Pseudo-Hermiticity with Unbounded Metric Operators and Quantum Mechanics
Ali Mostafazadeh

Pseudo-Hermitian operators form a special class of non-self-adjoint operators that were studied in the 1940-1960s, in the context of symmetrizable and quasi-Hermitian operators by mathematicians under various restrictive conditions. The results proved to be essentially out of reach for practical purposes. The interest in the subject was revived in the theoretical physics community in the early 2000s and led to the rediscovery and generalizations of the old concepts of symmetrizable and quasi-Hermitian operators. These developments mostly lacked mathematical rigor, but they found important applications in various areas of physics and subsequently triggered rigorous studies of the subject. In this talk I report a solution to a mathematical problem that plagued the recent wave of developments on the subject since its inception in 2002. It involves a construction that allows one to circumvent some of the recent negative results on the non-existence of bounded metric operators.

This talk consists of three parts. Part one provides a survey of basic concepts necessary for defining pseudo-Hermitian, quasi-Hermitian, and metric operators acting in a Hilbert space. Part 2 is a concise mathematical introduction to quantum mechanics that assumes no knowledge of physics. Part 3 is devoted to the description of the problem with unbounded metric operators and its solution.