KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Final Exam January 11, 2012

Duration of Exam: 110 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: _____

Surname: _____

Signature: _____

Section (Check One):

Section 1: E. Şule Yazıcı M-W-F (10:30) ___
Section 2: E. Şule Yazıcı M-W-F (11:30) ___
Section 3: Mehmet Sarıdereli M-W-F (13:30) ___
Section 4: Ali Göktürk M-W-F (15:30) ___
Section 5: Selda Küçükçifçi T-Th-F (12:30) ___

PROBLEM	POINTS	SCORE
1	15	
2	15	
3	15	
4	20	
5	15	
6	10	
7	15	
TOTAL	**105**	

1. (15 points) Let $\cos\left(4x - \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2}$. Find all solutions for x in the interval $[-2, 5]$.

$4x - \frac{\pi}{6} = A$ $\quad \cos A = \frac{-\sqrt{3}}{2} \rightarrow A_1 = \frac{5\pi}{6} \quad A_2 = \frac{7\pi}{6}$

or $\arccos A = \arccos \frac{-\sqrt{3}}{2}$

$A_1 = \frac{5\pi}{6} + 2\pi k$ $\qquad$ $A_2 = \frac{7\pi}{6} + 2\pi k$

$4x - \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi k$

$x_A = \frac{\pi + 2\pi k}{4} = \frac{\pi}{4} + \frac{\pi}{2}k$

$(k=0)$ $x_1 = \pi/4$

$(k=1)$ $x_2 = 3\pi/4$

$(k=2)$ $x_3 = 5\pi/4$

$(k=-1)$ $x_4 = \frac{-\pi}{4}$

$4x - \frac{\pi}{6} = \frac{7\pi}{6} + 2\pi k$

$x_B = \frac{\frac{4\pi}{3} + 2\pi k}{4} = \frac{\pi}{3} + \frac{\pi}{2}k$ $\quad (k=0)$

$x_5 = \frac{\pi}{3}$

$x_6 = \frac{5\pi}{6}$

$x_7 = \frac{4\pi}{3}$

$x_8 = \frac{-\pi}{6}$

We have 8 sowtions for x!

2. (15 points) Solve the following system using Gauss Jordan Elimination method. Write the solution set and determine if the system is **consistent, inconsistent, dependent or independent**.

$$\begin{cases} x_1 - 2x_2 + 2x_3 + 9x_5 = 2 \\ x_1 - 2x_2 + 2x_3 + x_4 + 5x_5 = 4 \\ 3x_1 - 6x_2 + 3x_3 + 6x_4 + 21x_5 = 3 \end{cases}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 2 & 0 & 9 & 2 \\ 1 & -2 & 2 & 1 & 5 & 4 \\ 3 & -6 & 3 & 6 & 21 & 3 \end{array}\right] \begin{matrix} (-R_1)+R_2 \to R_2 \\ \longrightarrow \\ (-3R_1)+R_3 \to R_3 \end{matrix} \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 0 & 9 & 2 \\ 0 & 0 & 0 & 1 & -4 & 2 \\ 0 & 0 & -3 & 6 & 6 & -3 \end{array}\right] \begin{matrix} \text{change} \\ \text{rows} \\ \longrightarrow \end{matrix} \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 0 & 9 & 2 \\ 0 & 0 & -3 & 6 & 6 & -3 \\ 0 & 0 & 0 & 1 & -4 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 2 & 0 & 9 & 2 \\ 0 & 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 1 & -4 & 2 \end{array}\right] \longrightarrow R_2(-2)+R_1 = R_1$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 4 & 5 & 0 \\ 0 & 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 1 & -4 & 2 \end{array}\right] \begin{matrix} R_3(2)+R_2 \Rightarrow R_2 \\ \longrightarrow \\ R_3(-4)+R_1 \Rightarrow R_1 \end{matrix} \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 21 & -8 \\ 0 & 0 & 1 & 0 & -6 & 5 \\ 0 & 0 & 0 & 1 & -4 & 2 \end{array}\right]$$

