MATH 101 MIDTERM 1 SOLUTION

1. Math 101- MT#1, Problem #1–Soln

Let $y = f(x) = -2x^2 + 12x - 16$.

(1) What are the x- and y-intercepts, if any?

SOLUTION. y-intercept =-16.

x-intercepts could be found by any of 2 methods.

Method 1. Quadratic formula: $x = \frac{-12 \pm \sqrt{12^2 - 4(-2)(-16)}}{2(-2)} = 2, 4$.

Method 2. Factor: $y = -2(x-2)(x-4)$.

Comment: many lost the sign from the -2 coefficient of x^2.

(2) Does the graph of $y = f(x)$ have a local minimum or local maximum (turning point)? If so, what are the x and y coordinates of the local minimum (or maximum)?

SOLUTION. Three different methods could be used here.

Method 1: complete the square: $y = -2((x-3)^2 - 9 + 8) = -2(x-3)^2 + 2$. Again, many students had trouble with the -2.

Method 2: differentiate and solve for $y' = 0$: $y' = -4x + 120$, so $x = 3$.

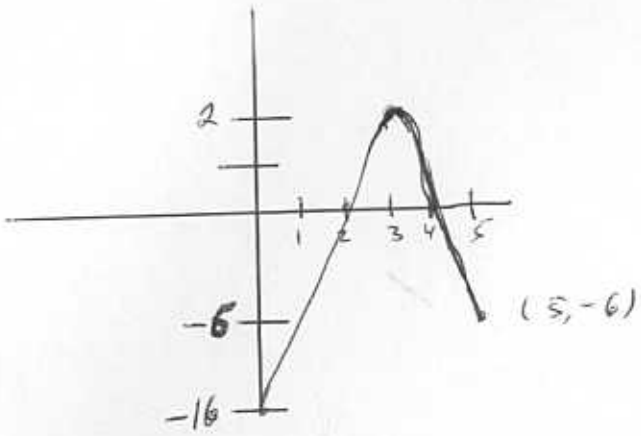
Method 3: use properties of quadratics: vertex is at $x = \frac{-B}{2A} = 3$. Now compute $y = 2$.

Both coordinates of the vertex must be given.

(3) What is the range of $f(x)$?

SOLUTION. $(-\infty, 2]$. Many students wrote $[2, -\infty)$.

(4) Sketch the graph of $y = -2x^2 + 12x - 16$ for $0 \leq x \leq 5$.



(5) Does this function have an inverse? If it does, give a restricted domain where the function has an inverse.

SOLUTION. No inverse because it the function is not one-to-one.

The function has an inverse on $(-\infty, 3]$ and on $[3, \infty)$.

2. Answer the following.

a) (3 points) Rewrite $\frac{2}{3} = \log_{125} 25$ in an equivalent exponential form.

$$125^{2/3} = 25$$

b) (6 points) What is the domain of the function defined by $y = \frac{\log(x+2)}{3} - 5$?

Which transformations (shifts,reflection,expansion and contraction) of $y = \log x$ in what order would lead to this?

Domain : $x+2 > 0$

because domain of log is $(0,\infty)$

$\Rightarrow x > -2$ or $(-2,\infty)$

Transformations:
(There is more than one correct solution)

1. Horizontal shift to left, 2 units
2. Contraction with $\frac{1}{3}$
3. Vertical shift down, 5 units

Find the numerical value of x in parts c) through e)

c) (3 points) $x = \frac{\log_3 e^5}{\log_3 \sqrt{e}} = \frac{5 \log_3 e}{\frac{1}{2} \log_3 e} = \frac{5}{1/2} = 10$

d) (3 points) $x = \log_4(1/2)$

$$4^x = \frac{1}{2} \Rightarrow 2^{2x} = 2^{-1} \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$$

e) (5 points) $\ln x + \ln(x-2) = 2\ln 2 + \ln 6$

$$\ln[x(x-2)] = \ln 2^2 + \ln 6 = \ln(4 \cdot 6)$$

$$\Rightarrow x(x-2) = 24$$

$$\Rightarrow x^2 - 2x - 24 = 0$$

$$\Rightarrow (x-6)(x+4) = 0 \Rightarrow x = 6 \text{ or } \cancel{x = -4}$$

cannot be as $x > 0$ in $\ln x$.

MATH 101 MT #1

Q.3 Solutions

(a) $\cos\left(\frac{3\pi}{4}\right) = \cos(\pi - \pi/4) = -\cos(\pi/4) = -1/\sqrt{2}$. (other sol^s possible)

(b) $\operatorname{arcsec}(-2) = \theta \Rightarrow \sec(\theta) = -2 \Rightarrow \frac{1}{\cos\theta} = -2 \Rightarrow \cos\theta = -1/2$

$\theta = \frac{2\pi}{3} + 2n\pi$, $\frac{4\pi}{3} + 2n\pi$. Domain of $\operatorname{arcsec}(x) = [0, \pi/2) \cup (\pi/2, \pi]$

$\Rightarrow \boxed{\theta = 2\pi/3}$ //

(c) $\cos(\underbrace{\arctan(\sqrt{3})}_{\theta})$ $\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \pi/3 + n\pi$. Domain of $\arctan(x) = [-\pi/2, \pi/2]$

$\Rightarrow \theta = \pi/3 \Rightarrow \cos(\pi/3) = 1/2$.

