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Bargaining and exclusivity in a borrower–lender relationship

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Abstract This paper presents a stylized model of a borrower–lender relationship where funds are gradually invested in a project with uncertain return. We show that an exclusive financing relationship arises endogenously in equilibrium due to initial lender's superior information on the project's progress. The analysis also identifies a novel distortionary effect of exclusivity and the consequent loss of future rents on the ex-ante choices of the borrower. When she chooses the amount of funds to be initially invested in the project, the borrower chooses to overinvest making the future rent extraction by the initial lender as costly as possible.

Keywords Relationship financing · Exclusivity · Adverse selection · Bargaining · Endogenous outside options

JEL Classification C78 · D82 · G24 · G32

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1 Introduction

Cash constrained project owners often raise external financing from lenders who can monitor the progress of their projects. Such relationship financing typically involves an insider lender who establishes close ties with the borrower after an initial funding and receives information on the future prospects of the project. The presence of an insider like a venture capitalist or a bank specialized in 'relationship financing' can resolve agency problems: the insider can act as an information intermediary in future financing rounds between the borrower and other financiers and mitigate the borrower's tendency to overinvest in a failing project (Admati and Pfleiderer 1994).

By tying future investment to the project's interim progress, the information generated by insiders serves as a discipline device for the borrower (Sahlman 1990). Despite this potential benefit, the close and exclusive involvement of an initial lender can also be costly from the borrower's point of view. Ideally, the borrower would like to have access to competing sources of financing and avoid an exclusive relationship with only one lender. In this paper, we develop a simple stylized model of a borrower–lender relationship where exclusive financing arises as an equilibrium feature due to the informational advantage of the initial lender. The project specific information that the initial lender acquires in his monitoring activities gives him bargaining power over the future financing round. Due to this superior information alone, the borrower continues to raise financing exclusively from the initial lender and leaves him surplus despite the availability of competitive outside lenders at the interim stage.

The model we present considers a project owner (the borrower) who needs financing to undertake a project with an uncertain return. The investment in the project is spread over two periods. Following an initial investment, which is optimally chosen by the borrower, the lender who provides initial financing and the borrower observe an information signal on the project's profitability. The two parties then bargain over the continuation terms to complete the project. The initial contract between the borrower and the initial lender does not impose any exclusivity clause. The initial lender may agree to provide the continuation funds or he may deny further financing. The borrower can also opt out from the bargaining table and seek all the continuation financing from competitive outside lenders. Since the outsiders do not observe the information signal, however, the terms they offer and hence the value of the borrower's outside option is determined *endogenously* by their beliefs on the signal realization.

The borrower's bargaining with a symmetrically informed inside lender and competitive but uninformed outside lenders generates an equilibrium with an exclusive financing relationship. In all cases where there is further investment, the initial lender is the party that provides financing and extracts part of the continuation surplus. The new information revealed to parties during the relationship creates an exit barrier to the borrower: the relationship *becomes* exclusive. We also analyze how the anticipation of an exclusive relationship with the initial lender affects the amount of initial investment optimally chosen by the borrower. Due to its effect on the subsequent bargaining game for continuation financing, the initial investment level becomes a strategic choice for the borrower: she makes the initial lender invest as much as possible subject to a participation constraint. Consequently, there is overinvestment in the project before information revelation. This overinvestment result is novel and complements a well known underinvestment result by Rajan (1992). By considering a setting where the borrower chooses the amount of funds to raise, not the effort as in Rajan, we show that the anticipation of an exclusive financing relationship causes too much initial investment whereas in Rajan it causes underinvestment in effort.

We should also note that our analysis of the bargaining game between the borrower and the initial lender may be of interest in itself. We model this bargaining situation as an alternating offers bargaining game with outside options where the value of the borrower's outside option is determined by the beliefs of outside lenders who can provide continuation funds. An important aspect is that the outsiders do not observe the information revealed to the insiders, and therefore their beliefs are determined endogenously in equilibrium by the bargaining behavior of the insiders. It turns out that the availability of an outside option for the borrower (i.e., the fact that there is no exclusivity clause between the borrower and the initial lender) becomes completely irrelevant in determining the bargaining outcome: through a mechanism similar to the one present in the 'lemons problem' the borrower becomes informationally captured by the initial lender.

Related literature The multi-stage financing problem we study relates our paper to the relationship banking literature as it meets all the three criteria of relationship banking as identified by Boot (2000): the relationship financier receives information beyond what is publicly available; information gathering takes place over time through multiple interactions with the borrower; and finally this information remains private. The closest paper in this literature is the seminal paper by Rajan (1992). In that paper, however, the bargaining game in the second stage is not explicitly modeled. Exclusive financing and surplus extraction by the initial lender may occur with exogenously specified beliefs by outside financiers: the outsiders cannot condition their beliefs regarding the profitability of the project on any action taken by the insiders. In the equilibrium Rajan describes the borrower may switch to other lenders, whereas in our model exclusivity occurs with probability one. Another difference is that Rajan focuses how the loss of rents in the future distorts the borrower's effort incentives in the first period (it causes too little effort), whereas we describe how it distorts the amount of initial investment (too much capital is raised and invested initially).