$x_5 = t \quad x_2 = y$

$x_5 = t$

$x_4 = 2 + 4t$

$x_3 = 5 + t$

$x_2 = y$

$x_1 = -8 - 21t + 2y$

$x_1 - 2x_2 + 21x_5 = -8$

$x_3 - x_5 = 5$

$x_4 - 4x_5 = 2$

} solution set

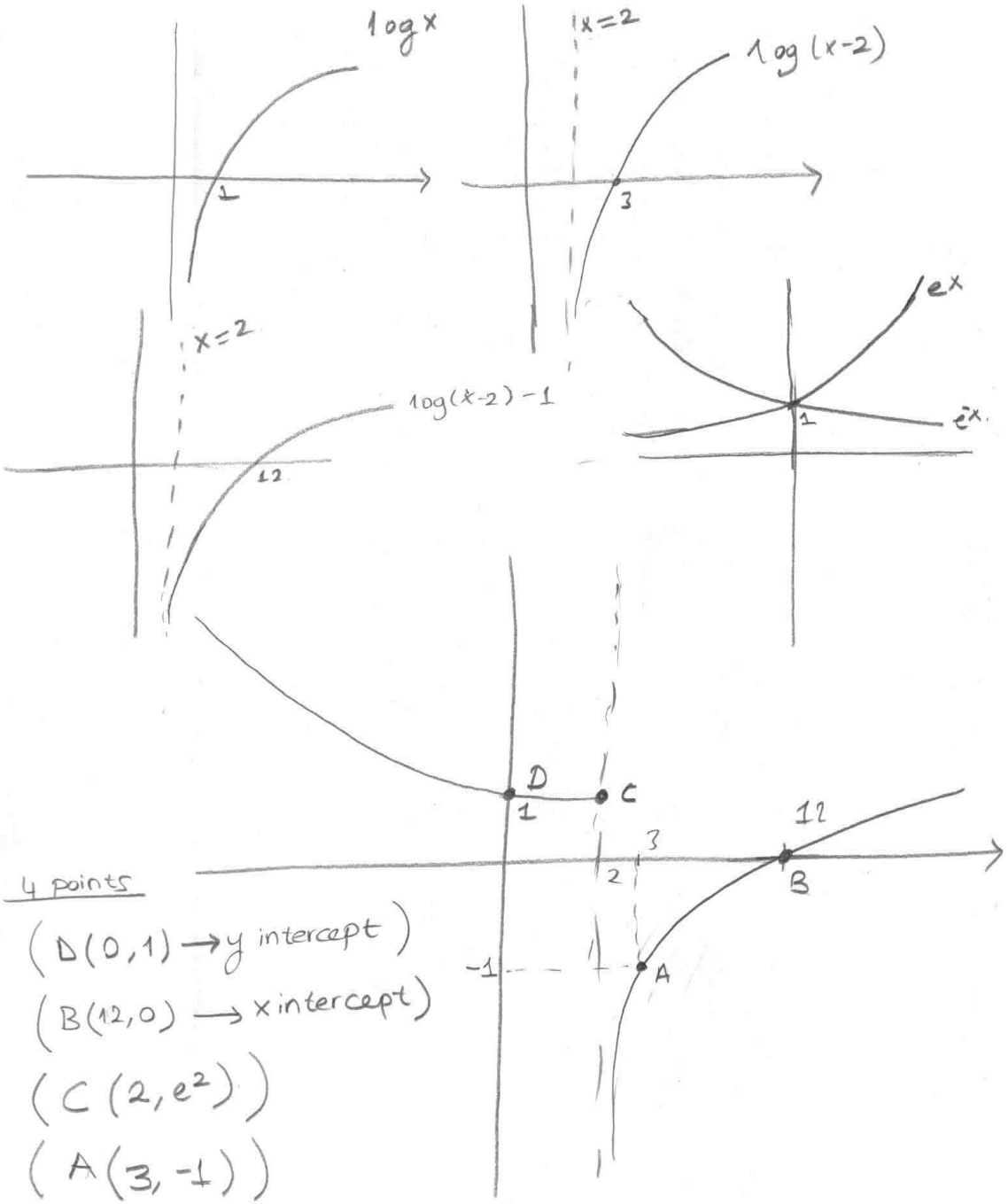
$$\{ -8 - 21t + 2y, y, 5 + t, 2 + 4t, t \}$$

dependent, consistent.

