KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Final Exam January 19, 2010

Duration of Exam: 120 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: ______________________________

Surname: ______________________________

Signature: ______________________________

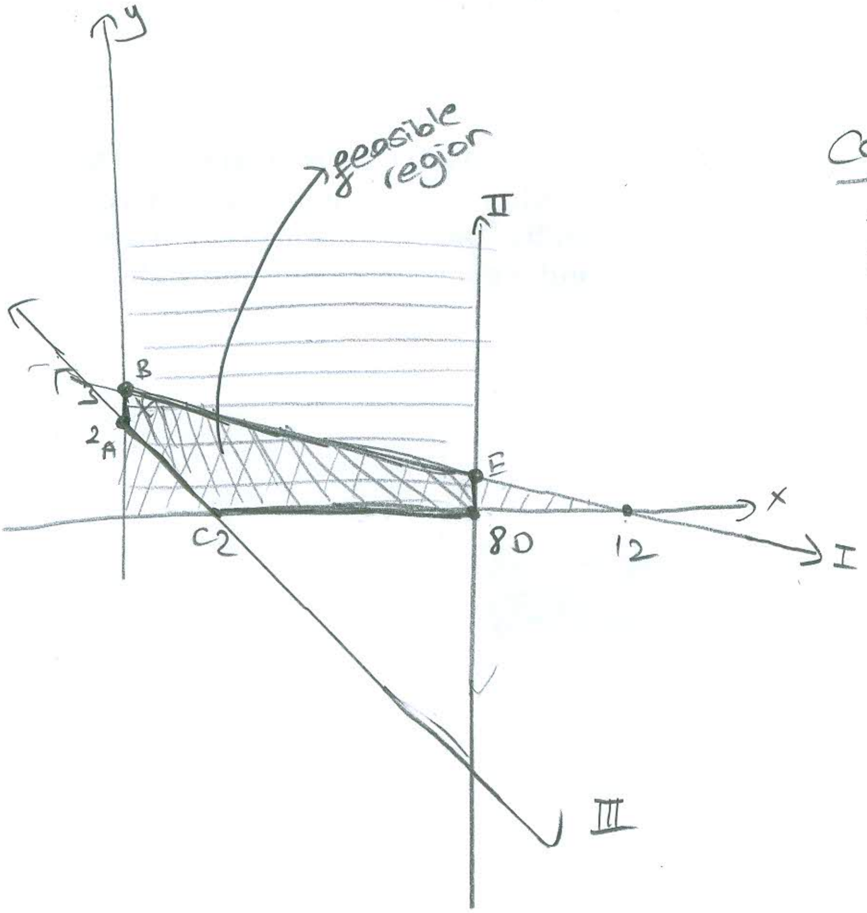
Section (Check One):

Section 1: Selda Küçükçifçi M-W (12:30) ___
Section 2: Selda Küçükçifçi M-W (15:30) ___
Section 3: E. Şule Yazıcı M-W(11:00) ___
Section 4: E. Şule Yazıcı M-W(14:00) ___
Section 5: Mehmet Sarıdereli T-Th(15:30) ___

PROBLEM	POINTS	SCORE
1	15	
2	14	
3	16	
4	12	
5	16	
6	20	
7	12	
TOTAL	**105**	

1. (15 points) Using the geometric approach maximize and minimize the objective function $z = x + 5y$ subject to the constraints

$$\begin{aligned} x + 4y &\le 12 && \mathrm{I} \\ x &\le 8 && \mathrm{II} \\ x + y &\ge 2 && \mathrm{III} \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$



Corner points	$z = x+5y$
A(0,2)	$z=10$
B(0,3)	$\boxed{z=15}$ ← max. value
C(2,0)	$\boxed{z=2}$ ← min. value
D(8,0)	$z=8$
E(8,1)	$z=13$

2. (14 points) (a) Solve the system $\begin{cases} x_1 + x_2 - x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 2 \\ x_1 + x_3 &= 3 \end{cases}$

using the fact that the inverse of $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is $\underbrace{\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{bmatrix}}_{=M^{-1}}$.

