KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm I March 12, 2019

Duration of Exam: 90 minutes

INSTRUCTIONS: You can use calculators in the exam. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: _____

Surname: KEY

Signature: _____

Section (Check One):

Section 1: Selda Küçükçiftçi Tu-Th (11:30) ___
Section 2: Ayberk Zeytin Tu-Th(11:30) ___
Section 3: Ayberk Zeytin Tu-Th(16:00) ___

PROBLEM	POINTS	SCORE
1	20	
2	27	
3	30	
4	15	
5	18	
TOTAL	**110**	

A list of formulas: $I = Prt$; $A = P(1+rt)$

$$A = P(1+i)^n;\ APY = (1+\tfrac{r}{m})^m - 1;\ A = Pe^{rt};\ APY = e^r - 1;$$

$$FV = PMT\tfrac{[(1+i)^n-1]}{i};\ PV = PMT\tfrac{[1-(1+i)^{-n}]}{i},\ \text{where } i = \tfrac{r}{m} \text{ and } n = mt$$

1. (20 points) A bank offers the following :

I. 24% simple interest
II. 23,5 % compounded semi-annually
III. 23% compounded quarterly
IV. 22,5% compounded monthly
V. 22% compounded continuously

Order the options from least profitable to most profitable. Answer : I < V < II < IV < III

$$APY_1 = 0.24$$

$$APY_2 = \left(1 + \frac{0.235}{2}\right)^2 - 1 = 0.24881$$

$$APY_3 = \left(1 + \frac{0.23}{4}\right)^4 - 1 = 0.25061$$

$$APY_4 = \left(1 + \frac{0.225}{12}\right)^{12} - 1 = 0.24972$$

$$APY_5 = e^{0.22} - 1 = 0.24608$$

2. Alp Emekli makes monthly payments of 320 TL to a retirement fund paying 1,9% per month. He retires after making 20 years of payment and agrees to be paid a monthly salary of 32 000 TL.

(a) (12 points) How much does Alp Emekli has in his retirement fund when he retires?

$$FV = 320 \frac{(1+0.019)^{240} - 1}{0.019}$$

$$= 1525\,570.41 \text{ TL.}$$

(b) (15 points) How long can Alp Emekli be paid the salary of 32 000 TL ?

$$1525\,570.41 = 32\,000 \frac{1-(1.019)^n}{0.019}$$

$$\frac{1525\,570.41 \times 0.019}{32\,000} = 1 - (1.019)^n$$

$$(1.019)^n = \left(1 - \frac{1525\,570.41 \times 0.019}{32000}\right)$$

$$n = \frac{\ln\left(1 - \frac{1525\,570.41 \times 0.019}{32\,000}\right)}{\ln 1.019} \cong 125.52$$

$n = 125$ months $\quad t = 10$ years & 5 months

3. A car enthusiast likes to purchase a supercar by taking out a loan of 1 000 000 TL to be amortized over 12 years subject to 26,4% interest compounded monthly. The loan is to be paid back by making monthly payments.

(a) (15 points) How much is each payment?

$$1\,000\,000 = PMT \quad \frac{1-\left(1+\frac{0.264}{12}\right)^{-144}}{\frac{0.264}{12}}$$

$$PMT = \frac{1000\,000 \times 0.022}{1-(1.022)^{-144}} = 23\,001.92 \text{ TL}.$$

(b) (15 points) Fill in the empty boxes of the amortization schedule of this loan :

Payment Number	Payment	Interest	Unpaid Balance Reduction	Unpaid Balance
0	—	—	—	1 000 000
1	23001.92	22000	1001.92	998 998.08
⋮ 99	⋮	⋮	⋮	⋮ 652 850.69
100	23 001.92	14362.72	8639.20	644211.49
⋮	⋮	⋮	⋮	⋮

$$PV = 23001.92 \quad \frac{1-(1.022)^{-45}}{0.022} = 652\,850.69$$

4. (15 points) Solve the following system of equations by using row operations :

$$\begin{cases} 2x + 3y &= 23 \\ -x + 5y &= -5 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 23 \\ -1 & 5 & -5 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -5 & 5 \\ 2 & 3 & 23 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -5 & 5 \\ 0 & 13 & 13 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & -5 & 5 \\ 0 & 1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 1 \end{array}\right]$$

So $x = 10$
$y = 1$.

5. (18 points) Use row operations to obtain a reduced matrix from:

$$\left[\begin{array}{cccc|c} 0 & 0 & -2 & 0 & 4 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 2 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -2 & 0 & 4 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 2 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & -4 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$