KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Exam 1 November 3, 2008

Duration of Exam: 75 minutes

INSTRUCTIONS: CALCULATORS ARE NOT ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: ANSWER KEYS

Signature: ________________________

Section (Check One):

Section 1: S. Küçükçifçi (Tue-Thu 11:00) ___
Section 2: S. Küçükçifçi (Tue-Thu 14:00) ___
Section 3: B. Coşkunüzer (Mon-Wed 9:30) ___
Section 4: B. Coşkunüzer (Mon-Wed 11:00) ___
Section 5: B. Özbağcı (Tue-Thu 11:00) ___

PROBLEM	POINTS	SCORE
1	15	
2	15	
3	20	
4	15	
5	15	
6	20	
TOTAL	**100**	

1. (15 points) A jazz concert brought in \$33000 on the sale of 1600 tickets. If the tickets sold for \$15 and \$25 each, how many of each type of ticket were sold?

Let x = # of tickets sold for \$15

y = # of tickets sold for \$25

$15x + 25y = 33000$ and $x + y = 1600$

$y = 1600 - x$

Thus, $15x + 25(1600 - x) = 33000$

$15x + 40000 - 25x = 33000$

$7000 = 10x$

$x = 700$

$y = 1600 - x = 900$

700 tickets sold for \$15

900 tickets sold for \$25

2. (15 points) (a) Find the slope of the line passing through the points $(2,5)$ and $(-1,6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{(6-5)}{(-1-2)} = -\frac{1}{3}$$

(b) Write an equation of the line in part (a).

$m = -\frac{1}{3}$ and $(x_1, y_1) = (2,5)$

$$y - y_1 = m(x - x_1) \Rightarrow y - 5 = -\frac{1}{3}(x-2)$$

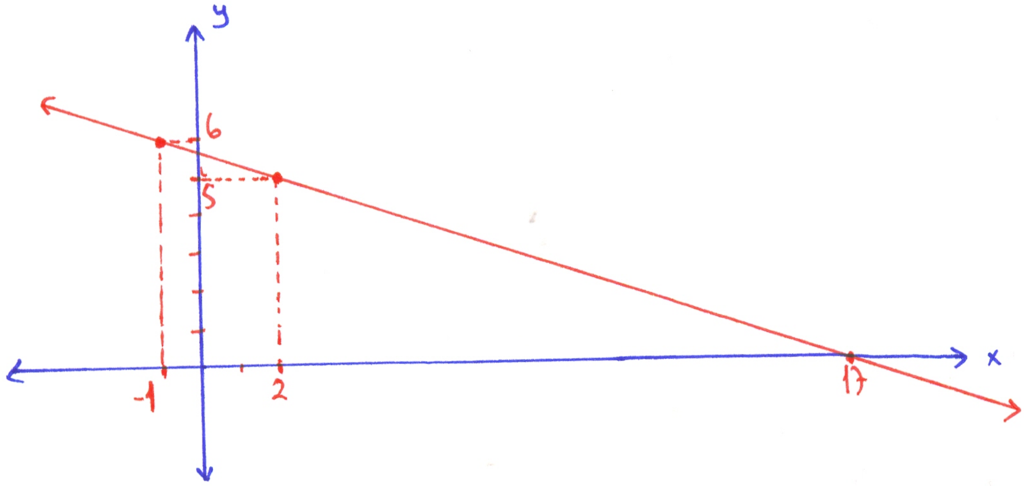
$$y - 5 = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{17}{3}$$

(c) Find the intercepts and draw the line in part (a).

x intercept: $y = 0 \Rightarrow 0 = -\frac{1}{3}x + \frac{17}{3} \Rightarrow \frac{1}{3}x = \frac{17}{3} \Rightarrow x = 17$

y intercept: $x = 0 \Rightarrow y = -\frac{1}{3}\cdot 0 + \frac{17}{3} \Rightarrow y = \frac{17}{3}$



3. (20 points) Consider the function $f(x) = \frac{-1}{2}x^2 + 2x - \frac{3}{2}$.

(a) Find the vertex.

$$f(x) = -\frac{1}{2}(x^2 - 4x) - \frac{3}{2}$$
$$= -\frac{1}{2}(x^2 - 4x + 4) + 2 - \frac{3}{2}$$
$$= -\frac{1}{2}(x-2)^2 + \frac{1}{2}$$

Vertex: $(h, k) = \left(2, \frac{1}{2}\right)$

(b) Find the line of symmetry (axis of symmetry).

$x = 2$

(c) Find x-intercept(s).

