
KOÇ UNIVERSITY
MATH 101 - FINITE MATHEMATICS
Midterm I October, 2015

Duration of Exam: 100 minutes

INSTRUCTIONS: You can use calculators in the exam. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive full credit. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS)** and sign your name, and indicate your section below.

Name: _____

Surname: _____

Signature: _____

Section (Check One):

- Section 1: Mine Çağlar M-W (8:30) _____
Section 2: Mine Çağlar M-W (10:00) _____
Section 3: Ayberk Zeytin Tu-Th(13:00) _____
Section 4: Ayberk Zeytin Tu-Th(16:00) _____

PROBLEM	POINTS	SCORE
1	20	
2	25	
3	22	
4	23	
5	15	
TOTAL	105	

A list of formulas: $I = Prt$; $A = P(1 + rt)$

$$A = P(1 + i)^n; APY = (1 + \frac{r}{m})^m - 1; A = Pe^{rt}; APY = e^r - 1;$$

$$FV = PMT \frac{[(1+i)^n - 1]}{i}; PV = PMT \frac{[1 - (1+i)^{-n}]}{i}, \text{ where } i = \frac{r}{m} \text{ and } n = mt$$

1. (a) (12 points) Solve the following equation for x

$$2 \log_b(x-3) - \log_b(2x-14) = \log_b x$$

where $b > 0, b \neq 1$.

$$\Rightarrow \log_b (x-3)^2 - \log_b (2x-14) = \log_b x$$

$$x \neq 3, x \neq 0, \\ x \neq 7$$

$$\Rightarrow \log_b \frac{(x-3)^2}{2x-14} = \log_b x$$

$$\Rightarrow (x-3)^2 = x(2x-14)$$

$$x^2 - 6x + 9 = 2x^2 - 14x$$

$$0 = x^2 - 8x - 9 = (x+1)(x-9)$$

$$x = -1 \text{ or } \boxed{x = 9}$$

can't be

(b) (8 points) Find x if $(x-1)^{202} = (x+1)^{101}$.

$$\Rightarrow \left((x-1)^2 \right)^{101} = (x+1)^{101}$$

$$\Rightarrow (x-1)^2 = x+1$$

$$x^2 - 2x + 1 = x + 1$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0 \text{ or } x = 3$$

2. (25 points) To expand your business you need a loan of 8000 TL. Your banker loans you the money at 12% compounded monthly which you agree to repay in 4 equal monthly payments.

(a) (7 points) Find your monthly payments.

$$8000 = \text{PMT} \frac{1 - \left(1 + \frac{0.12}{12}\right)^{-4}}{\frac{0.12}{12}}$$

$$\Rightarrow \text{PMT} = 2050.25 \text{ TL}$$

(b) (18 points) In the following amortization schedule, find the numbers A, B and C.

Payment Number	Payment	Interest	Unpaid Balance Reduction	Unpaid Balance
0	—	—	—	8000
1	2050.25	A	*	**
2	2050.25	***	B	
3				C
4	2050.25			0

$$A = 8000 \left(\frac{0.12}{12}\right) = 80 \text{ TL}$$

$$\frac{0.12}{12} = 0.01$$

$$\Rightarrow * = 2050.25 - 80 = 1970.25 \text{ TL}$$

$$\Rightarrow ** = 8000 - 1970.25 = 6029.75 \text{ TL}$$

$$\Rightarrow *** = 6029.75 (0.01) = 60.30 \text{ TL}$$

$$\Rightarrow B = 2050.25 - 60.30 = 1989.95 \text{ TL}$$

$$C = 2050.25 \frac{1 - (1 + 0.01)^{-1}}{0.01}$$

$$\Rightarrow C = 2029.95 \text{ TL}$$

3. (22 points) A person makes payments of 1500 TL at the end of each quarter into a savings account paying 6.5% compounded quarterly for 5 years.

(a) (12 points) How much is in the account at the end of 5 years?

$$FV = 1500 \frac{\left(1 + \frac{0.065}{4}\right)^{(5)(4)} - 1}{\frac{0.065}{4}}$$
$$= \$35115.67$$

(b) (10 points) Instead, if the person deposited 30,000 TL now into another account compounded continuously, what would be the interest rate in order to receive the same amount as in part (a) at the end of 5 years? Express your answer as percentage, rounded to three decimal places.

$$A = 30000 e^{r \cdot 5} = 35115.67$$

$$e^{5r} = \frac{35115.67}{30000}$$

$$5r = \ln\left(\frac{35115.67}{30000}\right)$$

$$\Rightarrow r \approx 0.03149$$

$$\Rightarrow r = 3.149\%$$

4. (23 points) Suppose you take out a loan of 200,000 TL mortgage for 10 years at an annual interest rate of 12% which you agree to repay in equal monthly payments.

(a) (7 points) Find your monthly payments.

$$200000 = \text{PMT} \frac{1 - \left(1 + \frac{0.12}{12}\right)^{-120}}{0.01}$$

$$10 \times 12 = 120$$

$$\frac{0.12}{12} = 0.01$$

$$\Rightarrow \text{PMT} \cong 2869.42 \text{ TL}$$

(b) (8 points) Find your unpaid balance at the end of 3 years.

$$\text{PV} = 2869.42 \frac{1 - \left(1 + 0.01\right)^{-7(12)}}{0.01}$$

7 yrs remaining

$$\cong 162,548.20 \text{ TL}$$

(c) (8 points) How much will you save in interest if you agree to refinance your remaining 7 years of debt with 8.4% annual interest rate, compounded monthly?

$$162,548.20 = \widetilde{\text{PMT}} \frac{1 - \left(1 + \frac{0.084}{12}\right)^{-84}}{\frac{0.084}{12}}$$

$$\Rightarrow \widetilde{\text{PMT}} = 2566.03 \text{ TL}$$

$$\text{In each payment, we save } 2869.42 - 2566.03 = 303.39 \text{ TL}$$

$$\text{Total interest saved: } 84 \times 303.39 = 25,484.76 \text{ TL}$$

5. (15 points) Consider the following system of linear equations,

$$\begin{aligned}2x + ty &= 6 \\ x + y &= 2\end{aligned}$$

where t is a real constant.

(a) (5 points) Find all t for which the system has no solution.

$$\begin{aligned}2x + ty &= 6 \\ x + y &= 2\end{aligned} \Rightarrow \begin{aligned}2x + ty &= 6 \\ + (-2x + 2y) &= -4 \\ \hline (t-2)y &= 2 \quad (*)\end{aligned}$$

If $t-2=0$, that is, $t=2$, then
the system has no solution for (x, y)

(b) (6 points) Find all t for which the above system has a unique solution.

If $t \neq 2$, then
 $y = \frac{2}{t-2}$ from (*)

and since $x+y=2$, we get $x = 2 - \frac{2}{t-2}$.

Therefore, for $t \in \mathbb{R} \setminus \{2\}$, the system
has a unique solution given by $(2 - \frac{2}{t-2}, \frac{2}{t-2})$.

(c) (4 points) Does there exist t for which the system has infinitely many solutions? Explain.

There are no such values of t because
one cannot reduce one equation to
 $0=0$
form.