Inversion Formulas in Truncated Data Ray-Tomography

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Abstract

In Ray-Tomography the goal is to recover a function in higher dimensions from knowledge of integrals of the function along lines. For example, in the Euclidean setting we let \( f \in C_0^\infty(\mathbb{R}^2) \), and for each straight line \( L \) in \( \mathbb{R}^2 \), we define the Ray-Transform of \( f \) along \( L \) as

\[
Rf(L) := \int_L f(l) \, dl \quad (dl = \text{Lebesgue measure in } L).
\]

The inversion problem consists of recovering \( f \) at all or some points of \( \mathbb{R}^2 \) from knowledge of \( Rf \) along all or some of the straight lines in \( \mathbb{R}^2 \). Or if \( \mathbb{H} \) is the Hyperbolic plane and \( f : \mathbb{H} \to \mathbb{R} \) is smooth and compactly supported, for each \( \gamma : \mathbb{R} \to \mathbb{H} \) arc-length parametrized geodesic we define the Ray-Transform of \( f \) along \( \gamma \) as

\[
Rf(\gamma) := \int_{\mathbb{R}} f(\gamma(s)) \, ds.
\]

Again, the inversion problem consists of recovering \( f \) at all or some points of \( \mathbb{H} \) from knowledge of \( Rf \) along all or some geodesics in \( \mathbb{H} \).

These two particular cases of Ray-Tomography are closely related to applications in medical imaging techniques, like Computed Tomography or Positron Emission Tomography [3], as well as to geological exploration techniques, appearing as the linearized problem in Travel Time Tomography [7].

The ray transform has been amply studied in general settings [8]. In addition to injectivity, support theorems and stability results for particular cases, inversion formulas have also been provided [2, 3, 5, 6]. Unfortunately, these inversion formulas require knowledge of \( Rf \) over all straight lines (or geodesics) to recover \( f \) at one given point, which is very inconvenient from the point of view of applications.

A different approach for the Euclidean case was proposed in [4]. The inversion of the Ray-Transform is reduced to a problem of inverting a one dimensional Hilbert transform. A new inversion formula, not requiring knowledge of all of \( Rf \), was obtained (eg. [4]) and injectivity and stability results for other cases of Euclidean truncated measurements followed [1].

In this talk we will first present how the Ray-Transform appears as a model of the measurements in the applications mentioned above. Then we will overview the non-locality issue of the previous inversion formulas. Afterwards we will go into more detail about the approach for the Euclidean setting introduced in [4], with the generalizations obtained in [1]. To conclude, we will present some new results generalizing the approach in [4] to the Hyperbolic plane setting.
References


