\]

where

\[
T = \begin{pmatrix}
\frac{b_{g1}^{\text{loc}} \times s_{g1}}{\left| b_{g1}^{\text{loc}} \times s_{g1} \right|} \\
\frac{b_{g2}^{\text{loc}} \times s_{g2}}{\left| b_{g2}^{\text{loc}} \times s_{g2} \right|}
\end{pmatrix}
\]

\[
G(\omega) = \frac{1}{2} \begin{pmatrix}
\left| b_{g1}^{\text{loc}} \right|^{2} S(\omega) b_{g1}^{\text{loc}} \\
\left| b_{g2}^{\text{loc}} \right|^{2} S(\omega) b_{g2}^{\text{loc}}
\end{pmatrix}
\]

The two observable directions of the \( \omega \) vector can be obtained from (7) by a fixed point iteration method. Under the assumption that the science interferometer baseline is parallel to the guide interferometer baseline, it has been shown that the unobserved component of \( \omega \) (the rotation about the axis aligned with the baseline orientation) has no effect on the estimation of the science baseline. If the baselines are not perfectly parallel, then there is a residual error in calculating the attitude of the science interferometer that results in a delay error. The unobserved component is called roll and the current SIM design considers another measurement to compensate this effect [9].

After \( \omega \) is obtained from (10), the science baseline in inertial frame can be calculated using the following equation

\[
b_{i} = b_{i}^{\text{loc}} + \omega \times b_{i}^{\text{loc}} + \frac{1}{2} \omega \times (\omega \times b_{i}^{\text{loc}}).
\]

The baseline estimation procedure is the critical component to the operation of the interferometer. In the following section, we study the propagation of the science and guide star position errors, metrology gauge errors, misalignment errors to the estimated science delay when the instrument is calibrated in space. The on-orbit calibration scheme is one of the options that is being considered for the SIM design to correct for the field-dependent errors. The procedure uses the delay measurements to estimate the instrument bias function. Details of the on-orbit calibration scheme and the residual errors due to the various error sources are presented in the following section.

III. ON-ORBIT CALIBRATION SCHEME

SIM derives its performance from precise (relative) distance measurements made between a set of fiducials. The errors introduced by the polarization effects due to rotation of corner cubes or by diffraction effects caused by the delay line motion are relatively static and repeatable. In this section, we study the type of errors that can essentially be parameterized by the position of an object in the instrument’s field of regard. Note that the observations discussed here are dedicated to calibrating the interferometers and have no science value.

The on-orbit calibration procedure has been developed using the delay measurements [11]. We briefly describe the calibration approach and present the simulation results. The basic measurement made by the instrument is the delay measurement \( d_{s} \),

\[
d_{s} = \left( s_{s}, b_{s} \right) + k_{s} + c(u,v)
\]

where \( s_{s} \) is the science star position vector, \( b_{s} \) is the interferometer baseline vector, \( k_{s} \) is the constant term and we have introduced \( c(u,v) \) as the field-dependent instrument measurement error. Here the independent variables \( u, v \) parameterize the star position vectors, \( s(u,v) = [u,v,\sqrt{1-(u^{2}+v^{2})}] \), and the baseline vector is oriented along the \( u \)-axis corresponding to the 15° × 15° field of regard.

Now, the objective is to learn the function \( c(u,v) \). If \( s_{s}, b_{s}, k_{s} \) are known exactly, (10) gives a direct measurement of \( c(u,v) \). However, \( s_{s} \) may be known to only a few milliarcsecond and the orientation of \( b_{s} \) may be known to perhaps an arcsec with an error of a
micron in its length, yielding and estimation of \( c \) that is grossly inadequate.

Although \( c(u, v) \) is practically unobservable from (10), the gradient \( \nabla c \), can be determined by canting and rolling the instrument by small angles (in the neighborhood of \( \pm 1^\circ \)) and taking the difference measurements. From these gradients the calibration error is then estimated using a least squares inversion. The proposed calibration scheme consists of the following steps.