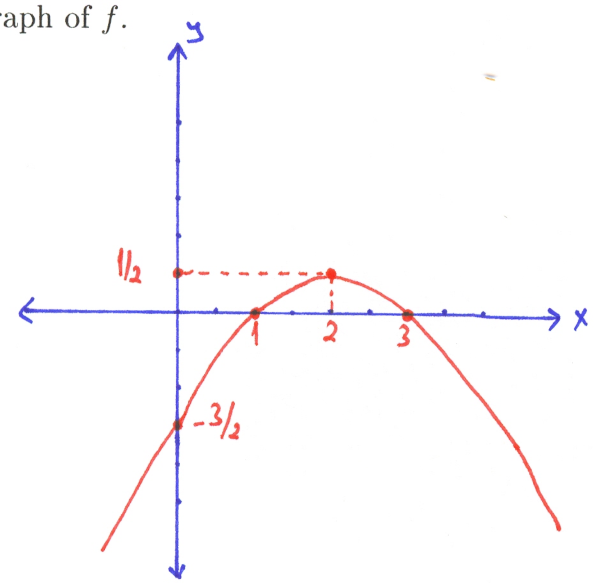
x intercept: $f(x) = 0 \Rightarrow -\frac{1}{2}(x-2)^2 + \frac{1}{2} = 0$

$$\frac{1}{2}(x-2)^2 = \frac{1}{2}$$
$$(x-2)^2 = 1$$

$x - 2 = \mp 1$

$x_1 = 1 \quad x_2 = 3$

(d) Sketch a graph of f.



(e) Find the range of f.

R: $\left(-\infty, \frac{1}{2}\right]$

4. (15 points) Answer the following questions.

(a) Solve the equation $3^{-2x} = \dfrac{9^{x-1}}{3^x + 3^x + 3^x}$ for x.

$$3^{-2x} = \frac{(3^2)^{x-1}}{3 \cdot 3^x}$$

$$3^{-2x} = \frac{3^{2x-2}}{3^{x+1}}$$

$$3^{-2x} = 3^{2x-2-(x+1)}$$

$$3^{-2x} = 3^{x-3}$$

$$-2x = x-3$$

$$3 = 3x$$

$$x = 1$$

(b) Solve the equation $\ln(3-2x) + 4 = 0$ for x.

$$\ln(3-2x)+4=0$$

$$\ln(3-2x) = -4$$

$$3-2x = e^{-4}$$

$$3-e^{-4} = 2x$$

$$x = \frac{3-e^{-4}}{2}$$

since $3-2x>0$

$$\frac{3}{2} > x$$

(c) Solve the equation $\log_b x - \log_b 2 = \log_b 3 - \log_b(x+5)$ for x.

$$\log_b x - \log_b 2 = \log_b 3 - \log_b (x+5)$$

$$\log_b\left(\frac{x}{2}\right) = \log_b\left(\frac{3}{x+5}\right)$$

$$\frac{x}{2} = \frac{3}{x+5}$$

$$x^2+5x = 6$$

$$x^2+5x-6=0$$

$$(x+6)(x-1)=0$$

$x=1$ since $x>0$

5. (15 points) Find $\tan \frac{5\pi}{12}$.

$$\tan \frac{5\pi}{12} = \frac{\sin \frac{5\pi}{12}}{\cos \frac{5\pi}{12}} = \frac{\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)}$$

$$\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \left(\sin\frac{\pi}{6} \cdot \cos\frac{\pi}{4}\right) + \left(\cos\frac{\pi}{6} \cdot \sin\frac{\pi}{4}\right)$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}(1+\sqrt{3})}{4}$$

$$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \left(\cos\frac{\pi}{6} \cdot \cos\frac{\pi}{4}\right) - \left(\sin\frac{\pi}{6} \cdot \sin\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

$$\tan \frac{5\pi}{12} = \frac{\frac{\sqrt{2}(1+\sqrt{3})}{4}}{\frac{\sqrt{2}(\sqrt{3}-1)}{4}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+2\sqrt{3}+1}{2}$$

$$(\sqrt{3}+1)$$

$$\tan \frac{5\pi}{12} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

6. (20 points) (a) Find $\cos(\sin^{-1}(\frac{-\sqrt{3}}{2}))$.

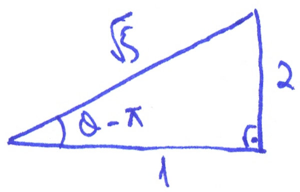
Let $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \theta$

$\sin\theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = -\frac{\pi}{3}$ since $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

(b) $\tan\theta = 2$, where $\theta \in [\pi, \frac{3\pi}{2}]$, find $\sin\theta$ and $\cos\theta$.

$\theta \in \left[\pi, \frac{3\pi}{2}\right] \Rightarrow$ III. quadrant



$\sin(\theta-\pi) = -\sin(\theta) = \frac{2}{\sqrt{5}}$

$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\cos(\theta-\pi) = -\cos(\theta) = \frac{1}{\sqrt{5}}$

$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$