Math 450 Winter 2017

Homework #4

You don't need to return the question with $*$ sign.

(*) Read 17.7 and 17.9-17.10

(1) Exercises 17.3 Questions 17a, 17b*, 17c (You need to read Example 3 for this question)

(2) Exercises 17.4 Questions 2a, 2b, 2e

(3) Exercises 17.5 Questions 2a, 2b, 3a, 3d, 3f

(4) Exercises 17.5 Questions 4a, 4d

(5) Answer each of the below and verify your answer.

(a) Construct a sequence of functions $s_n(x)$ and a function $s(x)$ such that $s_n(x)$ uniformly converges to $s(x)$ on $x \in [1, 2]$.

(b) Construct a sequence of functions $s_n(x)$ and a function $s(x)$ such that $s_n(x)$ converges to $s(x)$ pointwise on $x \in [1, 2]$ but does not converge uniformly.

(c) Let $h(x)$ be defined on $x \in [0, 2]$ by

$$h(x) = \begin{cases} 0 & x < 1 \\ 1 & x = 1 \\ 2 & x > 1 \end{cases}$$

Is it possible to construct a sequence of continous functions which converges uniformly to $h(x)$ on $(0, 2)$. If so construct one, if not explain.

(d) Let $h(x)$ be as in part (c). Is it possible to construct a sequence of continous functions which converges pointwise to $h(x)$ on $(0, 2)$. If so construct one, if not explain.