Math 324, Spring 2013. **Quiz #2**

Problem 1 (10 Pts). Let S be the helicoid given by the following parametric equation

$$r(u,\theta) = (u\cos\theta, u\sin\theta, \theta), \qquad 0 \leqslant u \leqslant \sqrt{3}, \qquad 0 \leqslant \theta \leqslant \pi.$$

Calculate the surface integral

$$\int\!\!\int_S y\,\mathrm{d}S$$

$r_u(u,\theta) = (\cos\theta, \sin\theta, 0)$ $\quad r_\theta(u,\theta) = (-u\sin\theta, u\cos\theta, 1)$

$|r_u \times r_\theta| = |(\sin\theta, -\cos\theta, u\cos^2\theta + u\sin^2\theta)| = \sqrt{1+u^2}$

$$\begin{aligned}
\int\!\!\int_S y\,\mathrm{d}S &= \int_0^{\sqrt{3}}\int_0^{\pi} u\sin\theta\,\sqrt{1+u^2}\,\mathrm{d}\theta\,\mathrm{d}u \\
&= \int_0^{\sqrt{3}} 2u\,\sqrt{1+u^2}\,\mathrm{d}u \\
&= \frac{2}{3}(1+u^2)^{3/2}\Big|_0^{\sqrt{3}} \\
&= \frac{2}{3}(2^3 - 1) \\
&= \frac{14}{3}
\end{aligned}$$

Answer = $\frac{14}{3}$

Problem 2 (15 Pts). Answer each by filling in the boxes. Show your work.

(a) Maximum rate of change of $f(x,y)=y\sqrt{x}$ at $(1,2)$ is $\boxed{\sqrt{2}}$

$$\begin{aligned}\nabla f(x,y) &= \left(\frac{y}{2\sqrt{x}},\sqrt{x}\right)\\ \nabla f(1,2) &= (1,1)\\ |\nabla f(1,2)| &= \sqrt{2}\end{aligned}$$

(b) If $z(s,t)=t\cos(s\ \ln t)$, then $\frac{\mathrm{d}z}{\mathrm{d}s}$ is

$\boxed{\frac{\mathrm{d}z}{\mathrm{d}s}= -t\,(\ln t\,)\sin(s\ln t)}$

(c) Let C be the curve given by parametric equations $x=t, y=t^2, z=t^3$, $0\leqslant t\leqslant 2$. Evaluate the line integral

$\boxed{\int_C 3xyz\,\mathrm{d}z = \quad 2^9}$

$$\begin{aligned}\int_C 3xyz\,\mathrm{d}z &= \int_0^2 3tt^2t^33t^2\,\mathrm{d}t\\ &= \int_0^2 9t^8\,\mathrm{d}t\\ &= t^9|_0^2\\ &= 2^9\end{aligned}$$