1. Consider the following game of “divide the dollar.” There is a dollar to be split between two players. Player 1 makes an offer (an offer by Player 1 specifies how much he would like Player 2 to have). Without observing Player 1’s offer, Player 2 specifies what would be an acceptable offer. Players’ choices have to be in increments of 25 cents, i.e., 0 cents, 25 cents, 50 cents, 75 cents, and $1. If Player 1’s offer is at least as large as what is acceptable to Player 2, then there is an agreement. Otherwise we say that there is no agreement. If there is an agreement Player 2 gets the amount offered by Player 1, while Player 1 gets the rest of the dollar. If there is no agreement neither player gets anything. Write down the strategic form of this game.

2. Consider the situation represented by the following bimatrix:

\[
\begin{array}{c|ccc}
 & L & M & R \\
\hline
U & 1,0 & 2,5 & -2,-1 \\
D & 2,1 & 2,1 & -1,0 \\
\end{array}
\]

(a) Write down the strategic form of this game.
(b) Is there a strictly dominant strategy equilibrium of this game? Explain.
(c) Is there a weakly dominant strategy equilibrium of this game? Explain.

3. Consider the following simple auction scenario. Two individuals, player 1 and player 2, are competing in an auction to obtain a valuable object. Each player bids in a sealed envelope, without knowing the bid of the other player. The bids must be in multiples of $100 and the maximum that they can bid is $500. The object is worth $400 to player 1 and $300 to player 2. The highest bidder wins the object. In case of a tie, Player 1 gets the object. The winner pays a price $p$ to be specified below. So, if the value of the object for player $i$ is $x$ and player $i$ wins the object her payoff is $x - p$. If she does not win the object her payoff is zero.

- Case 1 (First Price Auction): In this case, the winner of the object pays whatever she bids.
• Case 2 (Second Price Auction): In this case, the winner of the object pays whatever the other player bids. (If there are more than 2 players, the rule is that the winner pays the second highest bid. Since we have only two players the two rules are equivalent.)

Answer the following questions for both Case 1 and Case 2:

(a) Write down the strategic form.

(b) Is there a strictly dominant strategy equilibrium of this game? Explain.

(c) Is there a weakly dominant strategy equilibrium of this game? Explain.

(d) What are the action profiles that survive Iterated Elimination of Strictly Dominated actions? Explain.

(e) What are the action profiles that survive Iterated Elimination of Weakly Dominated actions? Explain.

(f) Is the game dominance solvable?

4. Consider the following model of price competition. Two firms set prices in a market whose demand curve is given by the equation

\[ Q = 6 - p \]

where \( p \) is the lower of the two prices. If firm 1 is the lower priced firm, then it is firm 1 that meets all of the demand; conversely, the same applies to firm 2 if it is the lower priced firm. In case they post the same price they each get half the market. Prices can be only be quoted in dollar units, such as 0, 1, 2, 3, 4, 5, or 6 dollars. Suppose, furthermore, that costs of production are zero for both firms, and each firm aims to maximize its own profits.

(a) Write down the strategic form of this game.

(b) Is there a strictly dominant strategy equilibrium of this game? Explain.

(c) Is there a weakly dominant strategy equilibrium of this game? Explain.

(d) What are the action profiles that survive Iterated Elimination of Strictly Dominated actions? Explain.

(e) What are the action profiles that survive Iterated Elimination of Weakly Dominated actions? Explain.

(f) Is the game dominance solvable?