The insider lender in our model can also be viewed as a venture capitalist who becomes involved with the project's progress. However, we should note that our model is too stylized to cover the rich set of roles venture capitalists play as inside investors [for a detailed description of the role of venture capitalists as informed investors, see Kaplan and Stromberg (2003) and Hege et al. (2003)]. The most closely related paper in the venture capital literature is Admati and Pfleiderer (1994). They focus on the problem of truthful revelation of interim information to outsiders when financing of the project is staged. By committing to provide a fixed fraction of continuation funds in exchange for a fixed fraction of project returns, the insider venture capitalist in Admati and Pfleiderer (1994) acts as an information intermediary and ensures optimal continuation. The presence of the insider in their paper credibly signals private information to outsiders and lures them in the project. Although they do not study it, they also mention the possibility of the insider extracting part of the continuation surplus due to his superior information, which is the focus of our paper.

This paper is also related to a literature which focuses on multi-stage financing problems. In Neher (1999), the investor faces a hold-up problem and adopts a gradual investment schedule to build up a collateral, gradually improving her bargaining position. Similarly, Pitchford and Snyder (2004) analyze a hold-up problem where the static game has no equilibrium with positive investment, whereas there is an equilibrium in the dynamic game with the investor making gradually decreasing (in contrast to Neher's gradually increasing investment sequence) investments. In Wang et al. (2004) multi-stage financing serves as a mechanism to control risk and mitigate moral hazard.

The plan of the paper is as follows. The next section describes the model. Section 3 presents the analysis. Section 4 considers some extensions of the basic framework. Section 5 concludes.

2 The model

In this section, we describe a stylized model of a borrower–lender relationship.

2.1 Agents, payoffs and timing

There are three dates, $T = 0, 1$, and 2. At date 0, a risk neutral borrower has a project that requires a fixed investment of \$1. If the project is undertaken, an initial investment of $x \in (0, 1)$ is made at date 0. Subsequently, at date 1 an information signal on the project's prospects is revealed, and the parties involved decide whether to complete the project or not. If completed, at date 2 the project generates a stochastic gross return Y, which is a non-negative random variable with distribution function F. We denote the expected value of Y with respect to F by $\mathbb{E}[Y]$.

The borrower (B) does not have any funds to finance the project and hence needs external financing from a lender (L). The capital market is composed of a continuum of risk neutral lenders who require a competitive rate of return normalized to zero. Therefore, at date 0, the borrower has monopoly over the particular project in question when she takes it to a lender . B has a discount factor $\beta_B \in (0, 1)$ and L has a discount factor $\beta_L \in (0, 1)$ across dates 0, 1, and 2. In order to describe a multi-period relationship between B and L, we make the following assumptions.

(A1) The completion of the project is spread over two periods. B initially chooses to raise only a fraction $x \in (0, 1)$ of the required capital. The initial investment x is publicly observable.

The initial investment takes the project to an interim stage where more information on its profitability is revealed. In particular, at date 1 the two parties, B and the lender L who provided the first period investment observe a payoff relevant information signal S which is a non-negative random variable jointly distributed with Y. Let G denote the signal's marginal distribution, $F(y|s)$ the conditional distribution of Y given $S = s$ and $\mathbb{E}[Y|s]$ the conditional expectation of Y given $S = s$. We assume that

(A2) For any strictly increasing function $u : \mathbf{R}_+ \to \mathbf{R}_+$,

$$\int_{\mathbf{R}_+} u(y)\, dF(y|s') > \int_{\mathbf{R}_+} u(y)\, dF(y|s) \quad \text{whenever } s' > s.$$

The above assumption is a first order stochastic dominance condition which implies that $\mathbb{E}[Y|s]$ is strictly increasing in s and therefore a higher realization of s implies better news on the prospects of the project.
(A3) The information signal S is not verifiable and it is privately observed only by B and the lender L who provided the first period investment.

The above assumption describes a situation where the two parties involved in the project in the first period obtain some soft information on the project's profitability. This might be information on the inherent quality of the project undertaken or the revelation of the ability of the borrower who runs the project.

In order to obtain the payoff Y from the project, which is to be realized at date 2, the remaining funds $1 - x$ has to be invested in the second period. This implies that following an initial investment of x and a signal realization s, the expected continuation surplus, from the point of view of the lender, is

$$R(s, x) \equiv \beta_L \mathbb{E}[Y|s] - (1 - x).$$

$R(s, x)$ is strictly increasing in the signal s and the initial investment x. We further assume that
(A4) For all $x \in (0, 1)$, there exists an $s^*(x) > 0$ such that $0 < G(s^*(x)) < 1$ and $R(s^*(x), x) = 0$.

