1. A scientist researches the survival rate of Caretta Caretta turtles and he thinks that the survival rate of a Caretta Caretta is a **linear-to-linear** function of its age. He knows that their survival rate never exceeds %90 and will approach arbitrarily close to %90 percent if a Caretta Caretta lives long enough. He also observed that at birth the survival rate is %20 and at the age of 1 it increases to %30.

(a) [7pts] Find the survival rate of a Caretta Caretta as a function of its age.

2 (hor asym = 90

$f(x) = \frac{90x + b}{x + d}$

2 ($f(0) = 20$ $\quad f(1) = 30$

$b = 20d$ $\quad 90 + b = 30 + 30d$

solving 2 equations give

3 ($b = 120$

$d = 6$

$f(x) = \frac{90x + 120}{x + 6}$

(b) [3pts] At what age a Caretta Caretta has the survival rate of %75 ?

1 ($f(x) = 75$

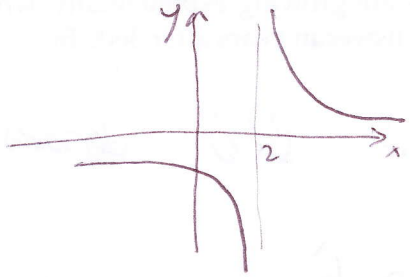
$\frac{90x + 120}{x + 6} = 75$

2 ($x = 22$

2.

$$f(x) = \frac{1}{x-2}$$

(a) [2pts] Sketch the graph of $f(x)$ (Hint: consider it as a linear-to-linear function).



2

hor asym = 2

hor asym = 0

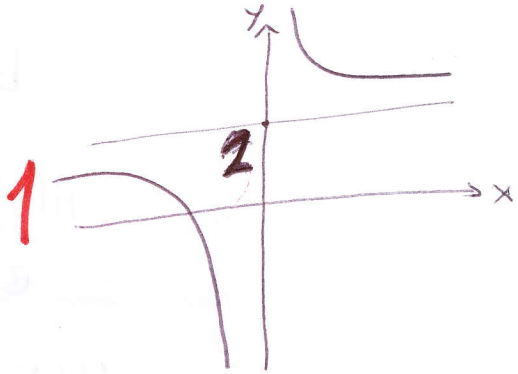
(b) [6pts] find the inverse function $f^{-1}(x)$, sketch the graph of $f^{-1}(x)$ find the domain and the range of $f^{-1}(x)$.

3

$x = \frac{1}{y-2}$

$xy - 2x = 1$

$f^{-1}(x) = y = \frac{1+2x}{x}$



1

1 domain of $f^{-1}(x)$: all real numbers except 0

1 range of $f^{-1}(x)$: all real numbers except 2

(c) [2pts] Finally, calculate the composition $f(f^{-1}(x))$=?

1st way: $f(f^{-1}(x)) = x$ by definition of inverse function

2

2nd way: $\frac{1}{\frac{1+2x}{x} - 2} = \frac{1}{\frac{1}{x}} = x$

3. The population of the city of Hattusa has a doubling time of 150 years. In the Bronze Age at 3000 BC, Hattusa's population was 10,000, while the city of Babylon's population was 5,000. After 500 years, Hattusa's population was 30 percent more than the population of Babylon.

   Assuming the population in both cities are growing exponentially, when will the cities have equal populations? Express your answer in years after 3000 BC.

3

$H(x)$ and $B(x)$ denotes populations.

$H(x) = 10000\, b^x$

$b^{150} = 2 \Rightarrow b = 1.004632$

$H(x) = 10000\,(1.004632)^x$

$B(x) = 5000\, d^x$

3

$H(500) = 1.3 \cdot B(500)$

$10000 \cdot (1.004632)^{500} = 1.3 \cdot 5000 \cdot d^{500}$

$d = 1.0054976$

$B(x) = 5000\,(1.0054976)^x$

-1 if 1.3 not used appropriately

4

Solve for x:

$H(x) = B(x)$

$10000\,(1.004632)^x = 5000\,(1.0054976)^x$

$\left(\frac{1.0054976}{1.004632}\right)^x = 2$

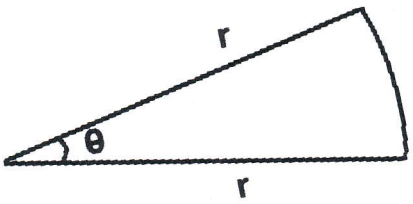
$x \ln\left(\frac{1.0054976}{1.004632}\right) = \ln 2$

$\therefore x \approx 804$ years

(This question is cancelled)

4. Jack has a pizza which shaped like a disc (circular with radius r). He takes one slice of it, the slice is shaped like a circular wedge. The slice has a perimeter of 2 feet, and has an area of $\frac{\pi}{4}$ square feet. → should be $1/4$

What is the value of the radius, r?



(Solution with corrected area $1/4$)

$$2r + \theta r = 2 \Rightarrow \theta = \frac{2-2r}{r}$$

$$\frac{\theta}{2} r^2 = 1/4$$

$$\frac{1}{2}\left(\frac{2-2r}{r}\right) r^2 = \frac{1}{4}$$

$$(1-r)\, r = \frac{1}{4}$$

$$r^2 - r + 1/4 = 0$$

$$r = \frac{-1 \pm \sqrt{1-1}}{-2} = \frac{1}{2}$$

Also note that when area is equal to $\frac{\pi}{4}$ as given then above solution gives a quadratic equation which has no solutions in real numbers.

5. (a) [5pts] Describe how the graph of the function $y = 2(3x - 12)^2 - 5$ differs from the graph of $y = x^2$.

$f(x) = x^2$

$y = 2 \cdot f(3x - 12) - 5$

$y = 2 \cdot f(3(x - 4)) - 5$

1

1 1st — hor dilation by 3 compression

1 2nd — hor shift by +4 units (that is to the right by 4) direction

1 3rd — ver dilation by 2 expansion

1 4th — ver shift by 5 to downwards

(b) [5pts] Solve the following for x

$10^{\log_2(x^2)} = 3$

5

$\log_{10} 10^{\log_2(x^2)} = \log_{10} 3$

$\log_2(x^2) = \log_{10} 3$

$x^2 = 2^{\log_{10} 3}$

$x = \mp\sqrt{2^{\log_{10} 3}}$

$x = \mp 1.179815$

one solution is sufficient for full credit