KOÇ UNIVERSITY

MATH 101

Final May 24, 2008

Duration of Exam: 90 minutes

INSTRUCTIONS: No books, no notes, no questions, and talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

Surname, Name: ANSWER KEY

Student ID no:

Signature:

Instructor's Name:HALUK ORAL

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

Problem 1 (20 pts) Evaluate the following limits.

(a)(5 pts) $\lim\limits_{x\to 0} \frac{1-Cos x}{x}$

(b)(5 pts) $\lim\limits_{x\to \infty} \frac{Sin x}{x}$

(c)(5 pts) $\lim\limits_{x\to 2} \frac{x^2-3x+2}{x^2-1}$

(d)(5 pts) $\lim\limits_{x\to \infty} \sqrt{x+2}-\sqrt{x}$

a) $\lim\limits_{x\to 0} \frac{1-\cos x}{x} = \lim\limits_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)} = \lim \frac{\sin^2 x}{x(1+\cos x)} = 0$

b) $-1 \leq \sin x \leq 1 \Rightarrow \frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$

$\lim\limits_{x\to\infty} {}^{-1}/_{x} \leq \lim\limits_{x\to\infty} \frac{\sin x}{x} \leq \lim\limits_{x\to\infty} {}^{1}/_{x}$

$0 \leq \lim\limits_{x\to\infty} \frac{\sin x}{x} \leq 0 \Rightarrow \lim\limits_{x\to\infty} \frac{\sin x}{x} = 0$

c) $\lim\limits_{x\to 2} \frac{x^2-3x+2}{x^2-4} = \lim\limits_{x\to 2} \frac{(x-2)(x-1)}{(x-2)\,(x+2)} = \lim\limits_{x\to 2} \frac{x-1}{x+2} = \frac{1}{4}$

d) $\lim\limits_{x\to\infty} \sqrt{x+2}-\sqrt{x} = \lim\limits_{x\to\infty} \frac{(\sqrt{x+2}-\sqrt{x})(\sqrt{x+2}+\sqrt{x})}{\sqrt{x+2}+\sqrt{x}}$

$= \lim\limits_{x\to\infty} \frac{x+2-x}{\sqrt{x+2}+\sqrt{x}} = \lim\limits_{x\to\infty} \frac{2}{\sqrt{x+2}+\sqrt{x}} = 0$

Problem 2 (20 pts)

$$\text{Let } f(x) = \begin{cases} \frac{x^2+x-6}{x-2} & x < 2 \\ a-b & x = 2 \\ \frac{bx-2b}{x-2} & x > 2 \end{cases}$$

(a)(10pts) For which value of b f(x) has a limit at x=2?

(b)(10pts) For which value of a f(x) is continuous at x=2?

a) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2} \frac{b(x-2)}{x-2} = b$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = 5$

So, $\underline{\underline{b=5}}$

b) $\lim_{x\to 2} f(x) = a-b = f(x) \Rightarrow \quad 5 = a-5 \quad \Rightarrow \quad a = 10$

Problem 3 (20 pts) Let $f(x) = x^2 - x + 3$

(a)(6pts) Evaluate $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ (Show all your work!)

(b)(7pts) Write the equation of the tangent line of f(x) at x=4

(c)(7pts) At which value of x the tangent line of f(x) is parallel to the line $3x - 5y + 4 = 0$?

a) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-(x+h)+3-(x^2-x+3)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x}-h+\cancel{3}-\cancel{x^2}+\cancel{x}-\cancel{3}}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2-h}{h} = \lim_{h\to 0} 2x+h-1 = 2x-1$

b) $m = 2\cdot(4)-1 = 7$ $(4,15)$ is passing point.

$f(4) = 16-4+3 = 15$

$y-15 = 7(x-4) \Rightarrow y = 7x-28+15 \Rightarrow y = 7x-13$

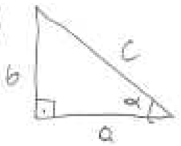
c) $3x-5y+4=0 \Rightarrow 5y = 3x+4 \Rightarrow y = \frac{3}{5}x + \frac{4}{5}$

the slope of $3x-5y+4=0$ is $\frac{3}{5}$

So, $2x-1 = \frac{3}{5} \Rightarrow x = 4/5$

Problem 4(a)(10pts) Prove that $Sin^2\alpha + Cos^2\alpha = 1$ for any α.

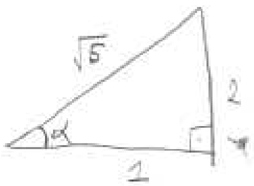
(b)(10pts)*If* $0 \leq \alpha \leq \frac{\pi}{2}$ and $Tan\alpha = 2$, then calculate $Cos\alpha$, $Sin\alpha$, $Cot\alpha$, $Cos2\alpha$, $Sin2\alpha$, $Tan2\alpha$

a) 

Let α be any angle

$\sin\alpha = \frac{b}{c}$ $\cos\alpha = \frac{a}{c}$ and $c^2 = b^2 + a^2$

$\Rightarrow \sin^2\alpha + \cos^2\alpha = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$

b) 

Notice that 2α has in second quadrant.
So $\cos 2\alpha$, $\tan 2\alpha$ are negative