This assumption, together with Assumption 2, implies that there exist high enough signal values that makes continuation the efficient choice. Also, the distribution of the signal is assumed to be independent from the initial investment or any action possibly taken by the borrower and the lender. In that sense, we abstract away from any moral hazard and asymmetric information issues between the two parties involved in the project.[1]

An important implication of the non-verifiability of the signal is that the two parties cannot write an enforceable clause in the date 0 contract that specifies a continuation or termination decision at date 1, contingent on the signal realization. The initial contract can only specify the first period investment x and payoffs to parties if the project is abandoned without further investment at date 1. L cannot make any ex ante signal contingent commitments to provide further financing or to abandon the project. Similarly, B cannot make any commitments to raise further financing exclusively from the initial lender. She can seek continuation financing from outside lenders. The novel modeling aspect we introduce is that the continuation decision and the source and the terms of continuation funds are determined by the bargaining between the B and L following the signal realization at date 1. We describe this bargaining situation next.

[1] For a model where the borrower manipulates the signal (window dressing) to secure continuation funds, see Cornelli and Yosha (2003). In Bergemann and Hege (1998) only the borrower observes the true interim state. In Rajan (1992), the likelihood of the good state depends on the borrower's effort. Bergemann and Hege (2005) consider a model where the entrepreneur, by controlling the allocations of funds into the project, also controls how fast the information on the project is revealed.

2.2 Bargaining over continuation

Upon observing the information signal, B and L are in a bilateral bargaining situation as two symmetrically informed parties. In case they choose to continue the project together, L provides $1 - x$ and the two parties specify a sharing rule over the final payoff. Both parties, however, can opt out from bargaining. The bargaining game we analyze is a standard alternating offers bargaining game with outside options. A novel feature is that the value of B's outside option is not exogenously given, but it is determined endogenously in equilibrium. The specifics of the bargaining game is as follows:

At date 1, a history is specified by the initial investment $x \in (0, 1)$. We denote the bargaining game after any such history by $\Gamma(x)$. Upon observing the signal s, the game starts with B making the first offer $\pi_B^0(Y) \in [0, Y]$. To this offer, L responds by accepting, opting out, or rejecting. [2]

Opting out by the initial lender If L opts out, he denies further financing and liquidates the project. In case of liquidation, the uncompleted project is sold at a scrap value all of which accrues to L. For simplicity we normalize the scrap value to zero. However, relaxing this assumption does not change our results in any important way. As we show later (see Remark 2), all we need is that when continuation is profitable, the scrap value of the project is never higher than the true continuation value.

If L rejects B's offer, he makes a counter-offer next period, $\pi_L^1(Y) \in [0, Y]$, to which now B responds by accepting, opting out or rejecting. If B rejects, she makes an offer $\pi_B^2(Y) \in [0, Y]$ at period 2 and the game continues in this fashion ad infinitum with B making an offer $\pi_B^t(Y)$ in periods $t = 0, 2, \ldots$, and L making an offer $\pi_L^t(Y)$ in periods $t = 1, 3, \ldots$. If payoff state at date 2 is $Y = y$ and the offer $\pi_i^t(Y)$ of $i \in \{L, B\}$ is accepted by $j \neq i$, then player i receives $\pi_i^t(y)$ at date 2. If neither party ever accepts an offer or ever opts out, both players receive zero payoff. Denote the amount of time that passes between each offer by $\Delta > 0$, and the discount rate for player $i \in \{L, B\}$ by $r_i > 0$. We define $\delta_i = e^{-r_i \Delta}$ as player i's discount factor in the bargaining game.

Opting out by the borrower If B opts out when responding to L's offer, he invites offers from outside lenders. In this case, we assume that L receives a zero payoff (see Remark 3). In exchange for providing continuation capital $1 - x$, the outside lenders make offers $b(Y) \in [0, Y]$ to B which specifies the amount that B is to receive if the project generates a payoff Y at date 2.[3] The outside lender whose offer has been accepted by B provides the continuation capital $1 - x$ and receives an expected payoff of $\beta_L \mathbb{E}[Y - b(Y) | I] - (1 - x)$ where I denotes the outsiders' information set which contains the event that B has opted out from bargaining with the initial lender. [4]

[2] There would be no qualitative change in our results if the lender were to be the first to make an offer.

[3] Notice that we impose a limited liability restriction, which requires that the borrower's payoff $b(Y)$ cannot be smaller than zero or greater than the realized payoff of the project.

[4] Our analysis of the bargaining game assumes that the outsiders do not observe the entire history of offers and accept/reject decisions of the bargaining parties. Our results do not change if this assumption is relaxed.

3 Analysis

We now solve the model backwards starting first with the bargaining game at date 1. Recall that the information signal is observable only to B and to the lender L who provides the initial funding. Therefore, if B opts out and seeks continuation financing from outside financiers, the terms of continuation financing (the outside offers she will receive) depend on the beliefs of the outsiders on the signal realization. Consequently, the value of B's outside option is determined by the equilibrium opting out behavior. The equilibrium concept that we employ is that of Perfect Bayesian equilibrium (PBE). Since the signal is unobservable, the outsiders can only condition on the event that B opts out in equilibrium, *as well as the initial contract* x, in forming their beliefs on the signal realization and hence their beliefs on the final payoff of the project. Since the only event in which the outsiders need to take an action, and hence their beliefs matter, is when B opts out and since they do not observe anything but that B has opted out in this event, there is only one information set of the outsiders, which we denote by I. For any function $u : \mathbf{R}_+ \to \mathbf{R}_+$, we denote the expected value of $u(Y)$ conditional upon observing B opting out by $\mathbb{E}[u(Y)|I]$.

