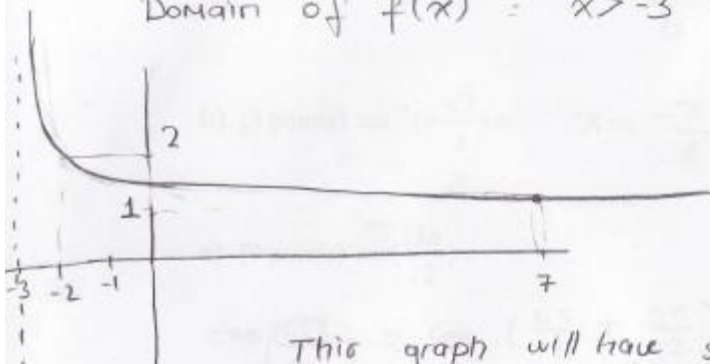


1.

a) (12 points) Sketch the graph of $f(x) = 2 + \log\left(\frac{1}{x+3}\right)$

Domain of $f(x) = x > -3$ or $(-3, +\infty)$



$$x = -2 \Rightarrow f(-2) = 2 + \log\left(\frac{1}{-2+3}\right) = 2 \quad (2, 2)$$

$$x = 7 \Rightarrow f(7) = 2 + \log\left(\frac{1}{7+3}\right) = 2 - \log 10 = 1 \quad (7, 1)$$

$$x = 97 \Rightarrow f(97) = 2 + \log\left(\frac{1}{97+3}\right) = 2 - 2 = 0 \quad (97, 0)$$

This graph will have such transformations from $f(x) = \log x$, respectively

- 1) horizontal shift to left 3 units
- 2) reflection about x -axis
- 3) vertical shift up, 2 units

b) Solve the following equations for x

i-) (6 points) $\log_2 x + \log_2(x+2) = 3$

$$3 = \log_2 8 \Rightarrow \log_2 x(x+2) = \log_2 8$$

$$x(x+2) = 8 \Rightarrow x^2 + 2x - 8 = 0$$

$$(x+4) \Rightarrow x = -4$$

$$(x-2) \Rightarrow x = 2$$

no negative value in logarithmic form

ii-) (3 points) $2x^2 e^x = 3x e^x$

$$(2x^2 - 3x) e^x = 0$$

this can not be zero

$$x(2x-3) = 0$$

$$x = 0, \quad x = \frac{3}{2}$$

iv-) (2 points) $\log_{25} x = \frac{3}{2}$

$$(25)^{\frac{3}{2}} = x$$

$$(5^2)^{\frac{3}{2}} = x$$

$$5^3 = 125 = x$$

iii-) (2 points) $\log_x 8 = -3$

$$\log_x 8 = -3$$

$$x^{-3} = 8 \Rightarrow \left(\frac{1}{x}\right)^3 = 2^3$$

$$x = \frac{1}{2}$$

v-) (2 points) $\log_{\frac{1}{4}} 64 = x$

$$\left(\frac{1}{4}\right)^x = 64$$

$$4^{-x} = 4^3$$

$$x = -3$$

2. Answer the following questions.

a) (3 points) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x = \frac{5\pi}{6}$ range $\cos^{-1}x$ is $(0, \pi)$

b) (3 points) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = x = -\frac{\pi}{6}$ range $\tan^{-1}x$ is $[-\pi/2, \pi/2]$

c) (9 points) $\sec\left(\frac{13\pi}{12}\right) =$

$$\cos\left(\frac{13\pi}{12}\right) = \cos\left(\frac{4\pi}{12} + \frac{9\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{3\pi}{4}\right)$$

$$\left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$$

$$\sec\left(\frac{13\pi}{12}\right) = \frac{1}{\cos\left(\frac{13\pi}{12}\right)} = \frac{2\sqrt{2}}{-1-\sqrt{3}} \cdot \frac{-1+\sqrt{3}}{-1+\sqrt{3}} = \frac{-2\sqrt{2}+2\sqrt{6}}{1-3} = \frac{\sqrt{2}-\sqrt{6}}{1}$$

