
KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm I March 6, 2018

Duration of Exam: 75 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS)** and sign your name, and indicate your section below.

Name: _____
Surname: _____
Signature: _____

KEY

Section (Check One):

- Section 1: E. Şule Yazıcı Tu-Th(10:00) _____
Section 2: Selda Küçükçifçi M-W (10:00) _____
Section 3: Altan Erdoğan M-W (11:30) _____

PROBLEM	POINTS	SCORE
1	20	
2	25	
3	15	
4	22	
5	20	
TOTAL	102	

A list of formulas: $I = Prt$; $A = P(1 + rt)$

$$A = P(1 + i)^n; APY = (1 + \frac{r}{m})^m - 1; A = Pe^{rt}; APY = e^r - 1;$$

$$FV = PMT \frac{(1+i)^n - 1}{i}; PV = PMT \frac{1 - (1+i)^{-n}}{i}, \text{ where } i = \frac{r}{m} \text{ and } n = mt$$

1. (20 points) Solve the following equations.

(a) $\frac{1}{2^x} = \frac{5}{8^{x+3}}$

$$8^{x+3} = 2^x \cdot 5$$

$$(2^3)^{x+3} = 2^x \cdot 5$$

$$\frac{2^{3x+9}}{2^x} = 5$$

$$2^{3x+9-x} = 5$$

$$2^{2x+9} = 5$$

$$(2x+9)\ln 2 = \ln 5$$

$$2x+9 = \frac{\ln 5}{\ln 2} = \log_2 5$$

$$x = \frac{1}{2} (\log_2 5 - 9)$$

(b) $\log_x(6-x) = 2$

$\log_x(6-x) = 2$ is equivalent to $6-x = x^2$

$$x^2 + x - 6 = 0$$

$$(x-2)(x+3) = 0$$

$$x = 2, -3$$

But: $x > 0$, so we exclude $x = -3$

Hence: $x = 2$ is the only solution

2. (25 points) Suppose that you have the following two options of investment:

Option 1: You may purchase a home for \$100,000 by signing a 20-year mortgage at 8% interest compounded monthly with monthly payment of \$A. You expect that the house will be worth \$350,000 at the end of 20 years.

Option 2: You may put \$A at the end of each month for 20 years into an account earning 6% compounded monthly.

which option would you prefer? Justify your answer.

First, we should find \$A, i.e. we should find PMT in the PV formula:
PV = \$100,000

$$n = 20 \times 12 = 240 \text{ months}$$

$$i = \frac{0.08}{12}$$

$$100,000 = PMT \frac{1 - \left(1 + \frac{0.08}{12}\right)^{-240}}{\frac{0.08}{12}}$$

$$A = PMT = 100,000 \times \frac{0.08}{12} \times \frac{1}{1 - \left(1 + \frac{0.08}{12}\right)^{-240}} = \$836.44$$

Option 2: For $n = 20 \times 12 = 240$, $i = \frac{0.06}{12}$, $PMT = 836.44$, by

FV formula,

$$FV = 836.44 \frac{\left(1 + \frac{0.06}{12}\right)^{240} - 1}{\frac{0.06}{12}} = \$386,469.49$$

we should choose the second option as

$$\$386,469.49 > \$350,000$$

3. (15 points) How long will it take you to pay off a 100,000 mortgage financed by 10%

interest compounded semiannually, if you are making 15,000 payments every six months?

For $PV = \$100,000$, $PMT = \$15,000$, $i = \frac{0.10}{2} = 0.05$, by PV

formula, we have:

$$100,000 = 15,000 \frac{1 - (1 + 0.05)^{-n}}{0.05}$$

$$5 = 15 (1 - (1.05)^{-n})$$

$$\frac{1}{3} = 1 - (1.05)^{-n}$$

$$(1.05)^{-n} = \frac{2}{3}$$

$$-n \ln(1.05) = \ln\left(\frac{2}{3}\right) \Rightarrow n = \frac{-\ln(2/3)}{\ln(1.05)} = 8.31, \quad n = 9 \text{ semiannuals}$$

4 years and 6 months

4. (22 points) You want to save some money for your retirement by making monthly

payments to an account that pays 9% compounded monthly. You deposit \$100 for the first

12 months and increased your deposits to \$200 for the next 12 months. How much money

do you have in the account at the end of this two year period?

For $n = 24$, $i = \frac{0.09}{12}$, $PMT = \$100$,

$$FV_1 = 100 \frac{\left(1 + \frac{0.09}{12}\right)^{24} - 1}{\frac{0.09}{12}} = \$2,618.85$$

we deposit extra \$100 for 12 months. For $n = 12$, $PMT_2 = \$100$,

$i = \frac{0.09}{12}$, we have:

$$FV_2 = 100 \frac{\left(1 + \frac{0.09}{12}\right)^{12} - 1}{\frac{0.09}{12}} = \$1,250.76$$

At the end of this two period, we have:

$FV_1 + FV_2 = \$3,869.61$ in the account.

5. (20 points) (a) If you invest \$5000 in an account that pays 5% compounded continuously,

How much money will you have in the account at the end of 20 years?

$$\begin{aligned} A &= 5,000 e^{0.05 \times 20} \\ &= 5,000 e \\ &= 13,591.41 \end{aligned}$$

(b) What is the Annual Percentage Yield (APY) for annual interest (nominal) rate of 6% compounded quarterly?

For $r = 0.06$, $m = 4$,

$$\begin{aligned} \text{APY} &= \left(1 + \frac{0.06}{4}\right)^4 - 1 \\ &= 0.06136 \end{aligned}$$

$$\text{APY} = 6.136\%$$