Math 450 Winter 2017

Homework #6

You don't need to return the question with $*$ sign.

(*) Read 18.6, 19.1 -19.3

(1) Dowload heat.m matlab code from class website and run it. Explain what is happening in the very first part (heuristically): Which part is changing fast and why ?

(2) Exercises 18.3 Questions 5, 9

(3) Answer the following for the heat conduction problem for a rod which is modelled by

$$\begin{aligned} \boldsymbol{L}[u] &= \alpha^2 u_{xx} - u_t = 0 \\ BC &: \quad u(0,t) = u_1, \quad u(L,t) = u_2, \quad 0 < t < \infty \\ IC &: \quad u(x,0) = f(x), \quad 0 < x < L \end{aligned}$$

(a) Set $u_1 = 10,\ u_2 = 30,\ f(x) = x$ and find $lim_{t\to\infty} u(\frac{L}{3}, t)$

(b) Set $u_1 = 10,\ u_2 = 30,\ f(x) = H\left(x - \frac{L}{2}\right)$ and find $lim_{t\to\infty} u(\frac{L}{3}, t)$. Explain why the answer you found is different (or equal) to the answer of part (a).

(c) Set $u_1 = 0,\ u_2 = 0,\ f(x) = sin(\frac{2\pi}{L}x)$. Suppose that $L = 100$ and there are two mosquitos sitting at $x = 0$ and $x = 0.1$, respectively. Let's assume mosquitos can feel any change in temperature. When will each mosquito feel any temperature change (and specify the change: cooling vs warming) ? Verify your answer mathematically.

(4) Solve the following problem

$$\begin{aligned} \boldsymbol{L}[u] &= u_{xx} - u_t = 0 \\ BC &: \quad u_x(0,t) = 0, \quad u_x(L,t) = 1, \quad 0 < t < \infty \\ IC &: \quad u(x,0) = f(x), \quad 0 < x < L \end{aligned}$$

(5) Exercises 18.3 Question 15

(6) Exercises 18.4 Question 1, 6, 10