KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm II December 20, 2014

Duration of Exam: 75 minutes

INSTRUCTIONS: Calculators may be used on the test. No cell phones, no books, no notes, and no talking are allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print** (use **CAPITAL LETTERS**) **and sign your name, and indicate your section below.**

Name: ______________________

Surname: KEY

Signature: ______________________

Section (Check One):

Section 1: Selda Küçükçifçi M-W (8:30) ___
Section 2: Selda Küçükçifçi M-W (10:00) ___
Section 3: Haluk Oral M-W(13:00) ___
Section 4: Haluk Oral M-W(16:00) ___
Section 5: Ayberk Zeytin T-Th(8:30) ___

PROBLEM	POINTS	SCORE
1	25	
2	20	
3	15	
4	15	
5	30	
TOTAL	105	

1. (a) (18 points) Find the inverse of the matrix

$$M = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|ccc} 7 & -3 & -3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_1(-1).]{R_1 \leftrightarrow R_2 \ \&} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & -1 & 0 \\ 7 & -3 & -3 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_1 + R_3 \to R_3]{R_1(-7) + R_2 \to R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 4 & -3 & 1 & 7 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2(\frac{1}{4}) \to R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -3/4 & 1/4 & 7/4 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{array}\right] \xrightarrow[R_2 + R_3 \to R_3]{R_2 + R_1 \to R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3/4 & 1/4 & 3/4 & 0 \\ 0 & 1 & -3/4 & 1/4 & 7/4 & 0 \\ 0 & 0 & 1/4 & 1/4 & 3/4 & 1 \end{array}\right] \xrightarrow{R_3 \cdot 4 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -3/4 & 1/4 & 3/4 & 0 \\ 0 & 1 & -3/4 & 1/4 & 7/4 & 0 \\ 0 & 0 & 1 & 1 & 3 & 4 \end{array}\right] \xrightarrow[R_3(\frac{3}{4}) + R_2 \to R_2]{R_3(\frac{3}{4}) + R_1 \to R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 3 \\ 0 & 1 & 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & 1 & 3 & 4 \end{array}\right] \qquad \text{So } M^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

(b) (7 points) Using the inverse of M in part (a) solve

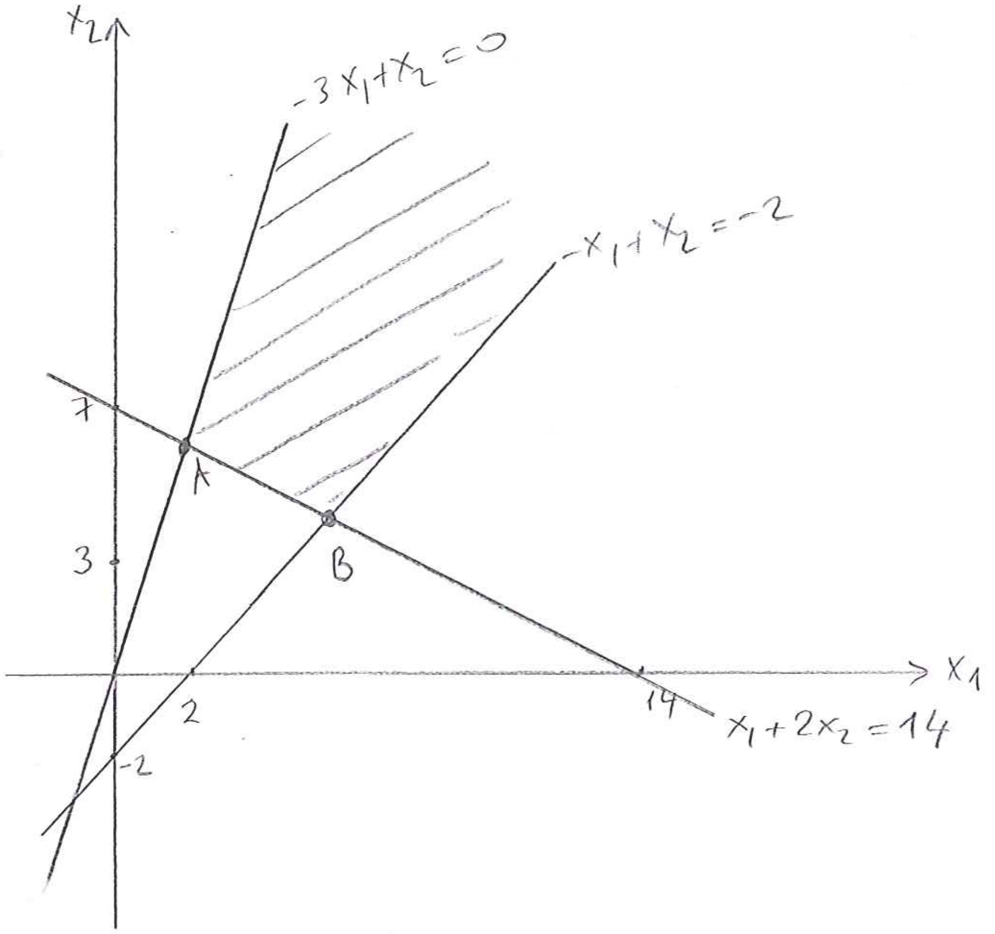
$$\begin{cases} 7x_1 - 3x_2 - 3x_3 &= 5 \\ -x_1 + x_2 &= 6 \\ -x_1 + x_3 &= 8 \end{cases}$$

$$M \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = M^{-1} \cdot \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 47 \\ 53 \\ 55 \end{bmatrix}.$$