We first analyze how the values of the outside options are determined in equilibrium. The characterization of the opting out behavior of L is straightforward, since outsiders do not take any action if L opts out and liquidates the project.

Proposition 1 *In any PBE of* $\Gamma(x)$, *L liquidates and opts out on the equilibrium path if* $s < s^*(x)$ *and does not liquidate if* $s > s^*(x)$ *.*

Proof Fix an initial investment level $x \in (0, 1)$ and suppose that $s < s^*(x)$. Then, the most that L can get by continuing bargaining is the whole expected continuation surplus $R(s, x) < R(s^*(x), x) = 0$, whereas he can get zero if he opts out and liquidates. Therefore, if $s < s^*(x)$ then L liquidates on the equilibrium path. Suppose now that $s > s^*(x)$ but L opts out on the equilibrium path at time t after observing s. This implies that the payoff from accepting the equilibrium offer of B, $\pi_B^t(Y)$, must be smaller than his outside option of zero. Note that for $s > s^*(x)$, we have $R(s, x) > 0$. But, then B, upon observing a signal $s > s^*(x)$, could offer $\widehat{\pi}_B^t(Y)$ with $\mathbb{E}\left[\widehat{\pi}_B^t(Y)|s\right] = R(s, x)/2\beta_L > 0$ at time t, which L would certainly accept. This would yield B a strictly positive payoff rather than her equilibrium payoff of zero if L liquidates. Therefore, $\pi_B^t(Y)$ cannot be an equilibrium offer, yielding a contradiction. □

Remark 1 Proposition 1 and the fact that the equilibrium value of the outside option of L is equal to zero continue to be true, even if we were to assume that the liquidation value, rather than being zero, is a function $Q(Y, x)$ with $Q(Y, x) \leq \beta_L Y - (1 - x)$, for all $x \in (0, 1)$. This condition requires that the liquidation value can never be higher than the true continuation surplus. One can justify such an assumption by the human capital value of B in the project. In case of liquidation, B no longer works for the project. Then, the condition $Q(Y, x) \leq \beta_L Y - (1 - x)$ merely means that the new owners of the uncompleted project who pay for the scrap value cannot generate a payoff higher than the true continuation value of the project with B.

We now turn to the equilibrium value of the outside option of B. If B opts out, the outside lenders make competitive offers $b(Y) \in [0, Y]$ which specify B's payoff for each payoff state. In exchange for providing the continuation funds $1 - x$, the outside lender holds a claim which pays her $y - b(y)$ if the payoff realization at date 2 is $Y = y$. Our next result shows that, in any equilibrium in which $y - b(y)$ is strictly increasing in y,[5] the equilibrium value of B's outside option is zero, regardless of the information signal s.

Proposition 2 *In any PBE of* $\Gamma(x)$ *in which* $y - b(y)$ *is strictly increasing,* (a) *B opts out on the equilibrium path if* $s < s^*(x)$, (b) *B does not opt out on the equilibrium path if* $s > s^*(x)$, (c) *The equilibrium outside offer is* $b(y) = 0$ *for all* y. *Therefore, the equilibrium value of B's outside option is zero for every signal realization.*

Proof See the Appendix. □

The main idea is that a borrower with a high signal suffers from adverse selection in the sense that if she were to attempt to seek financing from outside lenders, she would be believed by the outsiders to have received a signal low enough to make the expected continuation surplus negative from the perspective of outsiders. For this reason, the borrowers with a signal $s > s^*(x)$, i.e., precisely the ones for whom continuation is profitable, never opt out on the equilibrium path.

Remark 2 So far we have assumed that if the borrower receives continuation funds from an outside financier, the initial lender receives a payoff of zero. It would be more natural to assume that the initial lender still holds a claim over the project's return, even if he does not participate in continuation finance. Let us denote this claim by $\pi_0(Y) \in [0, Y]$ and assume that at date 0 the borrower and the lender bargain over $\pi_0(Y)$ as well as the initial investment, x. Let us also assume that this claim is observable to the outside parties. It is easy to show that Proposition 2 goes through without modification for any PBE in which $y - \pi_0(y) - b(y)$ is strictly increasing. In other words, the borrower's outside option at date 1 is zero for any signal realization and therefore she never opts out in equilibrium. The rest of our results do not change either, since the borrower never receives outside funding and thus $\pi_0(y)$ is never exercised in equilibrium.

Remark 3 An alternative bargaining specification would be allowing B to make an offer to the outside lenders when she opts out, rather than the outsiders making the offers. In this case, B might have a chance to signal her information through the offer she makes. Even if B is not able to reveal her information fully, she may still try to signal that she has received a signal $s > s^*(x)$, which makes continuation profitable. We analyze this possibility in Sect. 4 and show that our full exclusivity result goes through under this specification of the bargaining game as well.

