KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

FINAL May 27, 2017

Duration of Exam: 100 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, and talking allowed. You must always **explain your answers** and show **your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: ____________________

Surname: KEY

Signature: ____________________

Section (Check One):

Section 1: Selda Küçükcifci M-W(14:30) ___
Section 2: E. Şule Yazıcı M-W (13:00) ___
Section 3: E. Şule Yazıcı M-W (11:30) ___

PROBLEM	POINTS	SCORE
1	15	
2	20	
3	20	
4	15	
5	16	
6	16	
TOTAL	102	

1. (15 points) Use Cramer's rule to find the value of x_2 in the following system of linear equations.

$$\begin{cases} x_1 + 3x_2 - x_3 + 3x_4 &= 1 \\ 2x_2 + 3x_3 - 2x_4 &= 0 \\ x_1 + 3x_2 + 2x_3 + x_4 &= 2 \\ x_2 + 2x_4 &= 0 \end{cases}$$

$$\begin{vmatrix} 1 & 3 & -1 & 3 \\ 0 & 2 & 3 & -2 \\ 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -1 & 3 \\ 0 & 2 & 3 & -2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 3 & -2 \\ 0 & 3 & -2 \\ 1 & 0 & 2 \end{vmatrix} = 2(6) + (-6+6)$$

$$= 12.$$

$$\begin{vmatrix} 1 & 1 & -1 & 3 \\ 0 & 0 & 3 & -2 \\ 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 & -1 \\ 0 & 0 & 3 \\ 1 & 2 & 2 \end{vmatrix} = 2\left((0-6)+3\right) = -6$$

$$X_2 = \frac{-6}{12} = -\frac{1}{2}.$$

2. (20 points) Solve the following linear programming problem

$$\text{Minimize } C = 10x_1 + 7x_2 + 12x_3$$

$$\text{subject to } \begin{aligned} x_1 + x_2 + 2x_3 &\geq 7 \\ 2x_1 + x_2 + x_3 &\geq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

$$X = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 2 & 1 & 1 & 4 \\ \hline 10 & 7 & 12 & 1 \end{array}\right] \qquad X^T = \left[\begin{array}{cc|c} 1 & 2 & 10 \\ 1 & 1 & 7 \\ 2 & 1 & 12 \\ \hline 7 & 4 & 1 \end{array}\right]$$

Maximize : $P = 7y_1 + 4y_2$

$$\begin{cases} y_1 + 2y_2 \leq 10 \\ y_1 + y_2 \leq 7 \\ 2y_1 + y_2 \leq 12 \\ y_1, y_2 \geq 0. \end{cases} \qquad \begin{cases} y_1 + 2y_2 + X_1 = 10 \\ y_1 + y_2 + X_2 = 7 \\ 2y_1 + y_2 + X_3 = 12 \\ -7y_1 - 4y_2 + P = 0 \\ y_1, y_2, X_1, X_2, X_3 \geq 0. \end{cases}$$

$$\left[\begin{array}{cccccc|c} 1 & 2 & 1 & 0 & 0 & 0 & 10 \\ 1 & 1 & 0 & 1 & 0 & 0 & 7 \\ 2 & 1 & 0 & 0 & 1 & 0 & 12 \\ \hline -7 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} 10 \\ 7 \\ 6 \\ \\ \end{array} \longrightarrow \left[\begin{array}{cccccc|c} 1 & 2 & 1 & 0 & 0 & 0 & 10 \\ 1 & 1 & 0 & 1 & 0 & 0 & 7 \\ 1 & 1/2 & 0 & 0 & 1/2 & 0 & 6 \\ \hline -7 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cccccc|c} 0 & 3/2 & 1 & 0 & -1/2 & 0 & 4 \\ 0 & 1/2 & 0 & 1 & -1/2 & 0 & 1 \\ 1 & 1/2 & 0 & 0 & 1/2 & 0 & 6 \\ \hline 0 & -1/2 & 0 & 0 & 7/2 & 1 & 42 \end{array}\right] \begin{array}{c} 4 \cdot \frac{2}{3} = 8/3 \\ 2 \\ 12 \\ \\ \end{array} \longrightarrow \begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & X_1 & X_2 & X_3 \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 0 & 1 & -3 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 & 2 \\ 1 & 0 & 0 & -1 & 1 & 0 & 5 \\ \hline 0 & 0 & 0 & 1 & 3 & 1 & 43 \end{array}\right] \end{array}$$