(d) $-1 \le \sin(x) \le 1 \Rightarrow 1/3 \le f(x) = \frac{1}{2 - \sin(x)} \le 1 \Rightarrow R = [1/3, 1]$.

(e) $\sin 2x = 2\sin x \cos x$

$2\sin x \cos x = \frac{1}{2}\frac{\sin(x)}{\cos(x)} \Rightarrow \sin(x)\left[4\cos x - \frac{1}{\cos x}\right] = 0$.

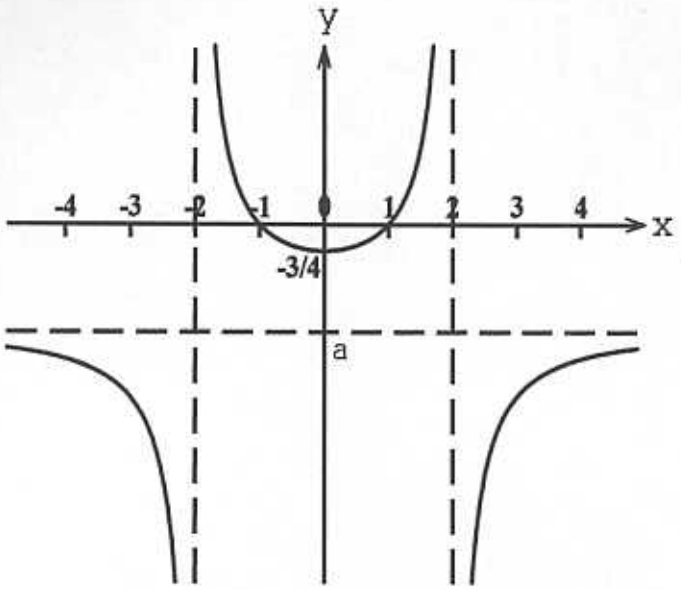
$\Rightarrow \sin x = 0$ or $\cos x = \pm 1/2$

$\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

$\cos x = 1/2 \Rightarrow x = \pi/3, 5\pi/3$

$\cos x = -1/2 \Rightarrow x = 2\pi/3, 4\pi/3$

4-) The graph of $f(x)=\dfrac{-3x^2+bx+c}{x^2+d}$ is given below.



a) (3 points) Write the x and y- intercepts of $f(x)$ using the graph.

$x=\pm 1 \rightarrow$ x intercepts

$y=-3/4 \rightarrow$ y intercept

b) (2 points) Find a shown in the graph.

$a=-3$

$\lim_{x\to\pm\infty} \dfrac{-3x^2+bx+c}{x^2+d} = -3$

$\dfrac{x^2\left(-3+\frac{b}{x}+\frac{c}{x^2}\right)}{x^2\left(1+\frac{d}{x^2}\right)} \approx -3$

c) (2 points) Write the equation of the horizontal asymptote.

$y=-3$

d) (4 points) Write the equations of the vertical asymptotes.

$x=2$ and $x=-2$

e) (9 points) Find the unknowns b, c and d using the information given on the graph.

$(x-2)(x+2)=x^2-4=x^2+d$ so $d=-4$

$f(0)=\frac{-3}{4}$ $\quad \frac{-3}{4}=\frac{c}{d}$ so $c=3$

$f(1)=0$ $\quad f(1)=\dfrac{-3+b+c}{1+d}=0$ $\quad b+c=3$ $\quad b+3=3$ $\quad b=0$

5. You sell your old car for a 24-month note for 10,000 YTL at 10% simple interest.

a) (5 pts) How much interest would you receive after the 2 year period?

$\text{Interest} = P \cdot r \cdot t = 10{,}000 \times 0.1 \times 2 = 2000 \text{ YTL}$

b) (3 pts) How much would you receive in total after 2 years?

$FV = P(1+r \cdot t) = P + \text{Interest} = 12{,}000 \text{ YTL}$

c) (7 pts) You need the money sooner than the maturation date of the note and sell the note to a friend for 11,000 YTL after a year. What annual interest rate will your friend receive from the investment?

For the friend: $P = 11{,}000$, $FV = 12{,}000$, $t = 1 \text{ year}$

$12{,}000 = 11{,}000(1+r)$

$\frac{12}{11} - 1 = r = \boxed{\frac{1}{11}}$

d) (5 pts) Was your friend wise to buy the note at this price? Give your reasoning.

No, because the original note gives 10% interest rate, that is 1/10, but my friend's investment gives 1/11, which is less than the original investment.