It is important to emphasize the difference between the way we model the continuation bargaining game and the way it is modeled in Rajan (1992). In his paper, after the signal is revealed to B and the initial lender (the insider bank in Rajan), B asks both the informed initial lender and an uninformed outsider to simultaneously submit a sealed

[5] This would hold, for instance, in case of linear sharing rules where the offers of the outsiders are of the form $b(y) = \alpha y$ for some $\alpha \in [0, 1]$.

bid for continuation financing. This is an auction setting with simultaneous bidding by an informed and an uninformed bidder. A generic result is that due to winner's curse, no equilibrium exists in pure strategies. In equilibrium, borrowers may switch to other lenders. In contrast, in our model exclusivity occurs with probability one. Another modeling difference is that the auction setting in Rajan (1992) does not allow for the outsider to condition his beliefs regarding the profitability of the project on any action taken by the insiders. Therefore, his beliefs that determine his bid remain *exogenous*. In contrast, in our bilateral bargaining setting, the outsiders' beliefs are determined in equilibrium by the equilibrium strategies and Bayes Rule, and the mechanism through which exclusivity occurs is 'adverse selection' rather than the 'winner's curse'.

Now we can characterize the equilibrium payoffs of the date 1 bargaining game. Let $V_i^1(x, s)$ denote the equilibrium payoff of player $i \in \{L, B\}$ of the bargaining game $\Gamma(x)$ conditional upon the signal s. The following result shows that the unique equilibrium outcome is such that the project is liquidated for signal realizations below $s^*(x)$. Above that threshold level, the initial lender provides the continuation financing and captures part of the expected continuation surplus.

Proposition 3 *Let* $\eta \equiv r_L/(r_L + r_\beta)$. *The unique equilibrium payoffs of* $\Gamma(x)$ *conditional upon signal* s *is given by*

$$V_L^1(x, s) = \begin{cases} (1-\eta) R(s, x), & s > s^*(x) \\ 0, & s \leq s^*(x) \end{cases}$$

$$V_B^1(x, s) = \begin{cases} \eta \frac{\beta_B}{\beta_L} R(s, x), & s > s^*(x) \\ 0, & s \leq s^*(x) \end{cases}$$

as $\Delta \to 0$.

Proof See Appendix. □

We can now solve for the date 0 contract that specifies the initial investment level x. From Proposition 3, we know that for signal realizations above $s^*(x)$, L provides the remaining $1 - x$ and captures $(1-\eta)$ of expected continuation surplus, whereas B obtains the remaining η of continuation surplus. Since $s^*(x)$ is a function of initial investment x, one can write down the ex ante (date 0) payoff functions $V_L^0(x)$ (for L) and $V_B^0(x)$ (for B) as follows:

$$V_L^0(x) = (1-\eta)\beta_L \int_{s^*(x)}^{\infty} R(s, x) dG(s) - x, \quad (1)$$

$$V_B^0(x) = \eta \frac{\beta_B}{\beta_L} \int_{s^*(x)}^{\infty} R(s, x) dG(s). \quad (2)$$

The following Lemma, which is instrumental in characterizing the equilibrium initial investment, describes how date 0 payoff functions depend on the initial investment level x chosen by B.

Lemma 1 *The date* 0 *value function of L (B) is strictly decreasing* (*increasing*) *in the initial investment level* x.

Proof See Appendix. □

The intuition behind this proposition is as follows: First, notice that $s^*(x)$ is strictly decreasing in $x \in (0, 1)$, as $\mathbb{E}[Y|s]$ is strictly increasing in s (by Assumption 2) and $(1-x)/\beta_L$ is strictly decreasing in x. Consequently, higher initial investment levels shift $s^*(x)$ to the left and makes continuation more likely. The expected continuation surplus, $R(s, x)$, increases in x for a given s. Therefore, the ex ante expected continuation surplus $\int_{s^*(x)}^{\infty} R(s, x)dG(s)$ increases as x increases.[6] Note also that a higher initial investment x has no cost for B, and hence her expected value at date 0 is increasing in x. On the other hand, an increase in x increases the cost for L by an equal amount, whereas the benefit increases by less, because L can extract only a portion of the date 1 continuation surplus. For this reason, the net expected value of L at date 0 is decreasing in x.

Lemma 1 implies that, in designing the date 0 equilibrium contract, B will choose the highest possible initial investment level that satisfies L's ex ante (date 0) individual rationality constraint. In other words, *due to its effect on the date* 1 *bargaining game, the initial investment level* x *becomes a strategic variable for B*. Exclusivity, which arises endogenously during the course of the relationship, gives L an implicit option to further invest and extract surplus. By making this implicit option as costly as possible (through the highest possible level of initial investment), B forces L down to his reservation level. Our next result characterizes the equilibrium initial investment level x^*.

Proposition 4 *There is a unique equilibrium investment level* $x^* \in (0, 1)$ *characterized by*

$$x^* = \beta_L (1-\eta) \int_{s^*(x^*)}^{\infty} R(s, x^*)dG(s). \tag{3}$$

B's equilibrium payoff is strictly positive and is given by

$$V_B^0(x^*) = \frac{\eta}{1-\eta} \left(\frac{\beta_B}{\beta_L}\right)^2 x^* > 0$$

Proof See Appendix. □

The equilibrium initial investment level x^* described in Eq. (3) is such that the present value of L's future expected continuation rents (right hand side) is equal to cost of starting the relationship today (initial investment). The more surplus L expects to extract in the future, the higher is the initial investment before any information is revealed.