$C = 43$

$X_1 = 0$

$X_2 = 1$

$X_3 = 3$

3. (20 points) Give an example of the following 2-player zero sum matrix games.

a-) A 3×3 strictly determined game. Show that it is strictly determined by finding the value of the game. Determine if it is fair or not?

$$\begin{bmatrix} -2 & \boxed{15} & 1 \\ \boxed{0} & 3 & \boxed{8} \\ -10 & 2 & -4 \end{bmatrix}$$

saddle point is 0. So it is fair.

b-) A 2×2 non-strictly determined fair game.

$$\begin{bmatrix} \boxed{3} & -6 \\ -1 & \boxed{12} \end{bmatrix} \qquad \frac{ad-bc}{D} = \frac{0}{D} = 0.$$

is an example. There are infinitely many other solutions

c-) A 2×2 non-strictly determined unfair game.

$$\begin{bmatrix} \boxed{3} & -3 \\ -4 & \boxed{2} \end{bmatrix} \qquad \frac{ad-bc}{D} = \frac{-6}{D} \neq 0.$$

is an example. There are infinitely many other solutions.

4. (15 points) Use Gauss-Jordan elimination to find the inverse of the following matrix, if it exists.

$$M = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 1 & 3 & 4 & 8 \\ 3 & 1 & -1 & -2 \\ 0 & 2 & 2 & 4 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 6 & 1 & 0 & 0 & 0 \\ 1 & 3 & 4 & 8 & 0 & 1 & 0 & 0 \\ 3 & 1 & -1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 6 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -5 & -10 & -20 & -3 & 0 & 1 & 0 \\ 0 & 2 & 2 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 2 & 3 & -2 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & -5 & -10 & -8 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 1 \end{array}\right]$$

no inverse.

A list of formulas: $I = Prt$; $A = P(1 + rt)$

$$A = P(1+i)^n; \; APY = (1+\tfrac{r}{m})^m - 1; \; A = Pe^{rt}; \; APY = e^r - 1;$$

$$FV = PMT\frac{[(1+i)^n - 1]}{i}; \; PV = PMT\frac{[1-(1+i)^{-n}]}{i}, \text{ where } i = \tfrac{r}{m} \text{ and } n = mt$$

5. (16 points) Esen borrowed 2500 TL for 1 year from a bank with an annual interest rate 13.2% compounded monthly. At the end of the 6th month, Esen made a partial payment of 1000 TL. Determine the amount that Esen must pay on the due date.

2500 TL   6th   1st.

$$A = 2500\left(1 + \frac{0.132}{12}\right)^6 = 2669.60 \text{ TL}.$$

$$2669.60 - 1000 = 1669.60 \text{ TL}.$$

$$A = 1669.60\left(1 + \frac{0.132}{12}\right)^6 = 1782.87 \text{ TL}.$$

6. (16 points) A woman borrows 18000 TL at 9% compounded monthly which is to be amortized over 3 years in equal monthly payments. For tax purposes, she needs to know the amount of interest paid during the first year of the loan. Find the interest paid during the first year of the loan.

$$18000 = PMT \frac{1-\left(1+\frac{0.09}{12}\right)^{-36}}{\frac{0.09}{12}}$$

$$PMT = 572.40 \text{ TL}.$$

$$PV_{1^{st}\text{ year}} = 572.40 \frac{1-\left(1+\frac{0.09}{12}\right)^{-24}}{\frac{0.09}{12}} = 12529.35 \text{ TL}.$$

$$\text{interest paid during the } 1^{st} \text{ year} = 572.40 \times 12 - (18000 - 12529.35)$$

$$= 1398.15 \text{ TL}.$$