KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm I March 20, 2012

Duration of Exam: 75 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit.** Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: _____

Surname: KEY

Signature: _____

Section (Check One):

Section 1: E. Şule Yazıcı T-Th-F(10:30) ___
Section 2: E. Şule Yazıcı T-Th-F(13:30) ___
Section 3: Selda Küçükcifci M-W-F(11:30) ___
Section 4: E. Şule Yazıcı T-Th-F(14:30) ___

PROBLEM	POINTS	SCORE
1	14	
2	12	
3	20	
4	16	
5	18	
6	20	
TOTAL	100	

1. Calculate the following if exists.

a-) (7 points) $\sin(\frac{2013\pi}{2}) = ?$

$\sin\left(\frac{2013\pi}{2}\right) = \sin\left(1006\pi + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1.$

b-) (7 points) $\cot^{-1}(-1) = ?$

$\cot^{-1}(-1)$ must be in $(0, \pi)$

So $\cot^{-1}(-1) = 3\pi/4.$

2. (12 points) Find the domain of the function $f(x) = \log(\sin^2 x)$

$\sin^2 x > 0$

So $\sin x \neq 0$

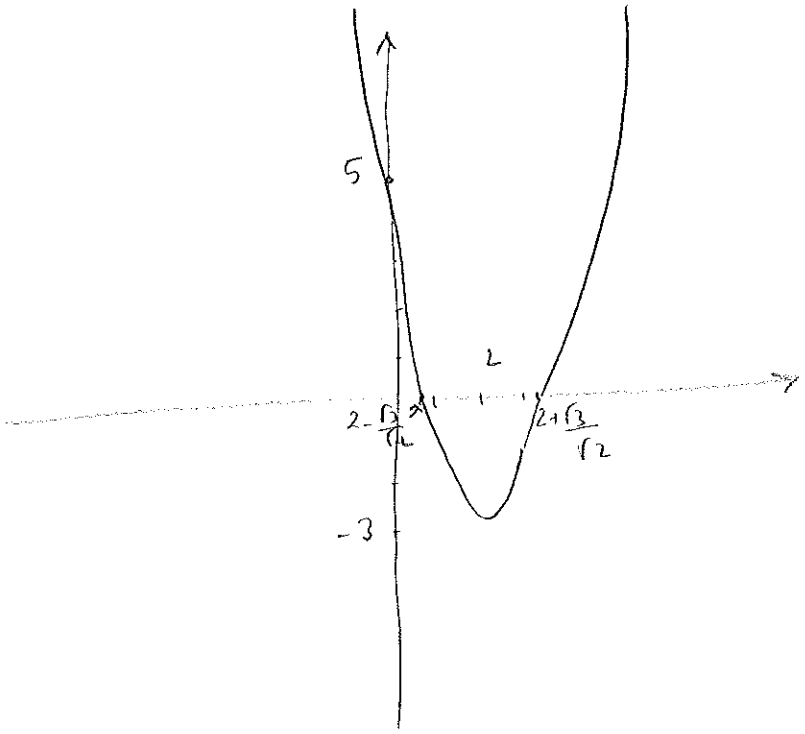
Domain: $\mathbb{R} \setminus \{\pi k : k \in \mathbb{Z}\}$

3. (20 points) If the vertex of a parabola is $(2, -3)$ and the parabola passes through the point $(4, 5)$ find the equation of the parabola. Sketch the graph of the parabola. Find the x and y- intercepts of the parabola.

$$y = a(x-2)^2 - 3$$

$$5 = a \cdot 2^2 - 3 \Rightarrow 4a = 8 \Rightarrow a = 2$$

So $y = 2(x-2)^2 - 3$.