[6] The effect of the decrease in $s^*(x)$ is negligible for small increases in x, since at $s^*(x)$ the continuation value $R(s^*(x), x)$ is equal to zero. A small increase in x leads to a higher expected continuation value only through its effect on $R(s, x)$.

An important question is whether the equilibrium level of the initial investment is socially optimal. To this end, let us define the expected social surplus at date 0 as

$$V_s^0(x) = \beta_s \int_{s^*(x)}^{\infty} \big(\beta_s \mathbb{E}[Y|s] - (1-x)\big) dG(s) - x,$$

where $\beta_s \in (0, 1)$ is the social discount factor. In the current setting the socially efficient investment level is not well defined, as V_s^0 is strictly decreasing and $x = 0$ can never be socially optimal. Nonetheless, Proposition 5 implies that, from the society's point of view, there is overinvestment in equilibrium.

Corollary 1 *The equilibrium level of initial investment is higher than the socially optimal level.*

Proof The social surplus is strictly decreasing in the level of initial investment, and hence there is a positive investment level that is smaller than x^* which yields a higher social surplus than does x^*. □

It is illustrative to discuss our overinvestment result in reference to the underinvestment result in Rajan (1992). In Rajan's model, the initial investment level that takes the project to the interim stage is fixed. What determines the ex ante likelihood of the good state is B's effort following this fixed investment. The project is continued at the good state, but the informed lender extracts part of this continuation surplus. Anticipating that she will only capture part of the benefits from her effort, B underinvests in effort. In our setting, the decision variable at date 0 is not B's effort, but the amount of capital to raise and invest. This initial investment level has no effect on the realization of the information signal, but for every signal realization, it determines the size of the continuation surplus. Since the entrepreneur anticipates exclusivity and the consequent loss of continuation surplus at date 1, she makes this rent extraction as costly as possible to the lender by making him overinvest. The initial investment is a strategic variable in our setting, not because it affects the ex ante likelihood of different information states, but because it is essentially the price of becoming an insider and extracting rents in the future.

4 Extension: signaling

In our specification of the bargaining game at date 1, we assumed that if B opts out, it is the outsiders who make offers to her. Alternatively, B could make contract offers $\pi_s(Y)$ (interpreted as the payoff B would get from the final payoff) to the outsiders. This brings about the possibility that she might be able to communicate the signal realization, and hence capture the entire expected surplus. In any equilibrium of this modified game that satisfies a similar monotonicity condition, i.e., $y - \pi_s(y)$ is strictly increasing in y, the value of the outside option of B is zero: borrowers who receive signals $s < s^*(x)$ opt out and receive a zero payoff, whereas those with signals $s > s^*(x)$ do not opt out.

To see this, first notice that if there is an equilibrium in which B opts out with an offer that is accepted by the outsiders after some signal realizations, then each of these offers must be different: equilibrium must be fully separating. This follows since for each signal s after which B opts out with an offer $\pi_s(Y)$ that is accepted, it must be the case that

$$\mathbb{E}\left[\pi_s(Y)\,|s\right] \geq \frac{R(s,x)}{\beta_L},$$

for otherwise, the initial lender L could make a more competitive offer which B would accept rather than opting out. Therefore, if there are two signals $s' > s''$ where the same offer $\pi(Y)$ is made, we would have

$$\mathbb{E}\left[Y-\pi(Y)\,|s\in\left\{s',s''\right\}\right]-\frac{1-x}{\beta_L} < \mathbb{E}\left[Y-\pi(Y)\,|s'\right]-\frac{1-x}{\beta_L} \leq 0$$

by Assumption 2, contradicting that the offer $\pi(Y)$ is accepted. Consequently, there is no equilibrium in which a borrower with a signal $s > s^*(x)$ opts out with an offer that is accepted. If there was such an equilibrium, then it would have to be fully separating and we would have

$$\mathbb{E}\left[\pi_s(Y)\,|s\right] = \frac{R(s,x)}{\beta_L} > 0.$$

Such an offer, however, would always be mimicked by borrowers with signals smaller than $s^*(x)$, who receive zero payoff in equilibrium and hence they have an incentive to make an offer which pay positive amounts at some states. This, in turn, would contradict with the hypothesis that $\pi_s(Y)$ is accepted in equilibrium. Therefore, the equilibrium outcome that is specified in Proposition 3 is robust to signalling opportunities by B.

5 Conclusion

In this paper, we present a simple model of a borrower–lender relationship and show how the information revealed to parties in the course of the relationship can act alone as an exit barrier and result in an exclusive relationship. The novelty of our analysis is that we explicitly analyze a bargaining game where the value of the outside option for the borrower (her ability to raise further financing from outsider lenders) is determined endogenously. Due to the informational asymmetry between the borrower and potential outside lenders, we show that the borrower is informationally captured by the initial lender and the relationship becomes exclusive. One other interesting insight that emerges from the analysis is that anticipation of an exclusive financing relationship distorts the initial investment to start the relationship. Since the borrower anticipates exclusivity and the consequent loss of surplus in the future, she makes this rent extraction as costly as possible to the lender by making him overinvest to start the relationship.

