Toric stacks, log stable maps, and Morse theory

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Suppose X is a complete toric variety with torus T and S is a one-parameter subgroup of T. Then the S orbit closure in X of any point of T determines a map from the projective line to X, which is in fact a log stable map. I will try to explain the idea of log stable maps in the talk. The component M of the space of log stable maps containing the maps thus constructed is itself a toric stack with torus T/S. We will describe the "fan" for M, as well as the fan for the universal domain curve. Q. Chen and M. Satriano proved that the coarse moduli space of M is the Chow quotient X/S, which is itself a toric variety studied by Kapranov, Sturmfels, and Zelevinsky. If one takes all these fans and "realizes" them "differentiably" instead of "as schemes over C," then all of this can be interpreted in terms of Morse theory and moduli spaces of broken flow lines.