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{=M} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow M \times M^{-1} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = M^{-1} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -6 \\ -2 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 = 5 \\ x_2 = -6 \\ x_3 = -2 \end{array}$$

(b) Use Gauss Jordan elimination to bring the following augmented matrix into their reduced form. Write the solution set for the corresponding system.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -6 \\ 0 & -3 & -3 & 18 \end{array}\right] \xrightarrow[3R_2 + R_3 \to R_3]{-R_2 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 12$

$x_2 + x_3 = -6 \Rightarrow x_3 = t \Rightarrow x_2 = -t - 6$

$0 = 0$

(consistent and dependent)

$S = \{ (12, -t-6, t) \mid t \in \mathbb{R} \}$

A list of formulas: $I = Prt$; $A = P(1+rt)$

$A = P(1+i)^n$; $APY = (1+\frac{r}{m})^m - 1$;

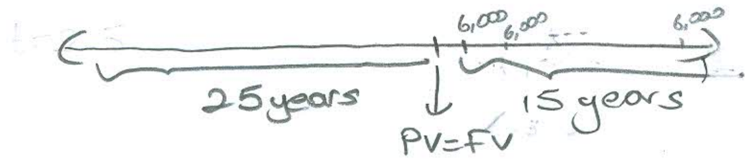
$A = Pe^{rt}$; $APY = e^r - 1$;

$FV = PMT\frac{[(1+i)^n - 1]}{i}$

$PV = PMT\frac{[1-(1+i)^{-n}]}{i}$, where $i = \frac{r}{m}$ and $n = mt$

3. (16 points) A person wants to establish an annuity for retirement purposes. He wants to make quarterly deposits for 25 years so that he can then make quarterly withdrawals of 6000 TL for 15 years. The annuity earns 6% interest compounded quarterly.

(a) How much will have to be in the account at the time he retires?



$$PV = 6000 \frac{\left(1-\left(1+\frac{0.06}{4}\right)^{-15\times 4}\right)}{\frac{0.06}{4}} = 236,281.61$$

(b) How much should be deposited each quarter for 25 years in order to accumulate the required amount?

$FV = 236,281.61$

$$236,281.61 = \left(\frac{\left(1+\frac{0.06}{4}\right)^{4\times 25} - 1}{\frac{0.06}{4}}\right) \times PMT$$

$\Rightarrow PMT = 1,032.69$

4. (12 points) Sketch the graph of the following functions by specifying at least 3 points on the graph

(a) $f(x) = \log_2(x+2)$

$x+2>0$

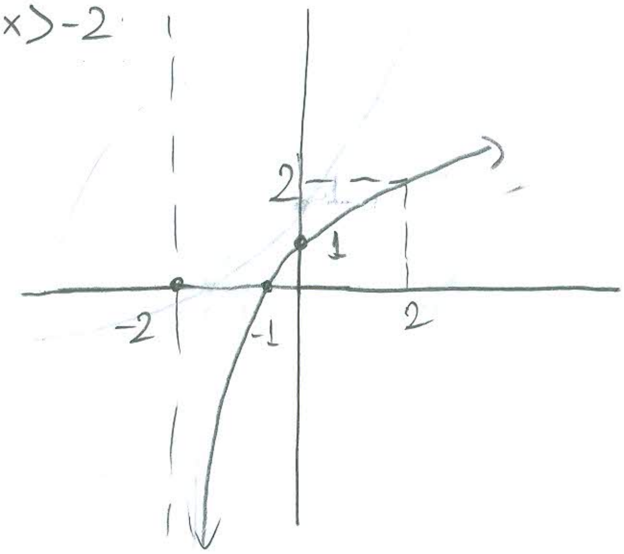
$x>-2$

$x=0 \Rightarrow y = \log_2 2 = 1$

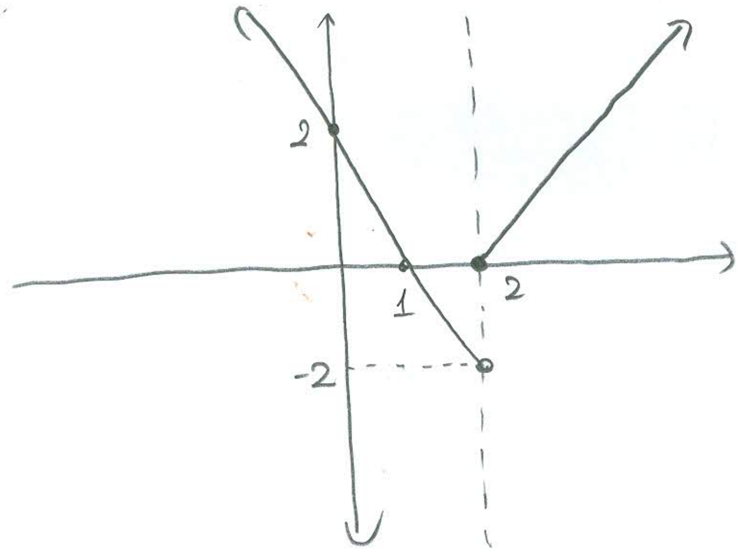
$y=0 \Rightarrow 0 = \log_2(x+2)$

$1 = x+2 \Rightarrow x = -1$

$x = 2 \Rightarrow \log_2 4 = 2$



(b) $f(x) = \begin{cases} 2-2x & if \quad x<2 \\ x-2 & if \quad x \geq 2 \end{cases}$



5. (16 points) There are 20 students in the a school.