2. (20 points) Determine the points minimizing and maximizing the objective function $Z = -3x_1 - 6x_2$, if exist, subject to the constraints

$$\begin{aligned} -x_1 + x_2 &\geq -2 \\ -3x_1 + x_2 &\leq 0 \\ x_1 + 2x_2 &\geq 14 \\ x_1,\ x_2, &\geq 0\ . \end{aligned}$$



$Z = -3x_1 - 6x_2$

A (2, 6) $\quad Z = -42$

B (6, 4) $\quad Z = -42$

There is no minimum value.

The maximum Z value is -42 and it is obtained at all the points in the line segment [A, B].

3. (15 points) Maximize the objective function $P = 2x_1 + x_2 + x_3$ subject to the constraints

$$\begin{aligned} x_1 + x_2 + 3x_3 &\leq 10 \\ 2x_1 + 4x_2 + 5x_3 &\leq 24 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

and determine the values of x_1, x_2, x_3 maximizing P.

$$X_1 + X_2 + 3X_3 + S_1 = 10$$

$$2X_1 + 4X_2 + 5X_3 + S_2 = 24$$

$$-2X_1 - X_2 - X_3 + P = 0$$

$$X_1, X_2, X_3, S_1, S_2 \geq 0$$

$$\begin{array}{cccccc|c} X_1 & X_2 & X_3 & S_1 & S_2 & P & \\ \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 3 & 1 & 0 & 0 & 10 \\ 2 & 4 & 5 & 0 & 1 & 0 & 24 \\ \hline -2 & -1 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[\substack{R_1(-2)+R_2 \to R_2 \\ R_1(2)+R_3 \to R_3}]{10/1=10 \quad 24/2=12} \left[\begin{array}{cccccc|c} 1 & 1 & 3 & 1 & 0 & 0 & 10 \\ 0 & 2 & -1 & -2 & 1 & 0 & 4 \\ \hline 0 & 1 & 5 & 2 & 0 & 1 & 20 \end{array}\right]$$

$\uparrow$

The maximum P value is 20.

$X_1 = 10$

$X_2 = 0$

$X_3 = 0$.

List of formulas: $I = Prt$; $A = P(1+rt)$

$A = P(1+i)^n$; $APY = (1+\frac{r}{m})^m - 1$; $A = Pe^{rt}$; $APY = e^r - 1$;

$FV = PMT\frac{[(1+i)^n-1]}{i}$; $PV = PMT\frac{[1-(1+i)^{-n}]}{i}$, where $i = \frac{r}{m}$ and $n = mt$

4. (15 points) Bank A, Bank B and Bank C have the same annual percentage yields (effective rates). If Bank A applies 8% interest compounded monthly, determine the annual nominal rates of Banks B and C given that Bank B applies interest compounded quarterly and Bank C applies interest compounded continuously. (Express your answers as a percentage correct to three decimal places.)

$$APY_A = \left(1+\frac{0.08}{12}\right)^{12} - 1$$

$$APY_B = \left(1+\frac{r_B}{4}\right)^{4} - 1$$

$$APY_C = e^{r_C} - 1$$

$$\left(1+\frac{0.08}{12}\right)^{12} - 1 = \left(1+\frac{r_B}{4}\right)^{4} - 1 \Rightarrow \left(1+\frac{0.08}{12}\right)^{12} = \left(1+\frac{r_B}{4}\right)^{4}$$

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$$\Rightarrow \left(1+\frac{0.08}{12}\right)^{3} = 1+\frac{r_B}{4} \Rightarrow r_B = \left[\left(1+\frac{0.08}{12}\right)^{3} - 1\right]4$$

$$\Rightarrow r_B = 0.08053 \Rightarrow 8.053\%.$$

$$\left(1+\frac{0.08}{12}\right)^{12} - 1 = e^{r_C} - 1 \Rightarrow e^{r_C} = \left(1+\frac{0.08}{12}\right)^{12}$$

$$\Rightarrow r_C = \ln\left(1+\frac{0.08}{12}\right)^{12} = 0.07973 \Rightarrow 7.973\%,$$

5. (a) (15 points) An account pays 9% interest compounded monthly. How long does it take to triple the deposit?

$$3P = A = P\left(1+\frac{0.09}{12}\right)^n$$

$$\ln 3 = n \ln\left(1+\frac{0.09}{12}\right)$$

$$n = \frac{\ln 3}{\ln\left(1+\frac{0.09}{12}\right)} \simeq 147.03$$

So $n = 148$ months.

$t = 12$ years and 4 months.

(b) (15 points) If you invest into the account paying 9% interest compounded monthly, in order to have \$120000 in 5 years what should be monthly periodic payment?

$$120\,000 = PMT \frac{\left(1+\frac{0.09}{12}\right)^{60} - 1}{\frac{0.09}{12}}$$

$$PMT = \frac{120\,000 \cdot 0.09}{12\left[\left(1+\frac{0.09}{12}\right)^{60} - 1\right]} = \$1591.00$$