6 Appendix

Proof of Proposition 2 Part (a) is easy to see as in this case the expected surplus, and hence the most that B can get by continuing bargaining, is negative, whereas she can get at least zero by opting out. Therefore, upon observing a signal $s < s^*(x)$, B opts out.

To prove (c), recall that the outsiders market is competitive and the required rate of return is zero. Therefore, if $\beta_L \mathbb{E}[Y|I] - (1-x) < 0$, then the equilibrium offer by the outsiders is $b(y) = 0$, for all y. If, on the other hand, $\beta_L \mathbb{E}[Y|I] - (1-x) \geq 0$, then the equilibrium offer $b(Y)$ by the outsiders satisfies

$$\mathbb{E}[b(Y)|I] = \frac{\beta_L \mathbb{E}[Y|I] - (1-x)}{\beta_L} \tag{4}$$

Now, suppose that there exists a set A which has positive measure under G such that $\min\{a : a \in A\} > s^*(x)$, and B opts out on the equilibrium path if she observes $s \in A$. With a slight abuse of notation, let A be the union of all such sets. Then, it must be the case that

$$\frac{\beta_B}{\beta_L} R(s,x) \leq \beta_B \mathbb{E}[b(Y)|s] \quad \text{for all } s \in A, \tag{5}$$

for otherwise, the initial lender L, upon observing an $s \in A$, could make an offer that would be certainly accepted by B and would give L a positive payoff rather than the equilibrium payoff of zero. Since $y - b(y)$ is strictly increasing, we have

$$\frac{\beta_B}{\beta_L} R(s,x) < \beta_B \mathbb{E}[b(Y)|s] \quad \text{for all } s < \min\{a : a \in A\},$$

by Assumption 2. Together with part (a), this implies that B opts out if, and only if, $s \leq \max\{a : a \in A\}$.[7] We will first show that $\beta_L \mathbb{E}[Y|I] - (1-x) < 0$, i.e., conditional on the event that B opts out, outsiders will always believe that expected continuation surplus is negative. Suppose, for contradiction, that $\beta_L \mathbb{E}[Y|I] - (1-x) \geq 0$. Then, we have

$$\begin{aligned} \mathbb{E}[Y - b(Y)|I] &= \mathbb{E}[Y - b(Y)|s \leq \max\{a : a \in A\}] \\ &= \frac{1-x}{\beta_L} \geq \mathbb{E}[Y - b(Y)|s = \max\{a : a \in A\}] \end{aligned}$$

by (4) and (5), which contradicts Assumption 2 and that A has positive measure. Therefore, $\beta_L \mathbb{E}[Y|I] - (1-x) < 0$, and hence $b(y) = 0$ for all y. If, on the other hand, the measure of the set of signals $s > s^*(x)$ such that B opts out is zero,

[7] We have not shown that the borrower with signal $s^*(x)$ opts out. But the proof goes through even if the borrower with signal $s^*(x)$ does not opt out. Also, the proof can easily be modified so that the claim is still true if $\min\{a : a \in A\}$ or $\max\{a : a \in A\}$ does not exist.

then we again have

$$\beta_L \mathbb{E}[Y|I] - (1-x) = \beta_L \mathbb{E}\left[Y|s \leq s^*(x)\right] - (1-x) < 0,$$

by $R(s^*(x), x) = 0$ and Assumption 1, and hence $b(y) = 0$ for all y.

Now, part (b) easily follows since

$$\frac{\beta_B}{\beta_L} R(s,x) > \beta_B \mathbb{E}[b(Y)|s] = 0 \quad \text{for all } s > s^*(x),$$

and hence B never opts out after observing a signal $s > s^*(x)$. □

Proof of Proposition 3 Let $s < s^*(x)$. Then, as it was shown by Propositions 1 and 2, the outside options of the parties are binding and the first player who gets the chance (L in our model) opts out and both players receive a zero payoff. If, on the other hand, $s = s^*(x)$, then whether the equilibrium path is characterized by opting out or not, both players receive an expected payoff of zero.

If $s > s^*(x)$, the value of outside options are zero in equilibrium. Therefore, the standard bargaining outcome results. In particular, in the unique (stationary and no-delay) equilibrium, the offers $\pi_i(Y)$ must satisfy

$$\beta_L \mathbb{E}[Y - \pi_B(Y)|s] - (1-x) = \delta_L [\beta_L \mathbb{E}[\pi_L(Y)|s] - (1-x)],$$
$$\beta_B \mathbb{E}[Y - \pi_L(Y)|s] = \delta_B \beta_B \mathbb{E}[\pi_B(Y)|s],$$

so that both players are indifferent between accepting an offer and rejecting (and making a counter-offer). Noting that $R(s,x) \equiv \beta_L \mathbb{E}[Y|s] - (1-x)$, these equations can be solved to obtain the equilibrium payoffs as

$$\beta_B \mathbb{E}[\pi_B(Y)|s] = \frac{\beta_B}{\beta_L} \frac{(1-\delta_L) R(s,x)}{(1-\delta_L \delta_B)},$$
$$\beta_L \mathbb{E}[Y - \pi_L(Y)|s] - (1-x) = \frac{\delta_L (1-\delta_B) R(s,x)}{(1-\delta_L \delta_B)}.$$

