1. \( N = \{1, 2\} \)
   \( A_1 = A_2 = \{0, .25, .50, .75, 1\} \)
   \[ u_1(a_1, a_2) = \begin{cases} 100 - a_1, & \text{if } a_1 \geq a_2 \\ 0, & \text{otherwise} \end{cases} \]
   \[ u_2(a_1, a_2) = \begin{cases} a_1, & \text{if } a_1 \geq a_2 \\ 0, & \text{otherwise} \end{cases} \]

   We can conveniently represent the strategic form with the following bimatrix:

   \[
   \begin{array}{ccccc}
   & 0 & .25 & .50 & .75 & 1 \\
   0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
   .25 & .75,.25 & .75,.25 & 0.0 & 0.0 & 0.0 \\
   .50 & .50,.50 & .50,.50 & 0.0 & 0.0 & 0.0 \\
   .75 & .75,.75 & .75,.75 & .75,.75 & 0.0 & 0.0 \\
   1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
   \end{array}
   \]

2. \( N = \{1, 2\} \)
   \( A_1 = \{U, D\} \), \( A_2 = \{L, M, R\} \)
   \[ u_1(U, L) = 1, u_1(U, M) = 2, u_1(U, R) = -2, u_1(D, L) = 2, u_1(D, M) = 2, u_1(D, R) = -1. \]
   \[ u_2(U, L) = 0, u_2(U, M) = 5, u_2(U, R) = -1, u_2(D, L) = 1, u_2(D, M) = 1, u_2(D, R) = 0. \]

   (a) We have to check if there is a strictly dominant action for each player. For player 1, action \( U \) is not strictly dominant, for action \( D \) does better when player 2 plays action \( L \). Action \( D \) is not strictly dominant either as action \( U \) does equally well when player 2 plays \( M \). This is all we need to answer this question as “No, there is not”, because we have to find a strictly dominant action for each player.

   (You may check to see that for player 2, action \( M \) is not strictly dominant, because although it strictly dominates \( R \), it does not strictly dominate \( L \). It is easy to check that actions \( L \) and \( R \) are not strictly dominant either.)

   (b) For player 1, action \( D \) weakly dominates action \( U \), because
   \[ u_1(D, L) \geq u_1(U, L) \]
   \[ u_1(D, M) \geq u_1(U, M) \]
   \[ u_1(D, R) \geq u_1(U, R) \]

   and at least one of the above inequalities holds strictly, e.g., \( u_1(D, L) > u_1(U, L) \).

   For player 2, action \( M \) weakly dominates both \( L \) and \( R \), because
   \[ u_2(U, M) > u_2(U, L) \]
   \[ u_2(D, M) \geq u_2(D, L) \]

   and
   \[ u_2(U, M) > u_2(U, R) \]
   \[ u_2(D, M) > u_2(D, R) \]

   Therefore, action profile \((D, M)\) is a weakly dominant strategy equilibrium.
3. I will assume that players can bid 0. It is okay if you have assumed otherwise.

**Case 1 (First Price Auction)**

(a) \( N = \{1, 2\}, A_1 = A_2 = \{0, 100, 200, 300, 400, 500\}\), and the payoff functions are specified by the following bimatrix

\[
\begin{array}{cccccc}
0 & 100 & 200 & 300 & 400 & 500 \\
0 & 400,0 & 0,200 & 0,100 & 0,0 & 0,-100 & 0,-200 \\
100 & 300,0 & 300,0 & 0,100 & 0,0 & 0,-100 & 0,-200 \\
200 & 200,0 & 200,0 & 200,0 & 0,0 & 0,-100 & 0,-200 \\
300 & 100,0 & 100,0 & 100,0 & 100,0 & 0,-100 & 0,-200 \\
400 & 0,0 & 0,0 & 0,0 & 0,0 & 0,0 & 0,-200 \\
500 & -100,0 & -100,0 & -100,0 & -100,0 & -100,0 & -100,0 \\
\end{array}
\]

(b) There is no strictly dominant strategy equilibrium, as neither player has a strictly dominant action.
(c) There is no weakly dominant strategy equilibrium, as neither player has a weakly dominant action.
(d) We can eliminate the strictly dominated actions in the following order:

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

after which there is no more strictly dominated actions. Therefore, all the action profiles in which player 1 and player 2 bid 0, 100, 200, 300, and 400 survive IESD actions. In other words the set of action profiles that survive IESD actions is

\[ \{0, 100, 200, 300, 400\} \times \{0, 100, 200, 300, 400\} \).