3. (15 points) Sketch the graph of the function

$$f(x) = \begin{cases} \log(x-2) - 1 & x > 2 \\ e^{-x} & x \leq 2 \end{cases}$$

by specifying at least 4 points on the graph. Find also x and y-intercepts.



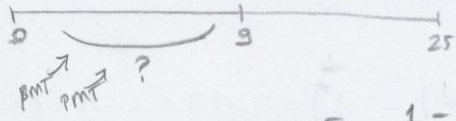
A list of formulas: $I = Prt$; $A = P(1+rt)$

$$A = P(1+i)^n;\ \ APY = (1+\tfrac{r}{m})^m - 1;\ \ A = Pe^{rt};\ \ APY = e^r - 1;$$

$$FV = PMT\tfrac{[(1+i)^n-1]}{i};\ \ PV = PMT\tfrac{[1-(1+i)^{-n}]}{i},\ \text{where } i = \tfrac{r}{m} \text{ and } n = mt$$

4. (20 points) Ahmet Uyank signed a 225,000 TL 25-year mortgage at 12.4% compounded monthly with Work Bank to buy his house 9 years ago. Whitebank now offers to refinance his mortgage at 9.9% compounded monthly. Ahmet Bey decides to accept the offer. His new 16-year mortgage will be in the amount of what he still owes Work Bank (after 9 years of making monthly payments to Work Bank).

(a) How much did Ahmet Bey pay Work Bank every month during the first nine years?

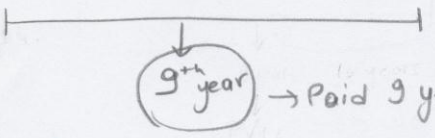


→ we need to calculate for 25 years!

$$225000 = PMT\ \frac{1-(1+\frac{0.124}{12})^{-12\times 25}}{\frac{0.124}{12}}$$

$$PMT = 2,436.52$$

(b) Show that the amount Ahmet Bey still owes Work Bank after nine years of monthly payments is 203,034.76 TL.



→ Paid 9 year so our debt is the PV for 16 years

$$PV_{16} = 2,436.52\ \frac{1-(1+\frac{0.124}{12})^{-12\times 16}}{\frac{0.124}{12}}$$

$$PV_{16} = 203,034.76 \text{ (prooved✓)}$$

(c) How much will Ahmet Bey have to pay Whitebank per month for the next 16 years?

new PV = 203,034.76 ⟷ 16 years.

$$203{,}034.76 = PMT \frac{1-\left(1+\frac{0.099}{12}\right)^{(16\times12)}}{\frac{0.099}{12}}$$

new PMT = 2,110.92

(d) How much will Ahmet Bey save by refinancing his mortgage?

$PMT_1 \times (16 \times 12) = 467{,}811.84$ = He would have paid this much

(PMT_1 ↓ 2,436.52)

$PMT_2 \times (16 \times 12) = 405{,}296.96$ = He paid this much

(PMT_2 ↓ 2,110.92)

467,811.84 − 405,296.96 = 62,514.88

He saved this much!

5. (15 points) Maximize the function $P = 10x_1 + 50x_2 + 10x_3$ subject to the constraints

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 22 \\ 3x_1 - x_2 + 2x_3 &\leq 24 \\ x_1 + x_2 + 3x_3 &\leq 36 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

