# KOÇ UNIVERSITY

# MATH 101 - FINITE MATHEMATICS

## Midterm 2 May 2, 2012

**Duration of Exam: 80 minutes**

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: _____

Surname: KEY

Signature: _____

Section (Check One):

Section 1: E. Şule Yazıcı T-Th-F(10:30) ___
Section 2: E. Şule Yazıcı T-Th-F(13:30) ___
Section 3: Selda Küçükcifci M-W-F(11:30) ___
Section 4: E. Şule Yazıcı T-Th-F(14:30) ___

| PROBLEM | POINTS | SCORE |
|---|---|---|
| 1 | 18 | |
| 2 | 16 | |
| 3 | 16 | |
| 4 | 19 | |
| 5 | 15 | |
| 6 | 16 | |
| TOTAL | 100 | |

A list of formulas: $I = Prt$; $A = P(1+rt)$

$A = P(1+i)^n$; $APY = (1+\frac{r}{m})^m - 1$; $A = Pe^{rt}$; $APY = e^r - 1$;

$FV = PMT\frac{[(1+i)^n-1]}{i}$; $PV = PMT\frac{[1-(1+i)^{-n}]}{i}$, where $i = \frac{r}{m}$ and $n = mt$

1. (18 points) Construct the amortization schedule for a loan of 1000 TL that is to be amortized in 4 equal monthly payments at 1% interest per month on the unpaid balance.

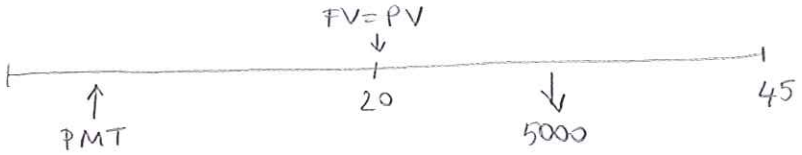
| Payment number | Payment | Interest | Unpaid balance reduction | Unpaid balance |
|---|---|---|---|---|
| 0 | | | | 1000 TL |
| 1 | 256.28 | 10 | 246.28 | 753.72 |
| 2 | 256.28 | 7.54 | 248.74 | 504.98 |
| 3 | 256.28 | 5.05 | 251.23 | 253.75 |
| 4 | 256.29 | 2.54 | 253.75 | 0 |
| Totals | 1025.13 | 25.13 | — | — |

$$1000 = PMT\left(\frac{1-(1+0.01)^{-4}}{0.01}\right)$$

$$PMT = \frac{10}{1-(1.01)^{-4}} = 256.28 \text{ TL.}$$

2. (16 points) A person wants to establish an annuity for retirement purposes. He wants to make quarterly deposits for 20 years so that he can then make quarterly withdrawals of 5000 TL for 25 years. The annuity earns 12% interest compounded quarterly.

(a) How much will have to be in the account at the time he retires?




$$PV = 5000 \; \frac{1 - \left(1 + \frac{0,12}{4}\right)^{-100}}{\frac{0,12}{4}}$$

$$PV = 157\,994.53 \text{ TL}.$$

(b) How much should be deposited each quarter for 20 years in order to accumulate the required amount?

$$FV = 157\,994.53$$

$$157\,994.53 = PMT \; \frac{\left(1 + \frac{0,12}{4}\right)^{80} - 1}{\frac{0.12}{4}}$$

$$PMT = \frac{157\,994.53 \times 0.03}{(1.03)^{80} - 1} = 491.64 \text{ TL}.$$

3. (16 points) A family has a 150000 TL, 30 year mortgage at 6.1% compounded monthly.

(a) Find the monthly payment.

$$150\,000 = PMT \; \frac{1-\left(1+\frac{0.061}{12}\right)^{-360}}{\frac{0.061}{12}}$$

$$PMT = 150000 \cdot \frac{0.061}{12} \cdot \frac{1}{1-\left(1+\frac{0.061}{12}\right)^{-360}}$$

$$PMT = 908.99 \text{ TL}.$$

(b) Find the unpaid balance after 10 years?

$$PV = 908.99 \; \frac{1-\left(1+\frac{0.061}{12}\right)^{-240}}{\frac{0.061}{12}}$$

unpaid balance after 10 years: $PV = 125\,861.92$ TL.

4. (19 points) Use Gauss Jordan elimination to bring the following augmented matrix into their reduced form. Write the solution set for the corresponding system.

