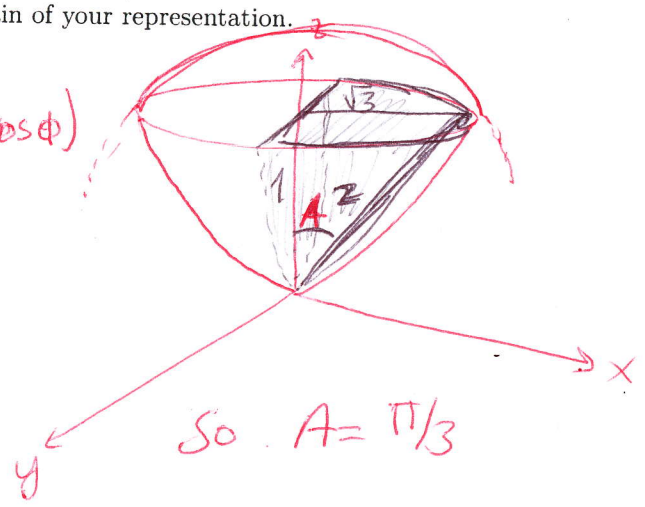
1. (a) Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the set $E = \{(x, y, z) \mid x \geqslant 0,\ x^2 + y^z \leqslant 3z\}$. Write also domain of your representation.

1st way

$r(\theta,\phi) = (2\sin\phi\cos\theta,\ 2\sin\phi\sin\theta,\ 2\cos\phi)$

$\{(\theta,\phi) : -\pi/2 \leq \theta \leq \pi/2,\ 0 \leq \phi \leq \pi/3\}$



So $A = \pi/3$

2nd way

$r(x,y) = (x, y, \sqrt{4 - x^2 - y^2})$

$\{(x,y) : x^2 + y^2 \leq 3,\ x \geqslant 0\}$

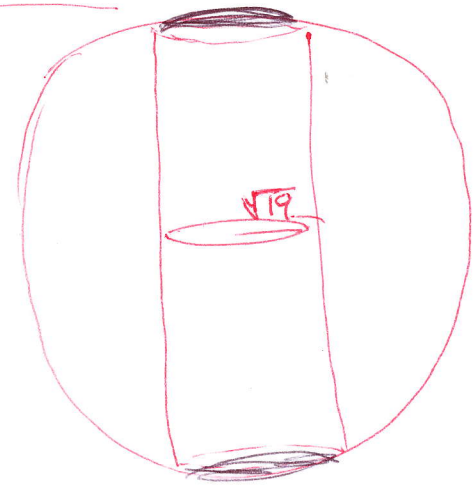
Answer =

(b) Setup an integral that calculates the surface area of the sphere $x^2 + y^2 + z^2 = 100$ that lies within the cylinder $x^2 + y^2 = 19$ in rectangular coordinates **(donot evaluate the integral)** Hint: you can use symmetry.

Area $= 2\int_{-\sqrt{19}}^{\sqrt{19}} \int_{-\sqrt{19-x^2}}^{\sqrt{19-x^2}} \frac{10}{\sqrt{100 - x^2 - y^2}}\, dy\, dx$

$z = f(x,y) = \sqrt{100 - x^2 - y^2}$

$\sqrt{1 + f_x^2 + f_y^2} = \dfrac{10}{\sqrt{100 - x^2 - y^2}}$

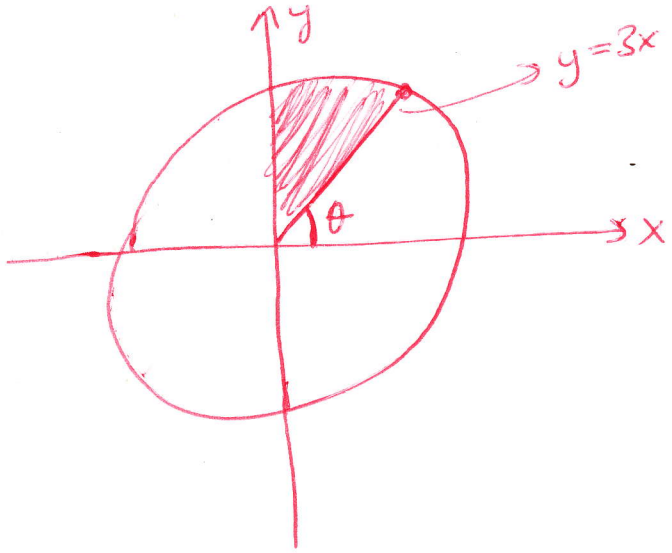


See quiz #1 solutions for details of calculation.

2. Evaluate

$$\iiint_E z\,dV,$$

where E is bounded by cylinder $x^2+y^2=9$ and the planes $x=0$, $y=3x$, and $z=0$ and $z=10$ in the first octant.