**Step 1** We rotate the instrument baseline \( \alpha \) (\( \alpha = \pm 1^\circ \)) degrees about the \( u \)-axis (roll) using the large angle maneuver capability of the attitude control system. Using the PFF algorithms, we make the delay measurements \( d_u^\pm(i, j) \) and \( d_v^\pm(i, j) \), \( i, j = 1, \ldots, N \), at the grid points, respectively. Then the application of (10) yields the relationship between the delays and the values of the calibration function at the rotated grid points

\[
d_u^\pm(i, j) = \langle s(i, j), U_u^\pm b^{loc} \rangle + c(u^\pm(i, j), v^\pm(i, j))
\]

\( i, j = 1, \ldots, N \)  

(11)

where \( s(i, j) = s(u, v) \), \( U_u^\pm \) is the rotation about the \( u \)-axis, and

\[
u^\pm(i, j) = u(i, j) \pm \alpha.
\]

(12)

**Step 2** We rotate the instrument baseline \( \beta \) (\( \beta = \pm 1^\circ \)) degrees about the \( v \)-axis (cant) using the ACS. We make the measurements \( d_v^\pm(i, j) \) and \( d_v^\pm(i, j) \), \( i, j = 1, \ldots, N \), at the grid points, respectively. As before, relation (10) yields

\[
d_v^\pm(i, j) = \langle s(i, j), U_v^\pm b^{loc} \rangle + c(u(i, j), v^\pm(i, j))
\]

\( i, j = 1, \ldots, N \)  

(13)

where \( U_v^\pm \) is the rotation about the \( v \)-axis, and

\[
v^\pm(i, j) = v(i, j) \pm \beta.
\]

(14)

**Step 3** We calculate approximations of the gradient of \( c(u, v) \) at the grid points

\[
\nabla c(i, j) = \left[ \frac{\partial c(i, j)}{\partial u}, \frac{\partial c(i, j)}{\partial v} \right]
\]

(15)

based on the measurements that were taken during the rotations of steps 1 and 2. Computing the central difference approximations:

\[
\frac{\partial c(i, j)}{\partial u} \approx \frac{1}{2\alpha}[c(u^+(i, j), v(i, j)) - c(u^-(i, j), v(i, j))]
\]

\( i, j = 1, \ldots, N \)  

(16)

\[
\frac{\partial c(i, j)}{\partial v} \approx \frac{1}{2\beta}[c(u(i, j), v^+(i, j)) - c(u(i, j), v^-(i, j))]
\]

\( i, j = 1, \ldots, N \)  

(17)

The right side of (18)–(19) is the measured approximation of \( \nabla c \).

**Step 4** Setting these estimates equal to the corresponding finite difference approximations of the spatial derivatives of \( c \) results in an overdetermined linear system that can be solved in a least-squares sense to compute the values of \( c \) at the grid points [10].

### A. Simulation Results

We present the simulation results obtained by performing the calibration scheme through steps 1–4. The true calibration function was derived from the estimates of the polarization and diffraction effects on the metrology systems. It is a 3rd-order polynomial in \( u \) and \( v \) plus a small-amplitude sinusoidal perturbation in each direction. Fig. 7 contains the surface plot of the true calibration function. Note that the field-dependent error defined by the true calibration function is included in the delay measurements of the science and guide interferometers. The \( c(u, v) \) in (10) corresponds to the science star field-dependent error whereas the guide star field-dependent errors affect the accuracy of the estimated science baseline, \( b_s \).

We start the calibration procedure by rotating the spacecraft so that the baseline vector \( b \) is collinear to \((1,0,0)\). Then, we select a grid consisting of \( N \times N \) stars in the \( 15^\circ \times 15^\circ \) field of regard \((N = 10)\). Assuming that, the star vector \( s \) is known a priori from the star catalogues within some (known) error bounds. After performing the steps 1–4, we calculate the residual error in the estimate of \( c \).

The accuracy of the characterization of \( c \) is determined by various parameters of the feed forward algorithm. This residual error does not include the linear calibration terms because these terms are inseparable from the baseline length and orientation [11]. In other words, the process of solving for the baseline vector absorbs the linear calibration terms and consequently this has no impact on the determination of the astrometric parameters of the stars. The residual error in the estimation of \( c \) can be expressed as

\[
e_{\text{residual}} = c(u, v) - \hat{c}(u, v)
\]

(20)

where \( \hat{c}(u, v) \) denotes the estimate of the \( c \) calculated using the calibration procedure.

Before presenting the simulation results, we describe each of the error sources that affect the.
accuracy of the PFF signal together with their magnitude estimates.

**Guide Star Position Error**: This error illustrates the second-order effect introduced by the guide star positions. The lack of accuracy in the guide star positions generates an error in the calculation of the spacecraft attitude which leads to a baseline estimation error. Although the star catalogue values include 20 mas error, the instrument can reduce the a priori star position errors by making delay measurements on the calibration stars and updating their positions as well as the baseline vectors [12]. Assuming that the initial error reduction process has taken place, we introduce 3–5 mas position errors in the a priori knowledge of the guide stars.

**Science Star Position Error**: This error represents the a priori knowledge error in the position of the science stars. The magnitude of this error is 2 mas.

