KOÇ UNIVERSITY

MATH 101

FINAL EXAM JANUARY 21, 2009

Duration of Exam: 120 minutes

INSTRUCTIONS:

- No books, no notes, no questions, and talking allowed.
- You must always explain your answers and show your work to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ANSWER KEY

Student ID no: __________

Signature: __________

(Check One):

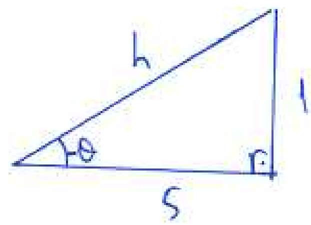
Section 1: Selda Küçükçifçi – TTh 11:00-12:15 : ___
Section 2: Selda Küçükçifçi – TTh 14:00-15:15 : ___
Section 3: Barış Coşkunüzer – MW 9:30-10:45 : ___
Section 4: Barış Coşkunüzer – MW 11:00-12:15 : ___
Section 5: Burak Özbağcı – MW 11:00-12:15 : ___

PROBLEM	1	2	3	4	5	6	7	8	TOTAL
POINTS	10	10	15	10	15	15	15	10	100
SCORE									

Problem 1 (10 pts) a) $\cos(\arctan(-\frac{1}{5})) = ?$

I. way: $\cos(\underbrace{\arctan(-\frac{1}{5})}_{\theta}) = ?$

$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$



$h^2 = 1 + 25 \Rightarrow h = \sqrt{26}$

$\cos\theta = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$

II. way: (By calculator)

$\cos(\arctan(-\frac{1}{5})) = 0{,}980580675$

b) Solve the following equation for x.

$$\ln(\ln x^e) = 2$$

$\ln x^e = e^2$

$e \cdot \ln x = e^2$

$\ln x = e$

$x = e^e$

Problem 2 (10 pts) An investor purchases 450 shares at \$21.40 a share, holds the stock for 26 weeks, then sells the stock for \$24.60 a share. Use the commission schedule below to find the annual rate of interest earned by the investment.

Principal	**Commission**
Under \$3,000	\$32 + 1.8% of principal
\$3,000-\$10,000	\$56 + 1% of principal
Over \$10,000	\$106 + 0.5% of principal

$450 \times 21.40 = \$\,9630$

Commission: $9630 \times 0{,}01 + 56 = \$\,152.3$

$P = 9630 + 152.3 = \$\,9782{,}30$

$450 \times 24.60 = \$\,11070$

Commission: $11070 \times 0{,}005 + 106 = \$\,161{,}35$

$A = 11070 - 161{,}35 = \$\,10908{,}65$

$A = P(1 + rt)$

$10908{,}65 = 9782{,}30\left(1 + r\,\frac{26}{52}\right)$

$10908{,}65 = 9782{,}30 + 4891{,}15r$

$r = 0{,}23028$

$r = 23{,}028\ \%$

Problem 3 (15 pts) A family has a \$80,000, 20 year mortgage at 8% compounded monthly.

a) Find the monthly payment.

$$80000 = PMT \frac{1-\left(1+\frac{0,08}{12}\right)^{-240}}{\frac{0,08}{12}}$$

$$PMT = 80000 \cdot \frac{0,08}{12} \cdot \frac{1}{1-\left(1+\frac{0,08}{12}\right)^{-240}} = \$\, 669,15$$

b) Suppose the family decides to add an extra \$100 to its mortgage payment each month starting with the very first payment. How long will it take the family to pay off the mortgage.

$$80000 = 769,15 \frac{1-\left(1+\frac{0,08}{12}\right)^{-n}}{\frac{0,08}{12}}$$

$$80000 \cdot \frac{0,08}{12} \cdot \frac{1}{769,15} = 1-\left(1+\frac{0,08}{12}\right)^{-n}$$

$$\left(1+\frac{0,08}{12}\right)^{-n} = 1 - 80000 \cdot \frac{0,08}{12} \cdot \frac{1}{769,15}$$

$$-n \cdot \ln\left(1+\frac{0,08}{12}\right) = \ln\left[1 - 80000 \cdot \frac{0,08}{12} \cdot \frac{1}{769,15}\right]$$

$$-n = \frac{\ln\left[1-80000 \cdot \frac{0,08}{12} \cdot \frac{1}{769,15}\right]}{\ln\left(1+\frac{0,08}{12}\right)}$$

$$-n = -177,93$$

178 months or 14 years 10 months

Problem 4 (10 pts) Write the solutions for the linear systems corresponding to the following augmented matrices.

a) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_2 = 2$

$x_3 = -1/3$

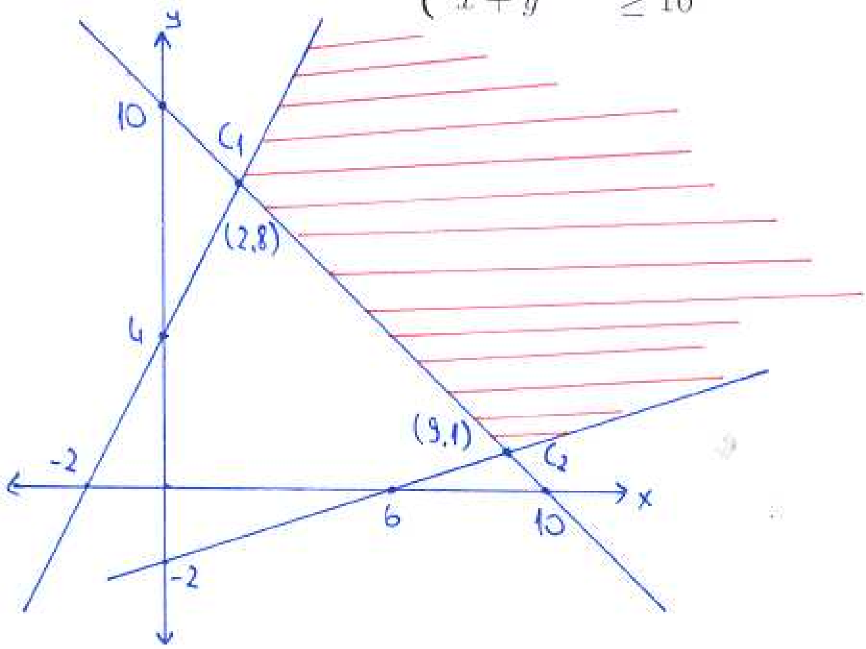
$x_1 = t \quad t \in \mathbb{R}$

So there are infinitely many solutions

b) $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right]$

No solution since $0 \neq 1$

Problem 5 (15 pts) a) Draw the region for $\begin{cases} 2x - y & \geq -4 \\ -x + 3y & \geq -6 \\ x + y & \geq 10 \end{cases}$



b) Use geometric method to minimize and maximize $P = 10x + 3y$

subject to $\begin{cases} 2x - y & \geq -4 \\ -x + 3y & \geq -6 \\ x + y & \geq 10 \end{cases}$

C_1 : $\begin{array}{l} 2x - y = -4 \\ x + y = 10 \end{array} \Rightarrow \begin{array}{l} x = 2 \\ y = 8 \end{array}$ $C_1(2,8)$

C_2 : $\begin{array}{l} -x + 3y = -6 \\ x + y = 10 \end{array} \Rightarrow \begin{array}{l} x = 9 \\ y = 1 \end{array}$ $C_2(9,1)$

$P = 10x + 3y \Rightarrow P(C_1) = 10 \cdot 2 + 3 \cdot 8 = 44 \Rightarrow$ min

$P(C_2) = 10 \cdot 9 + 3 \cdot 1 = 93$

Region unbounded $\Rightarrow$ No Max

Problem 6 (15 pts) Maximize $P = 4x_1 + 3x_2 + 2x_3$

$$\text{subject to} \quad \begin{cases} 3x_1 + 2x_2 + 5x_3 \leq 23 \\ 2x_1 + x_2 + x_3 \leq 8 \\ x_1 + x_2 + 2x_3 \leq 7 \\ x_1,\ x_2,\ x_3 \geq 0 \end{cases}$$

x_1	x_2	x_3	s_1	s_2	s_3	P	
3	2	5	1	0	0	0	23
2	1	1	0	1	0	0	8
1	1	2	0	0	1	0	7
-4	-3	-2	0	0	0	1	0