(a) $\left[\begin{array}{cccc|c} 1 & 4 & 2 & 0 & -3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right] \xrightarrow{R_2(-2)+R_1 \to R_1} \left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right] \xrightarrow{R_3 \cdot \frac{1}{2} \to R_3} \left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$$\begin{cases} x_1 + 4x_2 = -7 \\ x_3 = 2 \\ x_4 = 2 \end{cases}$$

$$\begin{aligned} x_1 &= -7 - 4t \\ x_2 &= t \\ x_3 &= 2 \\ x_4 &= 2 \end{aligned} \quad , t \in \mathbb{R}$$

$$S = \{(-7-4t, t, 2, 2) : t \in \mathbb{R}\}$$

(b) $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 3 & 4 \end{array}\right] \xrightarrow[R_1(-3)+R_3 \to R_3]{R_1(-2)+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & -2 \end{array}\right] \xrightarrow{R_2(-1) \to R_2} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{array}\right] \xrightarrow{R_2(-1)+R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{array}\right]$

$$\begin{cases} x_1 = 1 \\ x_2 = 1 \\ 0 = -2 \end{cases}$$

So there is no solution

$$S = \phi$$

5. (15 points) Let $A$ be a $3 \times 3$ matrix. Find $A$ if

$$\underbrace{\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}}_{B} A = \underbrace{\begin{bmatrix} 3 & 2 & 3 \\ 1 & 1 & 1 \\ -3 & -1 & -4 \end{bmatrix}}_{C}$$

$$A = B^{-1} \cdot C$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3+R_1 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2(-1)+R_3 \to R_3]{R_2(-2)+R_1 \to R_1} \underbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]}_{B^{-1}}$$

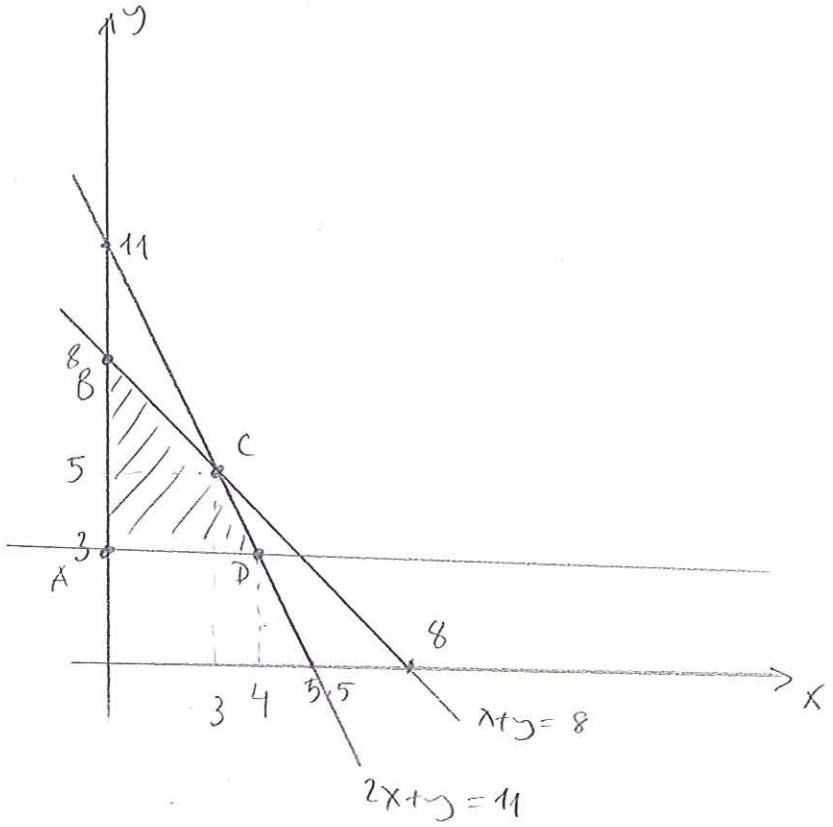
$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 3 \\ 1 & 1 & 1 \\ -3 & -1 & -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 3-2 & 2-2 & 3-2 \\ 1 & 1 & 1 \\ 3-1-3 & 2-1-1 & 3-1-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 0 & -2 \end{bmatrix}$$

6. (16 points) Consider the following system of linear inequalities.

$$\begin{aligned} 2x + y &\leq 11 \\ x + y &\leq 8 \\ x &\geq 0 \\ y &\geq 3 \end{aligned}$$

Sketch the feasible region and find the corner points.




A (0, 3)

B (0, 8)

C (3, 5)

D (4, 3)

$$C: \begin{cases} x+y=8 \\ 2x+y=11 \end{cases}$$

$x = 3$

$y = 5$