$\cos\alpha = \frac{1}{\sqrt{5}}$ $\sin\alpha = \frac{2}{\sqrt{5}}$ $.\cot\alpha = \frac{1}{2}$

$\cos 2\alpha = 2\cos^2\alpha - 1 = 2 \cdot \frac{1}{5} - 1 = -3/5$

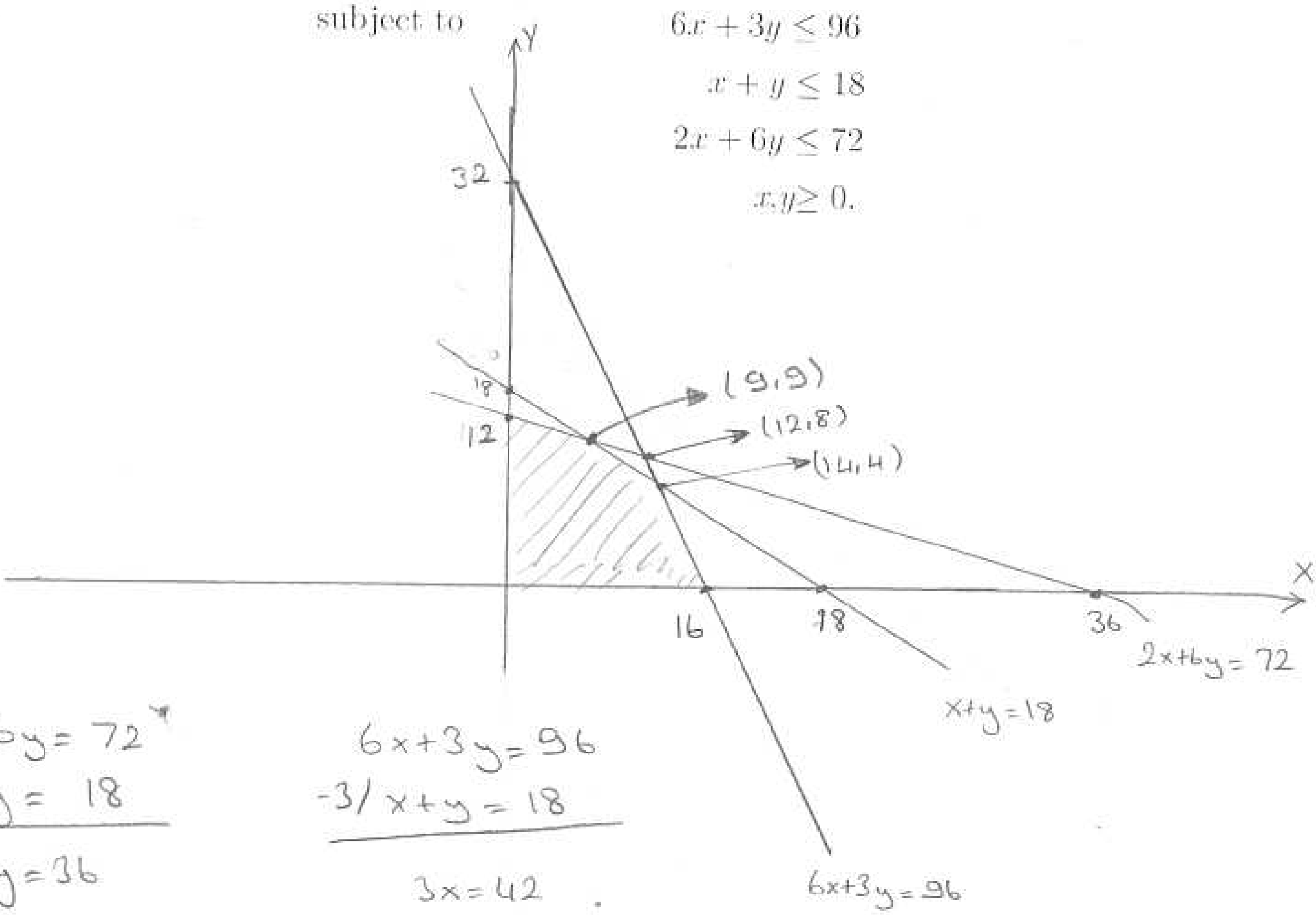
$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\frac{1}{\sqrt{5}}\frac{2}{\sqrt{5}} = 4/5$

$\tan 2\alpha = \frac{\frac{4}{5}}{-3/5} = \frac{-4}{3}$

Problem 5 (20 pts) Maximize $P = 80x + 70y$

subject to

$$6x + 3y \le 96$$
$$x + y \le 18$$
$$2x + 6y \le 72$$
$$x, y \ge 0.$$



$2x+6y=72$
$-2/\ x+y=18$

$4y=36$
$y=9$
$x=9$

$6x+3y=96$
$-3/\ x+y=18$

$3x=42$
$x=14$
$y=4$

$2x+6y=72$
$2/\ 6x+3y=96$

$-10x=-120$
$x=12$
$y=8$

So, the corner points are :

$(0,10) \Rightarrow P=700$
$(9,9) \Rightarrow P=1210$
$(14,4) \Rightarrow \boxed{P=1400}$
$(16,0) \Rightarrow P=1280$
$(0,0) \Rightarrow P=0$

MAXIMUM