y- intercept : $x = 0$ $\quad y = 5$

x- intercepts : $y = 0$ $\quad 2(x-2)^2 = 3$

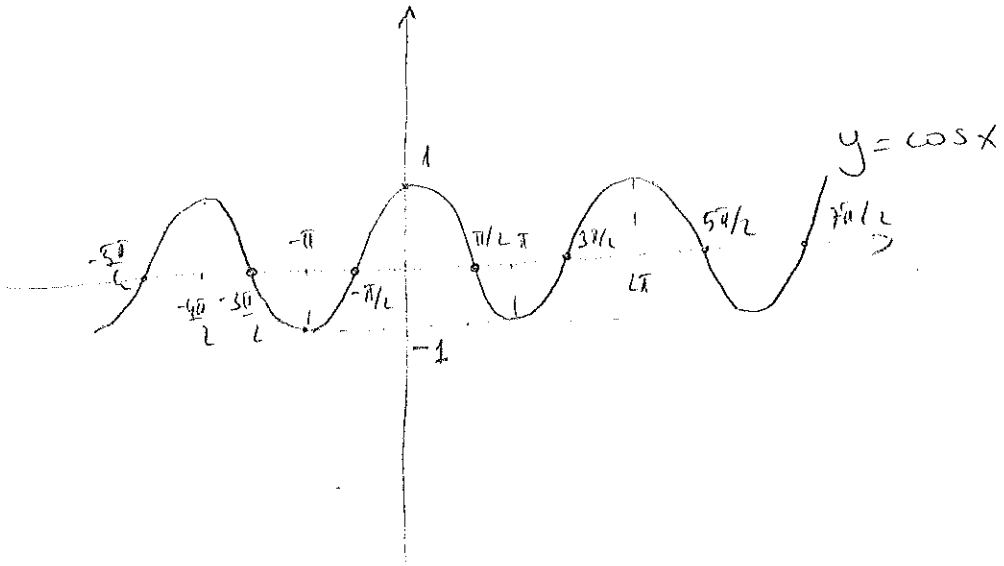
$(x-2)^2 = 3/2$

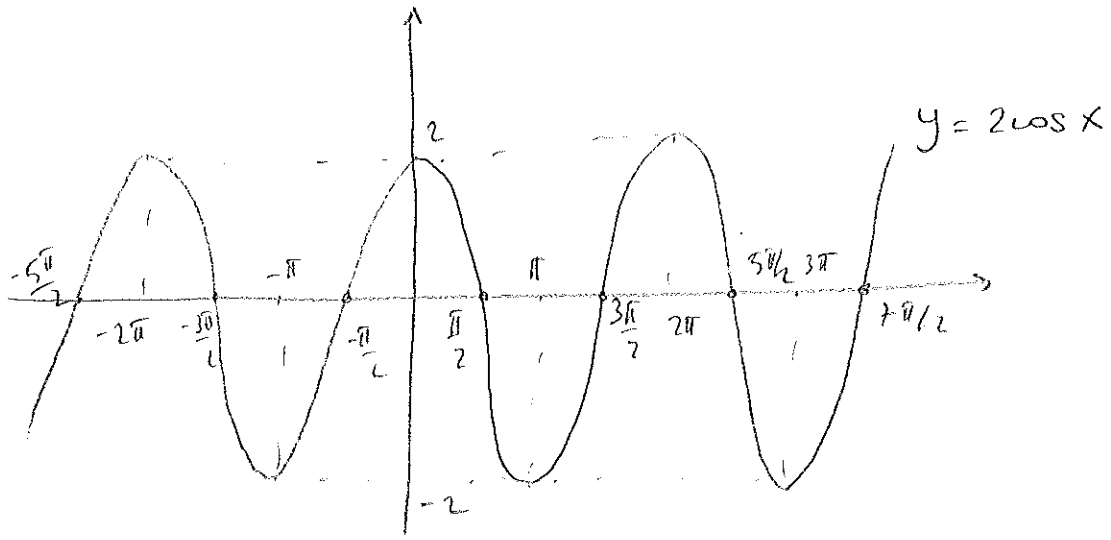
$x - 2 = \frac{\sqrt{3}}{\sqrt{2}}$ $\qquad x - 2 = -\frac{\sqrt{3}}{\sqrt{2}}$

$x = \frac{\sqrt{3}}{\sqrt{2}} + 2$ $\qquad x = -\frac{\sqrt{3}}{\sqrt{2}} + 2$

4. (16 points) Sketch the graph of $f(x) = -2\sin(x - \frac{\pi}{2})$ by using horizontal translation, vertical translation, stretch and shrink where it is appropriate. Indicate each step. Specify x and y intercepts.

$$f(x) = -2\sin\left(x - \frac{\pi}{2}\right) = 2\sin\left(\frac{\pi}{2} - x\right) = 2\cos x$$





5. (18 points) Solve the following equations

a-)

$$-\log_2(-x+1) = \log_2(-3x-1) - \log_2 7$$

$$\Rightarrow \log_2(-x+1)^{-1} = \log_2 \frac{-3x-1}{7}$$

$$\Rightarrow \frac{1}{-x+1} = \frac{-3x-1}{7}$$

$$\Rightarrow 7 = 3x^2 - 3x + x - 1$$

$$\Rightarrow 3x^2 - 2x - 8 = 0$$

$$\Rightarrow (3x+4)(x-2) = 0$$

$$\Rightarrow x = -\frac{4}{3} \qquad x = 2$$

since $x = 2$ makes $\log_2(-3x-1)$ undefined

$$S = \{-4/3\}.$$

b-)

$$(\log_2 x)^{\ln e} = (\ln e)^{(\log_2 x)}$$

$$(\log_2 x)^1 = 1^{\log_2 x}$$

$$\Rightarrow \log_2 x = 1$$

$$\Rightarrow x = 2$$

6. a-) (10 points) How long will it take 1000 TL to grow to 1060 TL if it is invested at 12% simple interest (nominal annual interest rate)?

$$I = 60 = Prt = 1000 \cdot 0.12 \cdot t$$

$$60 = 120t$$

$$t = \frac{1}{2} \text{ year}$$

$$\Updownarrow$$

6 months.

b-) (10 points) How much should you invest now to have 14641 TL in 2 years at 20% compounded semiannually?

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$14641 = P\left(1 + \frac{0.2}{2}\right)^{4}$$

$$14641 = P(1.1)^4$$

$$P = \frac{14641}{(1.1)^4} = \frac{14641}{1.4641} = \$10000$$

$(1.1)^2 = 1.21$

$(1.21)^2 = 1.4641$

$$\begin{array}{r} 121 \\ \times\ 121 \\ \hline 121 \\ 242 \\ +\ 121 \\ \hline 14641 \end{array}$$