Exotic 4-manifolds with small Euler characteristics

It is known that many simply connected, smooth topological 4-manifolds admit infinitely many smooth structures. The smaller the Euler characteristic, the harder it is to construct exotic smooth structure.

In this talk we present examples of symplectic 4-manifolds with same integral cohomology as $S^2 \times S^2$. We also discuss the generalization of these examples to $\#\{2n-1\}(S^2 \times S^2)$ for $n > 1$. As an application of these symplectic building blocks, we construct exotic smooth structure on small 4-manifolds such as $CP^2\#k(-CP^2)$ for $k = 2, 3, 4, 5$ and $3CP^2\#l(-CP^2)$ for $l = 4, 5, 6, 7$. We will also discuss an interesting applications to the geography of minimal symplectic 4-manifolds.