In Cyl Coord.

$$\int_0^{10}\int_{1.249_0}^{\pi/2}\int_0^3 z\; r\,dr\,d\theta\,dz.$$

$$= \left(\int_0^{10} z\,dz\right)\left(\int_{1.249_0}^{\pi/2}\int_0^3 r\,dr\,d\theta\right)$$

$\tan\theta = 3 \Rightarrow \theta = 1.249$

Note: this calculates the area of the wedge. Indeed you can also solve without triple integration

$$= 50 \cdot 1.4481$$

$$= 72.4$$

Answer = 72.4

3. A cylinder shaped object C sitting on xy-plane. The axis of the cylinder C is the z-axis, moreover its height is h and its radius is r.

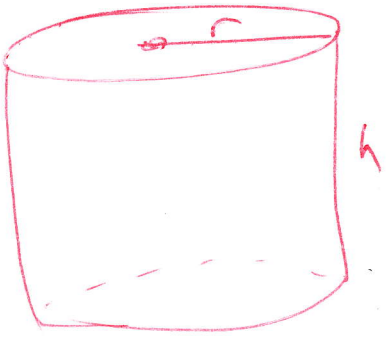
(a) Write down the set that defines C in spherical coordinates.

In spherical coord: $0 \le \rho$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$

$C = \{(\rho, \theta, \phi) \mid 0 \le \rho \sin\phi \le r,\ 0 \le \rho\cos\phi \le h,\ \}$

(b) Suppose object C has a density $\rho = e^{x^2+y^2}$, find its mass.

r is constant.



In Cyl. Coord

$$\int_0^h \int_0^{2\pi} \int_0^r e^{R^2} R\, dR\, d\theta\, dz$$

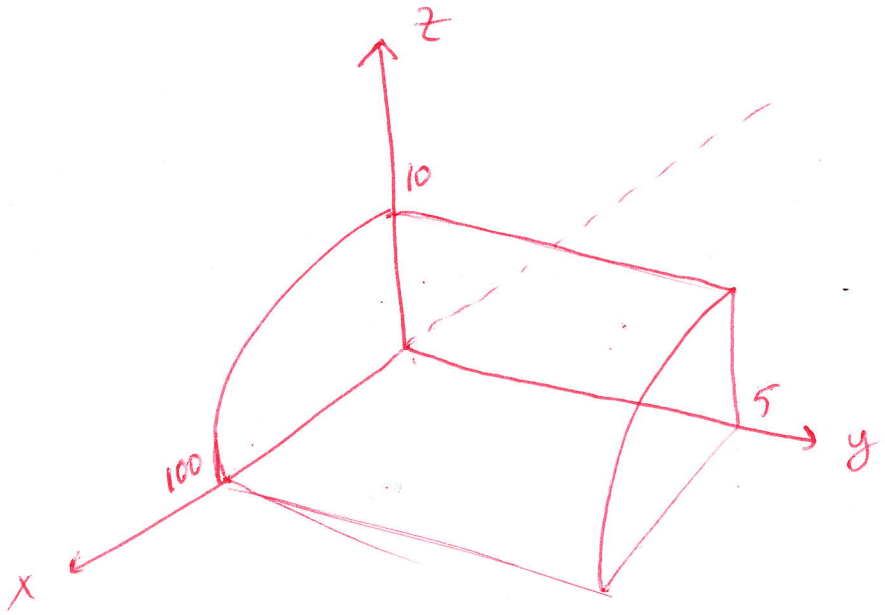
$$= \int_0^h \int_0^{2\pi} \frac{1}{2}\left(e^{r^2} - 1\right) d\theta\, dz$$

$$= \pi h \left(e^{r^2} - 1\right)$$

Mass of C = $\pi h \left(e^{r^2} - 1\right)$

4. Draw the solid that lies in the first octant and bounded by $x = 100 - z^2$, and $y = 5$. Then setup an integral that calculates its volume. The order of integration should be (~~dx dy dz~~) as below. Donot evaluate the integral.

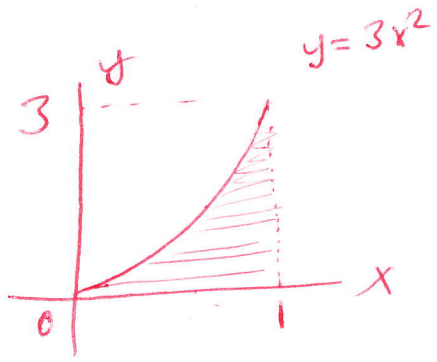
Draw the graph here :



Volume $= \int_{0}^{5} \int_{0}^{10} \int_{0}^{100-z^2} 1 \; dx\,dz\,dy$

5. Evaluate the following integral.

$$\int_0^3 \int_{\sqrt{\frac{y}{3}}}^1 \cos(x^3)\,dx\,dy$$



$$\int_0^1 \int_0^{3x^2} \cos(x^3)\,dy\,dx$$

$$= \int_0^1 3x^2 \cos(x^3)\,dx$$

$$= \sin(x^3)\Big|_0^1$$

Answer = $\sin(1)$