Recalling that $\delta_L = \exp(-\Delta r_L)$ and $\delta_B = \exp(-\Delta r_B)$, and taking limits, we have

$$\lim_{\Delta \to 0} \beta_B \mathbb{E}[\pi_B(Y)|s] = \frac{r_L}{r_L + r_B} \frac{\beta_B}{\beta_L} R(s,x) = \eta \frac{\beta_B}{\beta_L} R(s,x),$$
$$\lim_{\Delta \to 0} \left(\beta_L \mathbb{E}[Y - \pi_L(Y)|s] - (1-x)\right) = \frac{r_L}{r_L + r_B} R(s,x) = (1-\eta) R(s,x).$$

□

Proof of Lemma 1 First, notice that $s^*(x)$ is strictly decreasing in x, because $\mathbb{E}[Y|s]$ is strictly increasing in s and $(1-x)/\beta_L$ is strictly decreasing in x.

Since, expected continuation surplus $R(s,x)$ is strictly increasing in x, it is easy to see that $V_B^0(x)$ is also strictly increasing. A more precise description of the effect of x on B's date 0 value function $V_B^0(x)$ can be obtained by applying Leibniz integral rule:

$$\frac{d}{dx}V_B^0(x) = \eta\frac{\beta_B}{\beta_L}\left(\int\limits_{s^*(x)}^{\infty} dG(s) - \frac{ds^*(x)}{dx}(R(s^*(x), x))dG\left(s^*(x)\right)\right)$$
$$= \eta\frac{\beta_B}{\beta_L}\int\limits_{s^*(x)}^{\infty} dG(s)$$
$$= \eta\frac{\beta_B}{\beta_L}\left(1 - G\left(s^*(x)\right)\right) > 0,$$

by Assumption 4. Similarly, applying Leibniz integral rule to the value function of L yields:

$$\frac{d}{dx}V_L^0(x) = (1-\eta)\beta_L\left(1 - G\left(s^*(x)\right)\right) - 1 < 0.$$

□

Proof of Proposition 4 B has monopoly power over the project at date 0. She solves the following problem:

$$\max_{x\in(0,1)} V_B^0(x) \text{ s.t. } V_L^0(x) \geq 0.$$

If x^* is a solution to the above problem, then

$$V_L^0\left(x^*\right) = \beta_L(1-\eta)\int\limits_{s^*(x^*)}^{\infty} R(s, x^*)dG(s) - x^* = 0,$$

since $V_B^0(x)$ is strictly increasing and $V_L^0(x)$ is strictly decreasing in x. This yields (3). If equilibrium exists, then it is unique as $V_L^0(x)$ is strictly decreasing. To prove the existence of an $x^* \in (0, 1)$ such that $V_L^0(x^*) = 0$, first notice that V_L^0 is continuous, and that $\lim_{x\to 1} s^*(x) \geq 0$, and $\lim_{x\to 0} s^*(x) < \infty$ (by Assumption 4). Therefore, we have

$$\lim_{x\to 1} V_L^0(x) = (1-\eta)\beta_L \int\limits_{\lim\limits_{x\to 1} s^*(x)}^{\infty} \beta_L\mathbb{E}[Y|s]\,dG(s) - 1$$
$$\leq (1-\eta)\beta_L\int\limits_0^{\infty}\mathbb{E}[Y|s]\,dG(s) - 1$$
$$= (1-\eta)\beta_L\mathbb{E}[Y] - 1 < \beta_L\mathbb{E}[Y] - 1 \leq 0, \tag{6}$$

and

$$\lim_{x\to 0} V_L^0(x) = (1-\eta)\,\beta_L \int\limits_{\lim\limits_{x\to 0} s^*(x)}^{\infty} \left(\beta_L \mathbb{E}\left[Y|s\right]-1\right) dG(s) > 0. \tag{7}$$

The last inequality follows from the facts that $\lim_{x\to 0} s^*(x) < \infty$ and $\beta_L \mathbb{E}[Y|s]-1 > 0$ for all $s > \lim_{x\to 0} s^*(x)$. Continuity of V_L^0 and inequalities (6) and (7) imply that there exists an $x^* \in (0, 1)$ such that $V_L^0(x^*) = 0$. Finally, we have

$$\begin{aligned} V_B^0\left(x^*\right) &= \eta \frac{\beta_B}{\beta_L} \int\limits_{s^*(x^*)}^{\infty} R(s, x^*) dG(s) = \eta \frac{\beta_B}{\beta_L} \frac{V_L^0(x^*) + x^*}{(1-\eta)\,\beta_L} \\ &= \frac{\eta}{1-\eta} \left(\frac{\beta_B}{\beta_L}\right)^2 x^* > 0. \end{aligned}$$

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