**Fiducial Position Error**: As described in the earlier sections, in addition to high precision relative metrology, SIM uses an absolute metrology gauge. The absolute gauge measurements will be used to generate a matrix which converts relative metrology measurements to 3D position changes (Section IID). The accuracy of the absolute metrology gauge will limit the accuracy of the conversion matrix and hence the operating range of the relative metrology gauge. We simulate this effect by introducing an error on the order of 6–15 \( \mu \text{m} \) in the initial knowledge of the fiducial positions.

**Constant Term**: The instantaneous optical pathlength delay that is measured by the interferometer includes the so-called constant term that represents possible optical path differences between the light collected from the target object and the internal metrology. 0.5 microns of constant error is included in the delay measurements of the guide and science interferometers to simulate this effect.

Fig. 8(a) contains the residual error surface over a \( 15^\circ \times 15^\circ \) field of regard generated by performing the calibration scheme through steps 1–4 for the perfect case scenario. The “perfect” means the absence of the measurement errors and the a priori knowledge errors. The residual error (176 pm rms) that is observed in this configuration contains only the numerical and the discretization errors in the computations and represents the lower error bound for the proposed scheme.

Determining the upper error bound for the proposed scheme is a more involved study than the material presented here. It should be studied through Monte Carlo simulations so that a sufficient number of cases are covered to determine the particular effect of each error. This would not be possible with the current speed of the simulation model. Instead we present results from one case study that includes all the errors (see Fig. 8(b)). The rms residual surface increases to 697 pm. This implies that the procedure has the potential of being sufficiently accurate if the errors do not vary significantly from the given values.

The astrometric performance model provides all the required components (e.g., dynamics model, the ACS, and the PFF) for the instrument to perform realistic observational scenarios. This study not only investigates the feasibility of the on-orbit calibration scheme but also demonstrates the utilization of the astrometric performance model in a complicated observational scenario. The model is flexible enough to vary the parameters and explore the design trade space to meet the astrometric requirements.

### B. Cyclic Error Averaging Study

During the observations of SIM, the spacecraft will have some residual motion that is uncompensated by
the ACS. This motion causes the length of the two interferometer arms to change even though the star is stationary. To keep the interferometer centered on the starlight fringe, the delay line position is controlled to maintain equal pathlength between the two arms of the interferometer. This motion is monitored by metrology gauge beams and systematic errors occur due to the nonlinearities of the metrology sensors. These errors can be reduced by averaging the delay measurements during the postprocessing of the science data. In this section, the astrometric performance model is utilized to demonstrate the validity of this assumption.

The metrology gauges have a nonlinearity on the order of a nanometer with periodicity \( \lambda/2 \) (\( \lambda = 1.2 \times 10^{-6} \)), wavelength of light. The instantaneous error in the metrology measurement is modeled as

\[
e = \varepsilon \sin(kx) \tag{21}
\]

where \( \varepsilon \) is the magnitude and \( k = 4\pi/\lambda \) and \( x \) is the delay line motion. These errors are reduced automatically by averaging the gauge signal as the delay line moves through several \( \lambda \). For a given motion of the delay line, say \( x(t) \) over a time interval \([0, T]\), the average error is

\[
E(T) = \frac{1}{T} \int_{0}^{T} \sin(kx(t))dt, \quad x(0) = 0. \tag{22}
\]

Equation (22) shows that it is desirable for \( x(t) \) to be a fast linear motion to reduce the error. In the following case study, \( x(t) \) is determined by the
resulting motion of the delay line due to the residual motion of the SIM spacecraft that is uncompensated by the ACS system. Given the bandwidth of the ACS controller and the disturbance environment, the uncompensated motion of $\sim 1.25$ arcsec is observed for the spacecraft, yielding a 50 $\lambda$ motion of the delay line.

The instantaneous delay for the science interferometer can be written as (assuming that $k, c$ terms are zero)

$$d_s = \langle s_s, b_s \rangle + e_s$$

(23)

where $s_s$ is the science star, $b_s$ is the science baseline vector, and $e_s$ is the cyclic error due to the nonlinearity in the metrology gauges.

The same equation applies to the guide interferometers and the delay equation becomes

$$d_{gs} = \langle s_{gs}, b_{gs} \rangle + e_{gs}.$$  

(24)

In this study, we investigate the attenuation of the cyclic error due to the averaging in (22). This phenomenon is better understood when the cyclic errors are included only in the science interferometer. However, the guide interferometer cyclic errors propagate to the estimated delay in a nonlinear manner and their significance is not clearly understood. The astrometric performance model makes it possible to study these effects easily and provides a better understanding of the instrument behavior under the realistic operational environment. The following steps are taken to collect the data and various averaging methods are carried out after the simulation.

