KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm I November 8, 2014

Duration of Exam: 75 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: ______________________

Surname: KEY

Signature: ______________________

Section (Check One):

Section 1: Selda Küçükçifçi M-W (8:30) ___
Section 2: Selda Küçükçifçi M-W (10:00) ___
Section 3: Haluk Oral M-W(13:00) ___
Section 4: Haluk Oral M-W(16:00) ___
Section 5: Ayberk Zeytin T-Th(15:30) ___

PROBLEM	POINTS	SCORE
1	12	
2	16	
3	20	
4	37	
5	20	
TOTAL	105	

1. (12 points) Find any vertical and horizontal asymptotes of the graphs given below:

(a) $f(x) = \dfrac{9}{x^2 - 9}$ $= \dfrac{9}{(X-3)(x+3)}$

vertical asymptotes are $X = 3$ & $X = -3$

Since they do not make the numerator 0.

Horizontal asymptote is $y = 0$ since

$$\frac{9}{X^2 - 9} = \frac{\frac{9}{X^2} \to 0}{1 - \frac{9}{X^2} \to 0} \to 0$$

(b) $g(x) = \dfrac{x^2 - 8x + 7}{x^2 + 7x - 8}$ $= \dfrac{(x-7)(x-1)}{(x+8)(x-1)}$

vertical asymptote is $X = -8$ since

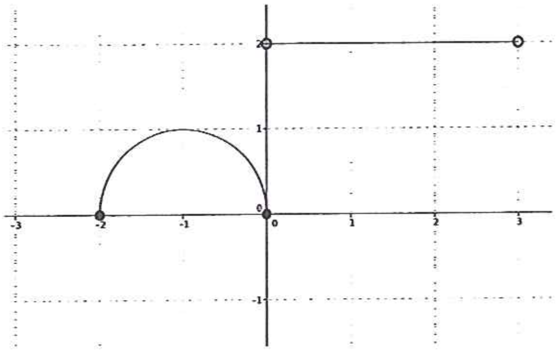
-8 does not make the numerator 0.

and $g(x)$ behaves as $y = \dfrac{X-7}{X+8}$ when $X \neq 1$, but close to 1.

Horizontal asymptote $y = 1$ since

$$\frac{X^2 - 8X + 7}{X^2 + 7X - 8} = \frac{1 - \frac{8}{X} \to 0 + \frac{7}{X^2} \to 0}{1 + \frac{7}{X} \to 0 - \frac{8}{X^2} \to 0} \to 1.$$

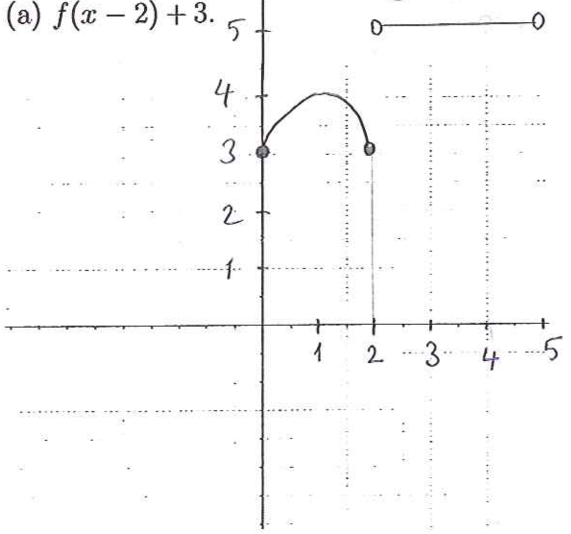
2. (16 points) A graph of the function $f(x)$ is given as:



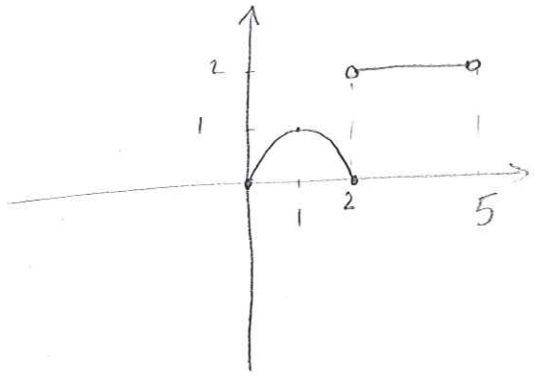
Sketch the graph of the following functions;

(a) $f(x-2)+3$.

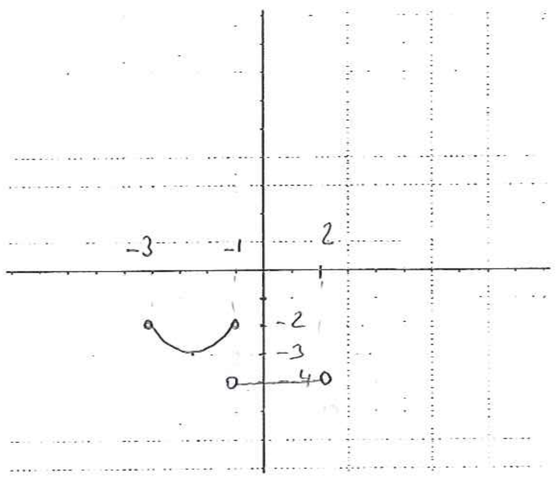
$y = f(x-2)+3$



$y = f(x-2)$

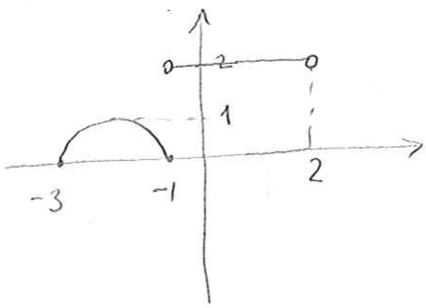


(b) $-f(x+1)-2$.