$x_1 + x_2 + x_3 + s_1 = 22$

$3x_1 - x_2 + 2x_3 + s_2 = 24$

$x_1 + x_2 + 3x_3 + s_3 = 36$

$-10x_1 - 50x_2 - 10x_3 + P = 0$

$x_1, x_2, x_3, s_1, s_2, s_3, P \geq 0$

→ pivot element

1	(1)	1	1	0	0	0	22
3	-1	2	0	1	0	0	24
1	1	3	0	0	1	0	36
-10	(-50)	-10	0	0	0	1	0

→ most negative number

$\rightarrow \frac{22}{1} = 22$

$\rightarrow \frac{24}{-1} = -24$

$\rightarrow \frac{36}{1} = 36$

$R_1 + R_2 \Rightarrow R_2$

$R_1(50) + R_4 \Rightarrow R_4$

$R_1(-1) + R_3 \Rightarrow R_3$

1	1	1	1	0	0	0	22
4	0	3	1	1	0	0	46
0	0	2	-1	0	1	0	14
40	0	40	50	0	0	1	1100

$x_1 = 0 \quad x_2 = 22$

$x_3 = 0 \quad s_1 = 0$

$s_2 = 46 \quad s_3 = 14$

$P = 1100$

Maximum value is 1100

6. (10 points) A textile company has 3 types of products type A, B and C. Each meter of type A is made from 2 kilograms of wool and 4 kilograms of cotton. Similarly each meter of type B is made from 3 kilograms of wool and 2 kilograms of cotton and type C is made from 5 kilograms of wool and 1 kilogram of cotton. In stock the company has 500 kilograms of wool and 300 kilograms of cotton. The machines can make at most 200 meters a day. The company makes a profit of 5 TL, 3 TL and 4 TL for each meter of type A, B and C, respectively. How many meter of each type should the company produce to maximize its daily profit?

Write the decision variables, appropriate equation(s) and inequalities so that this problem can be solved.

DO NOT SOLVE THE PROBLEM after you have written the equation(s) and inequalities.

	A	B	C	Total
W	2kg	3kg	5kg	500kg
Cot	4kg	2kg	1kg	300kg

x_1 meter A, x_2 meter B, x_3 meter C, P = profit

subject to constraints

$$2x_1 + 3x_2 + 5x_3 \leq 500$$
$$4x_1 + 2x_2 + x_3 \leq 300$$
$$x_1 + x_2 + x_3 \leq 200$$
$$x_1, x_2, x_3 \geq 0$$

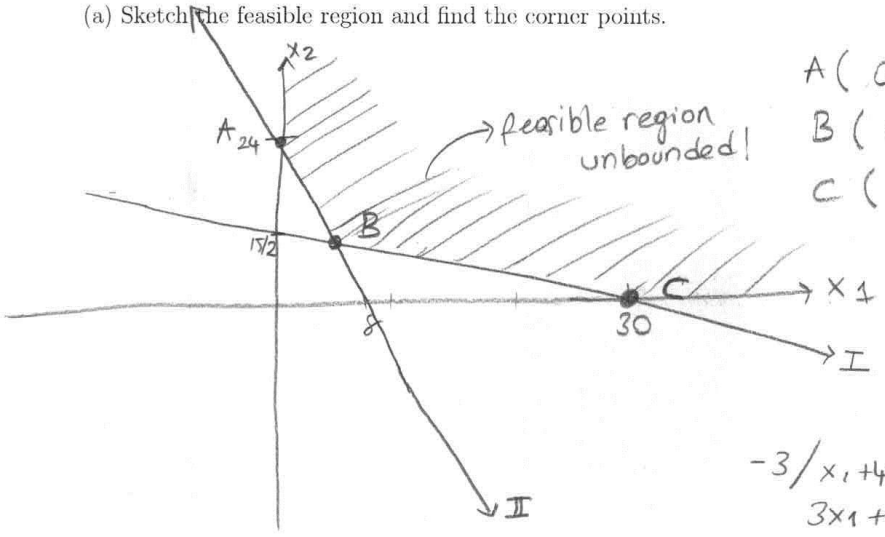
maximize the objective function

$$5x_1 + 3x_2 + 4x_3 = P$$

7. (15 points) Consider the objective function $z = x_1 + 5x_2$ subject to the constraints

$$\begin{aligned} x_1 + 4x_2 &\geq 30 \quad \text{I} \\ 3x_1 + x_2 &\geq 24 \quad \text{II} \\ x_1, x_2 &\geq 0 \quad \text{III} \end{aligned}$$

(a) Sketch the feasible region and find the corner points.



A (0,24)
B (6, 6)
C (30,0)

$-3/ x_1 + 4x_2 = 30$
$3x_1 + x_2 = 24$

$x_1 = 6 \quad x_2 = 6$

(b) If exist, find maximum and minimum values of z and where they occur.

A (0,24) → $z = x_1 + 5x_2$ → $z_1 = 0 + 24 \cdot 5 = 120$

B (6, 6) → $z_2 = 6 \cdot 1 + 5 \cdot 6 = 36$

C (30, 0) → $z_3 = 30 \cdot 1 + 5 \cdot 0 = 30$

no max value since it's unbounded

min value is $z_3 = 30$