KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Exam 2 December 15, 2008

Duration of Exam: 90 minutes

INSTRUCTIONS: Calculators are allowed on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: ANSWER KEY

Signature: ________________

Section (Check One):

Section 1: S. Küçükçifçi (Tue-Thu 11:00) ___
Section 2: S. Küçükçifçi (Tue-Thu 14:00) ___
Section 3: B. Coşkunüzer (Mon-Wed 9:30) ___
Section 4: B. Coşkunüzer (Mon-Wed 11:00) ___
Section 5: B. Özbağcı (Mon-Wed 11:00) ___

PROBLEM	POINTS	SCORE
1	15	
2	20	
3	15	
4	20	
5	15	
6	15	
TOTAL	100	

A list of formulas: $I = Prt$; $A = P(1+rt)$
$A = P(1+i)^n$; $APY = (1+\frac{r}{m})^m - 1$; $APY = r_e$: effective rate
$FV = PMT\frac{(1+i)^n-1}{i}$
$PV = PMT\frac{1-(1+i)^{-n}}{i}$, where $i = \frac{r}{m}$ and $n = mt$

1. (15 points) A newborn child receives a \$20000 gift toward a college education from her grandparents. How much will the \$20000 be worth in 17 years if it is invested at 7% compounded quarterly?

$P = \$20000$

$r = 7\% = 0{,}07$

$m = 4$

$t = 17$ years

$n = 17 \cdot 4 = 68$

$i = \frac{r}{m} = \frac{0{,}07}{4} = 0{,}0175$

$A = P(1+i)^n$

$A = 20000(1+0{,}0175)^{68}$

$A = \$\,65068{,}44$

2. (20 points) Michael deposits \$1000 every year into a retirement fund that earns 6% compounded annually. Due to the change in employment, these deposits stop after 15 years, but the account continues to earn interest until Michael retires 20 years after the last deposit is made. How much is in the account when Michael retires? How much total interest is gained?

$PMT = \$1000$

$r = 6\% = 0{,}06$

$m = 1$

$i = \frac{r}{m} = \frac{0{,}06}{1} = 0{,}06$

$t = 15$ years

$n = 15$

$FV = PMT\frac{(1+i)^n - 1}{i}$

$FV = 1000\frac{(1+0{,}06)^{15} - 1}{0{,}06}$

$FV = \$23275{,}97$

$A = P(1+i)^n \Rightarrow A = 23275{,}97(1+0{,}06)^{20}$

$A = \$\,74649{,}19$

Total deposit: $15 \times 1000 = \$\,15000$

Total interest: $74649{,}19 - 15000 = \$\,59649{,}19$

3. (15 points) A person purchased a house 5 years ago for \$200000. The house was financed by paying 20% down and signing a 20-year mortgage at 10% on the unpaid balance, compounded monthly. The owner now wishes to refinance the house because of a need for additional cash. If the new market value of the house is \$240000, how much cash will the owner receive after repaying the balance of the original mortgage?

Unpaid Balance : $200000 - 200000 \cdot (0{,}20) = \$ 160000$

$PV = \$160000 \quad , \quad i = \dfrac{0{,}10}{12} \quad , \quad n = 20 \cdot 12 = 240$

The monthly payment is :

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = 160000 \frac{\left(\frac{0{,}10}{12}\right)}{1-\left(1+\frac{0{,}10}{12}\right)^{-240}}$$

$PMT = \$ 1544{,}03$

To compute the unpaid balance after 5 years : (now)

$PMT = \$ 1544{,}03 \quad , \quad i = \dfrac{0{,}10}{12} \quad , \quad n = 15 \cdot 12 = 180$

Unpaid Balance : $PV = PMT \dfrac{1-\left(1+\frac{0{,}10}{12}\right)^{-180}}{\left(\frac{0{,}10}{12}\right)} = \$ 143683{,}48$

$240000 - 143683{,}48 = \$ 96316{,}52$

4. (20 points) (a) Find the inverse of the matrix

$$M = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 3 & 1 & 0 & 1 \end{array}\right] \sim$$

$R_1 + R_3 \to R_3$ $\qquad$ $-2R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right] \sim$$

$-R_3 \to R_3$ $\qquad$ $-3R_3 + R_1 \to R_1$

$-2R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -6 & 3 \\ 0 & 1 & 0 & 2 & -3 & 2 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right] \qquad M^{-1} = \begin{bmatrix} 4 & -6 & 3 \\ 2 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

(b) Using the inverse of M in part (a) solve

$$\begin{cases} x_1 + 3x_3 = 10 \\ x_2 + 2x_3 = 12 \\ -x_1 + 2x_2 = 16 \end{cases}$$

$$\begin{bmatrix} x_1 + 0.x_2 + 3x_3 \\ 0x_1 + x_2 + 2x_3 \\ -x_1 + 2x_2 + 0x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 16 \end{bmatrix}$$

$A \cdot x = B$

$\underbrace{A^{-1} \cdot A}_{I} \cdot x = A^{-1} \cdot B$

$x = A^{-1} \cdot B$

$$\underbrace{\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 10 \\ 12 \\ 16 \end{bmatrix}}_{B}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} \cdot B = \begin{bmatrix} 4 & -6 & 3 \\ 2 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 12 \\ 16 \end{bmatrix}$$

$$= \begin{bmatrix} 4.10 + (-6).12 + 3.16 \\ 2.10 + (-3).12 + 2.16 \\ (-1).10 + 2.12 + (-1).16 \end{bmatrix} = \begin{bmatrix} 16 \\ 16 \\ -2 \end{bmatrix}$$

$x_1 = 16$

$x_2 = 16$

$x_3 = -2$

5. (15 points) **Use Gauss-Jordan elimination method** to solve the following system of linear equations.

$$2x - 6y + 2z = 10$$
$$x - 4y + 2z = 3$$
$$4x - 14y + 6z = 16$$

$$\left[\begin{array}{ccc|c} 2 & -6 & 2 & 10 \\ 1 & -4 & 2 & 3 \\ 4 & -14 & 6 & 16 \end{array}\right] \underset{1/2 R_1 \to R_1}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 1 & -4 & 2 & 3 \\ 4 & -14 & 6 & 16 \end{array}\right] \underset{\substack{-R_1 + R_2 \to R_2 \\ -4R_1 + R_3 \to R_3}}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 0 & -1 & 1 & -2 \\ 0 & -2 & 2 & -4 \end{array}\right] \underset{-R_2 \to R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 0 & 1 & -1 & 2 \\ 0 & -2 & 2 & 4 \end{array}\right] \underset{\substack{2R_2 + R_3 \to R_3 \\ 3R_2 + R_1 \to R_1}}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 11 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x - 2z = 11$

$y - z = 2$

$z = t \quad , \quad t \in \mathbb{R}$

$x = 11 + 2t$

$y = 2 + t$

- consistent
- dependent
- infinitely many solution

6. (15 points) Find x and y so that

$$A.B = \begin{bmatrix} y & y \\ 2 & 1 \end{bmatrix}$$

where

$$A = \begin{bmatrix} x & -1 \\ 1 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} y & y \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2x-4 & x-1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} y & y \\ 2 & 1 \end{bmatrix}$$

Thus, $2x - 4 = y$

$x - 1 = y$

Solving a system by substitution,

$x - 1 = y \Rightarrow x = y + 1$

$2x - 4 = y \Rightarrow 2(y+1) - 4 = y$

$2y + 2 - 4 = y$

$y = \underline{\underline{2}}$

$x = y + 1 \Rightarrow x = 2 + 1 \Rightarrow \underline{\underline{x = 3}}$