(e) We can eliminate the weakly dominated actions in the following order:

<table>
<thead>
<tr>
<th>Player 1:</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2:</td>
<td>500</td>
</tr>
<tr>
<td>Player 1:</td>
<td>400</td>
</tr>
<tr>
<td>Player 2:</td>
<td>400</td>
</tr>
<tr>
<td>Player 1:</td>
<td>300</td>
</tr>
<tr>
<td>Player 2:</td>
<td>0</td>
</tr>
<tr>
<td>Player 1:</td>
<td>0</td>
</tr>
<tr>
<td>Player 2:</td>
<td>100</td>
</tr>
<tr>
<td>Player 1:</td>
<td>100</td>
</tr>
</tbody>
</table>

which leads to the unique outcome (200,200).

(f) The game is not dominance solvable.

**Case 2 (Second Price Auction)**

(a) \( N = \{1, 2\}, A_1 = A_2 = \{0, 100, 200, 300, 400, 500\}\), and the payoff functions are specified by the following bimatrix

\[
\begin{array}{cccccc}
0 & 100 & 200 & 300 & 400 & 500 \\
0 & 400,0 & 0,300 & 0,300 & 0,300 & 0,300 & 0,300 \\
100 & 400,0 & 300,0 & 0,200 & 0,200 & 0,200 & 0,200 \\
200 & 400,0 & 300,0 & 200,0 & 0,100 & 0,100 & 0,100 \\
300 & 400,0 & 300,0 & 200,0 & 100,0 & 0,0 & 0,0 \\
400 & 400,0 & 300,0 & 200,0 & 100,0 & 0,0 & -100,0 \\
500 & 400,0 & 300,0 & 200,0 & 100,0 & 0,0 & -100,0 \\
\end{array}
\]

(b) There is no strictly dominant strategy equilibrium, as neither player has a strictly dominant action.
(c) There is no weakly dominant strategy equilibrium, as neither player has a weakly dominant action. Notice that for both players, actions 300 and 400 weakly dominate every other action, but not each other.

(d) There is not strictly dominated action for either player and hence all the action profiles survive IESD actions.

(e) We can eliminate the weakly dominated actions in the following order:

<table>
<thead>
<tr>
<th>Player 1:</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2:</td>
<td>0</td>
</tr>
<tr>
<td>Player 1:</td>
<td>500</td>
</tr>
<tr>
<td>Player 2:</td>
<td>500</td>
</tr>
<tr>
<td>Player 1:</td>
<td>100</td>
</tr>
<tr>
<td>Player 2:</td>
<td>100</td>
</tr>
<tr>
<td>Player 1:</td>
<td>200</td>
</tr>
</tbody>
</table>

which leads to the following set of outcomes \(\{300,400\} \times \{200,300,400\}\). However, there are other orders of elimination which lead to different outcomes.

(f) The game is not dominance solvable.

4.   

(a)    
- \(N = \{1, 2\}\)
- \(A_1 = A_2 = \{0, 1, 2, 3, 4, 5, 6\}\). We will denote a typical element of \(A_i\) by \(p_i, i = 1, 2\).

\[
 u_1(p_1, p_2) = \begin{cases} 
 p_1 (6 - p_1), & \text{if } p_1 < p_2 \\
 \frac{p_1(6 - p_1)}{2}, & \text{if } p_1 = p_2 \\
 0, & \text{if } p_1 > p_2 
\end{cases}
\]

\[
 u_2(p_1, p_2) = \begin{cases} 
 p_2 (6 - p_2), & \text{if } p_2 < p_1 \\
 \frac{p_2(6 - p_2)}{2}, & \text{if } p_1 = p_2 \\
 0, & \text{if } p_2 > p_1 
\end{cases}
\]

We can represent the game with the following bimatrix.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>2.5</td>
<td>2.5</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.5</td>
<td>4.4</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.5</td>
<td>0.8</td>
<td>4.5</td>
<td>9.0</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.5</td>
<td>0.8</td>
<td>0.9</td>
<td>4.4</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>0.5</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
<td>2.5</td>
<td>5.0</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>0.5</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
<td>0.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

(b) There is no strictly dominant strategy equilibrium, as neither player has a strictly dominant action.

(c) There is no weakly dominant strategy equilibrium, as neither player has a weakly dominant action.

(d) All actions survive IESD actions.

(e) IEWD actions leads to the unique outcome (1,1).

(f) The game is not dominance solvable.