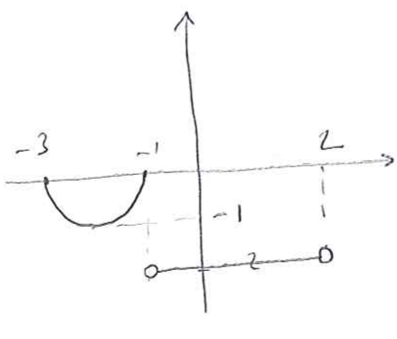


$y = -f(x+1)-2$

$y = f(x+1)$



$y = -f(x+1)$



3. Let $\log 2 = a$ and $\log 3 = b$.

(a) (18 points) Complete the below table by finding $\log x$ in terms of a and b.

x	1	2	3	4	5	6	8	9	10
$\log x$	0	a	b	$2a$	$1-a$	$a+b$	$3a$	$2b$	1

$\log 1 = 0$

$\log 2 = a$

$\log 3 = b$

$\log 4 = \log 2^2 = 2\log 2 = 2a$

$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - a$

$\log 6 = \log(2.3) = \log 2 + \log 3 = a+b$

$\log 8 = \log 2^3 = 3\log 2 = 3a$

$\log 9 = \log 3^2 = 2\log 3 = 2b$

$\log 10 = 1$

(b) (2 points) Can you write $\log 7$ in terms of a and b? Explain.

We can not write $\log 7$ in terms of a & b since 7 is a prime number.

4. (a) (25 points) Let $\sin x = \frac{2}{3}$, where $\frac{\pi}{2} < x < \pi$. Evaluate

(i) $\cos x$ $\quad \cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \frac{4}{9} = \frac{5}{9}$

Since $x \in \left(\frac{\pi}{2}, \pi\right)$ $\quad \cos x = -\frac{\sqrt{5}}{3}$.

(ii) $\cos(2x)$ $= \cos^2 x - \sin^2 x$

$= \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$

(iii) $\tan x$ $= \frac{\sin x}{\cos x} = \frac{\frac{2}{3}}{-\frac{\sqrt{5}}{3}} = \frac{-2}{\sqrt{5}}$ 1

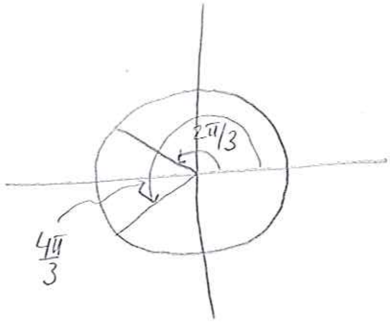
(iv) $\tan(2x)$ $= \frac{2\tan x}{1-\tan^2 x} = \frac{\frac{-4}{\sqrt{5}}}{1-\frac{4}{5}} = \frac{\frac{-4}{\sqrt{5}}}{\frac{1}{5}} = \frac{-20}{\sqrt{5}} = -4\sqrt{5}$ 2

(v) $\sin\left(\frac{\pi}{4} - x\right)$ $= \sin\frac{\pi}{4} \cdot \cos x - \cos\frac{\pi}{4} \cdot \sin x$

$= \frac{\sqrt{2}}{2} \cdot \frac{(-\sqrt{5})}{3} - \frac{\sqrt{2}}{2} \cdot \frac{2}{3}$

$= \frac{\sqrt{2}(-\sqrt{5}-2)}{6}$.

(b) (12 points) Solve $\cos(x + \frac{\pi}{2}) = \frac{-1}{2}$ in $[-2\pi, 2\pi]$.



$$\cos\left(x+\frac{\pi}{2}\right) = -\frac{1}{2}$$

$$x+\frac{\pi}{2} = \frac{2\pi}{3} + 2k\pi \qquad k \in \mathbb{Z}$$

or

$$x+\frac{\pi}{2} = \frac{4\pi}{3} + 2k\pi$$

Then $x+\frac{\pi}{2} = \frac{2\pi}{3} + 2k\pi$ or $x+\frac{\pi}{2} = \frac{4\pi}{3} + 2k\pi$

$$x = \frac{4\pi - 3\pi}{6} + 2k\pi \qquad x = \frac{8\pi - 3\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi$$

$k=0$ $\quad x = \frac{\pi}{6}$, $\quad x = \frac{5\pi}{6}$

$k=-1$ $\quad x = \frac{\pi}{6} - 2\pi = \frac{-11\pi}{6}$ $\quad x = \frac{5\pi}{6} - 2\pi = \frac{-7\pi}{6}$

$$S = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6} \right\}.$$

5. Consider

$$\begin{cases} x_1 + x_2 + x_3 &= 6 \\ x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 &= P \end{cases}$$

(a) (12 points) By using Gauss-Jordan elimination method find all values of P for which the above system has solution(s).

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & -1 & 2 \\ 2 & 3 & 0 & P \end{array}\right] \xrightarrow[R_1(-2)+R_3 \to R_3]{R_1(-1)+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -4 \\ 0 & 1 & -2 & P-12 \end{array}\right] \xrightarrow[R_2(-1)+R_3 \to R_3]{R_2(-1)+R_1 \to R_1}$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 0 & P-8 \end{array}\right]$$

the system has infinitely many solutions when $P-8=0$

that is $P=8$.

(b) (8 points) Solve the system for that value of P.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 3x_3 = 10$

$x_2 - 2x_3 = -4$

there are infinitely many solutions

they are

$x_1 = 10 - 3t$

$x_2 = -4 + 2t$

$x_3 = t$

$t \in \mathbb{R}$

$S = \{(10-3t, -4+2t, t) : t \in \mathbb{R}\}$.