$R_1 - \frac{3}{2} R_2 \to R_1$

$\frac{R_2}{2} \to R_2$

$R_3 - \frac{R_2}{2} \to R_3$

$R_4 + 2R_2 \to R_4$

0	1/2	7/2	1	-3/2	0	0	11
1	1/2	1/2	0	1/2	0	0	4
0	1/2	3/2	0	-1/2	1	0	3
0	-1	0	0	2	0	1	16

$R_1 - R_3 \to R_1$

$R_2 - R_3 \to R_2$

$2R_3 \to R_3$

$R_4 + 2R_2 \to R_4$

0	0	2	1	-1	-1	0	8
1	0	-1	0	1	-1	0	1
0	1	3	0	-1	2	0	6
0	0	3	0	1	2	1	22

$P = 22 - 3x_3 - s_2 - 2s_3 \quad \Rightarrow \quad x_3 = s_2 = s_3 = 0$

$x_1 = 1$

$x_2 = 6$

$s_1 = 8$

$\text{Max}\ (P) = 22$ at $(1, 6, 0)$

Problem 7 (15 pts) Find the following limits.

a) $\lim\limits_{x\to 3} \frac{x^2-5x+6}{x^2-6x+9}$

$$= \lim_{x\to 3} \frac{(x-2)\cancel{(x-3)}}{(x-3)\cancel{(x-3)}} = \lim_{x\to 3} \frac{x-2}{x-3}$$

the limit does not exist since

$$\lim_{x\to 3^+} \frac{x-2}{x-3} = +\infty \quad \text{and} \quad \lim_{x\to 3^-} \frac{x-2}{x-3} = -\infty$$

b) $\lim\limits_{x\to 0} \frac{x + \sin 3x}{\sin 3x}$

$$= \lim_{x\to 0} \frac{x}{\sin 3x} + \frac{\sin 3x}{\sin 3x} = \left(\lim_{x\to 0} \frac{x}{\sin 3x} \right) + 1$$

$$= 1 + \lim_{x\to 0} \frac{3x}{3 \cdot \sin 3x} = 1 + \lim_{x\to 0} \frac{1}{3} \cdot \frac{3x}{\sin 3x}$$

$$= 1 + \frac{1}{3} = \frac{4}{3}$$

c) $\lim\limits_{x\to -\infty} \frac{\sqrt{9x^6-x}}{x^3+1}$

$$= \lim_{x\to -\infty} \frac{\frac{\sqrt{9x^6-x}}{x^3}}{\frac{x^3+1}{x^3}} = \lim_{x\to -\infty} \frac{-\sqrt{9-\frac{1}{x^5}}}{1+\frac{1}{x^3}}$$

$$= -3$$

Problem 8 (10 pts) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ ax^2 - bx + 3 & 2 \le x < 3 \\ 2x - a + b & x \ge 3 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x \to 2^-} \frac{x^2-4}{x-2} = \lim\limits_{x \to 2^-} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4 \\ \\ f(2) = \lim\limits_{x \to 2^+} ax^2 - bx + 3 = 4a - 2b + 3 \end{array}\right\} \begin{array}{l} 4a - 2b + 3 = 4 \\ 4a - 2b = 1 \end{array}$$

$$\left.\begin{array}{l} \lim\limits_{x \to 3^-} ax^2 - bx + 3 = 9a - 3b + 3 \\ \\ f(3) = \lim\limits_{x \to 3^+} 2x - a + b = 6 - a + b \end{array}\right\} \begin{array}{l} 9a - 3b + 3 = 6 - a + b \\ 10a - 4b = 3 \end{array}$$

$$\begin{cases} 4a - 2b = 1 \\ 10a - 4b = 3 \end{cases}$$

$$\begin{cases} -8a + 4b = -2 \\ 10a - 4b = 3 \end{cases}$$

$2a = 1 \Rightarrow a = 1/2$

$2 - 2b = 1 \Rightarrow 2b = 1 \Rightarrow b = 1/2$