(a) In how many ways can we choose a president; a vise-president; a secretary and a member for a 4 person student council?

$$\underbrace{\binom{20}{4}}_{\text{Choose 4 people from 20 students}}\underbrace{\binom{4}{1}}_{\text{choose a president}}\underbrace{\binom{3}{1}}_{\text{(choose a vise-president)}}\underbrace{\binom{2}{1}}_{\text{a secretary}}\underbrace{\binom{1}{1}}_{\text{a member}} = \frac{\cancel{20}^{5} \cdot 19 \cdot \cancel{18}^{3} \cdot 17}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4}} \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 116{,}280$$

(b) In how many ways can we choose a library committee of 5 person, if Mary should be and; Mark should not be in the committee?

Mary + Mark + 18 students

$$\binom{18}{4} = \frac{18 \cdot 17 \cdot \cancel{16}^{2} \cdot \cancel{15}^{5}}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4}} = 3{,}060$$

6. (20 points) Using the simplex method maximize the objective function $P = 3x_1 + 2x_2 + x_3$ subject to the constraints

$$\begin{aligned} x_1 + x_2 + x_3 &\le 6 \\ x_1 - x_2 - x_3 &\le 3 \\ x_1,\ x_2,\ x_3 &\ge 0 \end{aligned}$$

$$x_1 + x_2 + x_3 + s_1 = 6$$

$$x_1 - x_2 - x_3 + s_2 = 3$$

$$-3x_1 - 2x_2 - x_3 + P = 0$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

$$\begin{array}{c c} & \begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 6 \\ 1 & -1 & -1 & 0 & 1 & 0 & 3 \\ \hline -3 & -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \xrightarrow[3R_2 + R_3 \to R_3]{-R_2 + R_1 \to R_1} \left[\begin{array}{cccccc|c} 0 & 2 & 2 & 1 & -1 & 0 & 3 \\ 1 & -1 & -1 & 0 & 1 & 0 & 3 \\ \hline 0 & -5 & -4 & 0 & 3 & 1 & 9 \end{array}\right]$$

pivot column

pivot column

$$\xrightarrow{\frac{R_1}{2} \to R_1} \left[\begin{array}{cccccc|c} 0 & 1 & 1 & 1/2 & -1/2 & 0 & 3/2 \\ 1 & -1 & -1 & 0 & 1 & 0 & 3 \\ \hline 0 & -5 & -4 & 0 & 3 & 1 & 9 \end{array}\right] \xrightarrow[5R_1 + R_3 \to R_3]{R_1 + R_2 \to R_2} \left[\begin{array}{cccccc|c} 0 & 1 & 1 & 1/2 & -1/2 & 0 & 3/2 \\ 1 & 0 & 0 & 1/2 & 1/2 & 0 & 9/2 \\ \hline 0 & 0 & 1 & 5/2 & 1/2 & 1 & 16.5 \end{array}\right]$$

$x_1 = 9/2$

$x_2 = 3/2$

$s_1 = 0$

$s_2 = 0$

$\boxed{P = 16.5}$

7. (12 points) Let $\sin(3x - \pi/5) = -0.5$. Find all solutions for x in the interval $[\pi, 3\pi/2]$.

Let $3x - \frac{\pi}{5} = A \Rightarrow \sin A = -\frac{1}{2}$

$$A = \pi + \frac{\pi}{6} + 2k\pi \qquad A = -\frac{\pi}{6} + 2k\pi$$

$$= \frac{7\pi}{6} + 2k\pi$$

$$3x - \frac{\pi}{5} = -\frac{\pi}{6} + 2k\pi$$

$$3x - \frac{\pi}{5} = \frac{7\pi}{6} + 2k\pi$$

$$3x = \frac{\pi}{30} + 2k\pi$$

$$3x = \frac{41\pi}{30} + 2k\pi$$

$$x = \frac{\pi}{90} + \frac{2}{3}k\pi$$

$$x = \frac{41\pi}{90} + \frac{2k\pi}{3}$$

$$k=2 \Rightarrow x = \frac{\pi}{90} + \frac{4\pi}{3} = \frac{121\pi}{90}$$

$$k = 1 \qquad x = \frac{101\pi}{90}$$