KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm I March 20, 2017

Duration of Exam: 75 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit.** Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: KEY

Surname:

Signature:

Section (Check One):

Section 1: Selda Küçükcifci M-W(14:30) ---
Section 2: E. Şule Yazıcı M-W (13:00) ---
Section 3: E. Şule Yazıcı M-W (11:30) ---

PROBLEM	POINTS	SCORE
1	24	
2	25	
3	15	
4	20	
5	20	
TOTAL	104	

A list of formulas: $I = Prt$; $A = P(1 + rt)$

$A = P(1 + i)^n$: $APY = (1 + \frac{r}{m})^m - 1$: $A = Pe^{rt}$; $APY = e^r - 1$;

$FV = PMT\frac{[(1+i)^n - 1]}{i}$; $PV = PMT\frac{[1-(1+i)^{-n}]}{i}$, where $i = \frac{r}{m}$ and $n = mt$

1. (24 points) Ziraat Bank offers an account with an APY of 2.243%.

(a) If the interest is compounded monthly, what is the equivalent annual nominal rate?

$$0,02243 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$1,02243 = \left(1 + \frac{r}{12}\right)^{12}$$

$$r = \left(\sqrt[12]{1,02243} - 1\right) \times 12 = 0,022220$$

$$2,22\ \%$$

(b) If you wish to have \$100,000 in this account after 8 years, what equal deposit should be made each month?

$$100\,000 = PMT \frac{\left(1 + \frac{0,0222}{12}\right)^{96} - 1}{\frac{0,0222}{12}}$$

$$PMT = \frac{\frac{2220}{12}}{\left(1 + \frac{0,0222}{12}\right)^{96} - 1} = \$\ 952,86$$

2. (25 points) A person purchases a \$160.000 home 20 years ago by signing a 30-year mortgage at 13.2% compounded monthly. The interest rates have dropped and the person wants to refinance the unpaid balance by signing a new 10-year mortgage at 8.4% compounded monthly. How much less interest will the person pay by this refinancing for the last 10 years?

$$160.000 = PMT \frac{1-\left(1+\frac{0,132}{12}\right)^{-360}}{\frac{0,132}{12}}$$

$$PMT = \frac{160.000 \times \frac{0,132}{12}}{1-\left(1+\frac{0.132}{12}\right)^{-360}} = 1794,97$$

$$\text{Unpaid balance after 20 years} = 1794.97 \frac{1-\left(1+\frac{0,132}{12}\right)^{-120}}{\frac{0,132}{12}} = 119\,272,89$$

$$PMT_2 = \frac{119\,272.89 \times \frac{0,084}{12}}{1-\left(1+\frac{0.084}{12}\right)^{-120}} = 1472.44$$

$$\text{Interest Saving} = (1794.97 - 1472.44) \times 12 \times 10 = 38\,703,6$$

3. (15 points) How long will it take to double your money if it is invested at 8% compounded monthly?

$$2P = P\left(1+\frac{0.08}{12}\right)^n$$

$$2 = \left(1+\frac{0.08}{12}\right)^n \qquad n = 105 \text{ months}$$

$$\ln 2 = \ln\left(1+\frac{0.08}{12}\right)^n$$

$$n = \frac{\ln 2}{\ln\left(1+\frac{0.08}{12}\right)}$$

4. (20 points) Solve the following system using Gauss Jordan elimination method.

$$\begin{cases} x_1 - 3x_2 + 2x_3 = 5 \\ 2x_1 - 5x_2 + x_3 = 3 \\ x_1 - x_2 - 4x_3 = 19 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 5 \\ 2 & -5 & 1 & 3 \\ 1 & -1 & -4 & 19 \end{array}\right] \begin{array}{l} -2R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 5 \\ 0 & 1 & -3 & -7 \\ 0 & 2 & -6 & 14 \end{array}\right] \begin{array}{l} 3R_2+R_1 \to R_1 \\ -2R_2+R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -7 & -16 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 0 & 28 \end{array}\right]$$

no solution

inconsistent system

$SS = \{\}$

5. (20 points) Determine whether the following matrices are in reduced form. If they are not, bring them in reduced form. Write the solution set for the corresponding systems. Determine if the system is consistent, inconsistent, dependent or independent.

10 points (a) $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 2 & -4 \end{array}\right]$ $-2R_2+R_3 \to R_3$ $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 = -1$

$x_2 = -2$

$SS = \{(-1,-2)\}$

Consistent, independent System

Unique Solution

(b) 10 points $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 6 \\ 1 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{array}\right]$ $-R_1+R_2 \to R_2$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 6 \\ 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 & 3 \end{array}\right]$ $-R_2 \to R_2$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 6 \\ 0 & 0 & +1 & 0 & +3 \\ 0 & 0 & 1 & 0 & 3 \end{array}\right]$ $-R_2+R_1 \to R_1$, $-R_2+R_3 \to R_3$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 + 2x_4 = 3$

$x_3 = 3$

$SS = \{(3-2x_4, x_2, 3, x_4) \mid x_2, x_4 \in \mathbb{R}\}$

Let $x_2 = t$ and $x_4 = s$

$SS = \{(3-2s, t, 3, s) \mid s, t \in \mathbb{R}\}$

Consistent

dependent system