**Step 1** Rotate the spacecraft so that the baseline vector $b_s$ is colinear to (1,0,0). Then select a science star in the field of regard. Assume that, the star vector $s_s$ is known perfectly.

**Step 2** Wait until the large residual amplitudes are attenuated by the ACS system (down to 1.25 arcsec peak-to-peak).

**Step 3** Stay at the nominal position. Make delay measurements using the PFF algorithm and analyze the residual science delay error by integrating over different time intervals. The error in estimating the science delay due to the science and guide cyclic error is defined as

$$e_{\text{delay}} = e_s + \langle s_s, (b_s - \hat{b}_s) \rangle$$

(25)

where $e_s$ is the science cyclic error and $\langle s_s, (b_s - \hat{b}_s) \rangle$ is the error in the estimation of the science baseline due to the guide cyclic errors.

**Simulation Results:** Fig. 9 contains the results from the cyclic error analysis study. The major changes in pathlength delay result from rotating the S/C about the $v$-axis. S/C attitude motion about the $v$-axis is plotted in Fig. 9(d). As can be seen from the figure, the observation time is divided into 10 s increments and the averaging is performed at every 10 s period. Fig. 9(a) shows the averaged residual error when the analysis is performed only with the science interferometer cyclic error. Fig. 9(b) shows the averaged residual error when the analysis is performed only with the guide interferometers cyclic errors. Finally, Fig. 9(c) shows the combined effect of the science and guide interferometers cyclic errors in the averaged residual error.

The numbers associated with each bar on the graph shows the attenuation factor corresponding to that averaging interval. It is obtained by dividing the resultant error by the nominal error ($\varepsilon = 3$ nm) that was chosen for this study. Note that after 10 s of integration time, the error at bar 1 (Fig. 9(a)) is attenuated to about $1/7.72$ times its initial value (e.g., $3 \times 10^{-9}/3.88 \times 10^{-10}$). It can be observed that we can benefit from the averaging methods when the delay line is moving with a fast linear motion (e.g., intervals 3, 4, 6, 7, 8, 9). The adverse effect can be observed for intervals 1, 2, 5, 10 when the delay line is moving slowly. We observe an interesting behavior for intervals 5 and 6. Although the spacecraft is moving about the same amount for both intervals, the resultant error is significantly different. A closer look at interval 5 reveals the fact that the delay line in fact does not move between 246–247 s and 248–249 s and causes a poor cyclic error averaging compared with interval 6.

Fig. 10(a) and (b) show the cyclic error averaging results when the integration time is altered. Fig. 10(a)
IV. DISCUSSION AND CONCLUSIONS

This paper describes the development and utilization of an “astrometric performance model” for the SIM spacecraft and the instrument. The model consists of a real-time dynamics formulation of the spacecraft and a real-time ACS. These are integrated with the major instrument subsystem of the external metrology and guide interferometers in MATLAB Simulink environment to simulate real-time observational scenarios of the instrument. The astrometric performance model can be utilized to verify the astrometric error budget and study the sensitivity of the PFF signal to various error sources.

The first observational scenario that was studied in this paper was the on-orbit calibration procedure. On-orbit calibration is one of the options that is being considered for the current SIM design to correct for the field-dependent errors to meet the astrometric requirements. Results of a particular case study were presented to demonstrate the viability of the on-orbit calibration procedure. The results indicate that the procedure has the potential of being sufficiently accurate and robust. This study also demonstrated the utilization of the astrometric model for predicting the performance of the instrument under various error sources.

The second test that was conducted by the astrometric model was to demonstrate the feasibility of the cyclic averaging concept that is used to reduce the metrology error. The cyclic error is a systematic error that occurs due to the nonlinearity of the metrology gauges. This error affects the accuracy of the PFF signal by polluting the measurement of the guide and science interferometers. However, averaging the final delay output can help to reduce this error in a certain extent. We have demonstrated the fact that this error is highly dependent on the interval that the integration takes place. The delay line motion plays a key role in the reduction of the cyclic errors at that particular interval.

The astrometric model has proven to be a valuable tool for predicting the instrument performance before the instrument is built. The simulation environment allows the navigation of the complex trade space for optimizing the instrument performance and the astrometric error budget. The astrometric model is used in the validation and verification process for the SIM Project. Currently the capabilities to handle various field-dependent and independent errors (e.g., Alignment errors, corner cube errors, thermal and dynamical deformations) are added to the simulation model to validate the instrument performance.
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