d) (6 points) If $\tan x = \frac{1}{2}$ and x is in $[\pi, \frac{3\pi}{2}]$; find $\sin x = ?$

$$\frac{\sin x}{\cos x} = \frac{1}{2} \quad \cos x = 2 \sin x$$

$$\sin^2 x + \cos^2 x = 1 \quad \Rightarrow 5 \sin^2 x = 1$$

$$\sin^2 x + 4 \sin^2 x = 1 \quad \sin^2 x = \frac{1}{5}$$

$$\sin x = -\frac{1}{\sqrt{5}}$$

because $x \in [\pi, \frac{3\pi}{2}]$

e) (6 points) Solve $\cos x = -\frac{\sqrt{3}}{2}$ where x is in $[-2\pi, 2\pi]$

$$x_1 = \frac{5\pi}{6} \quad x_2 = -\frac{5\pi}{6}$$

$$x_3 = \frac{7\pi}{6} \quad x_4 = -\frac{7\pi}{6}$$

3-) Let $f(x) = \frac{1}{\cot x}$.

a) (5 points) Find domain of $f(x)$.

$$\mathbb{R} / \left\{ \frac{\pi}{2} + k \mid k \in \mathbb{Z} \right\}$$

b) (3 points) Find x and y intercepts of $f(x)$.

no x-intercepts or y-intercepts...

c) (5 points) Write the equations of the vertical asymptotes of $f(x)$.

$$x = \frac{\pi}{2} + \pi k \quad \text{for all } k \in \mathbb{Z}$$

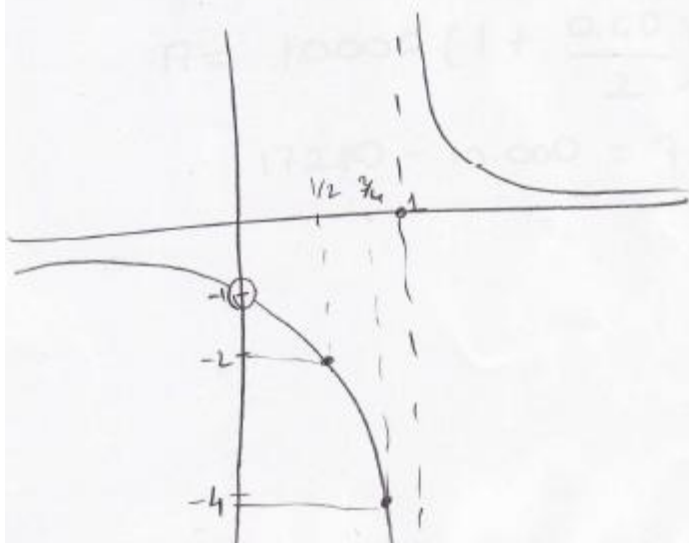
4-) (15 points) Sketch the graph of $f(x) = \frac{x}{x^2 - x}$.

Dom. $\mathbb{R} - \{0, 1\}$

no x-intercept or y-intercept.

vertical asymptote: $x=1$

horizontal asymptote $y=0$



$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\frac{1}{4} - \frac{1}{2}} = -2$$

$$f\left(\frac{3}{4}\right) = \frac{\frac{3}{4}}{\frac{9}{16} - \frac{3}{4}} = -4$$

5. You want to invest some money at the bank for a year. The bank offers three possible alternatives.

i-) 45% compounded annually

ii-) 40% compounded semiannually

iii-) 45% compounded continuously

a-) (12 points) Which of the above would be wise to choose? Explain your answer.
(Answer with no explanation would get no credit)

$$APY_1 = (1 + 0.45)^1 - 1 = 0.45$$

$$APY_2 = \left(1 + \frac{0.40}{2}\right)^2 - 1 = 0.44$$

first one is better than the second one because $APY_1 > APY_2$
and third one is better than the first one because
it is continuous, (and the same interest rate.)

b-) (6 points) If you choose option (b) and you deposit 10000YTL to the bank for 18 months, How much interest would you earn?

18 months = 3 semiannuals

$$A = 10000 \left(1 + \frac{0.40}{2}\right)^3 = 17280$$

$$17280 - 10000 = 7280 \text{ YTL.}$$
