KOÇ UNIVERSITY

MATH 101

Midterm 1 March 06, 2008

Duration of Exam: 60 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

Surname, Name: ANSWER KEY

Student ID no: ______________________

Signature: ______________________

Instructor's Name:HALUK ORAL

PROBLEM	POINTS	SCORE
1	15	
2	20	
3	20	
4	25	
5	20	
TOTAL	100	

Problem 1 (15 pts) Write the equations of the lines passing through

(a)(5 pts) (3,7) and (5,8)

(b)(5 pts) (2,3) and (2,4)

(c)(5 pts) (1,3) and (2,3)

a) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-7}{5-3} = 1/2$

$y - y_1 = m(x - x_1) \Rightarrow y - 8 = 1/2(x-5)$

$\Rightarrow y - 8 = \frac{x}{2} - \frac{5}{2} \Rightarrow y = \frac{x}{2} + \frac{11}{2}$

$\Rightarrow 2y - x - 11 = 0$

b) $m = \dfrac{4-3}{2-2} = \dfrac{1}{0}$ (undefined) So. $x = 2$ is the equation

c) $m = \dfrac{3-3}{2-1} = 0$ $y = 3$ is the equation.

Problem 2 (20 pts) Consider $f(x) = x^2 - 6x + 8$

(a)(5 pts) Find the vertex of the graph of the funtion.

(b)(5 pts) Find x and y-intercepts of the function.

(c)(5 pts) Find the domain and the range of the function.

(d)(5 pts) Sketch the graph of the function.

a) $x^2 - 6x + 8 = x^2 - 6x + 8 + 1 - 1 = (x-3)^2 - 1$ vertex: $(3, -1)$

b) y-intercept : $f(0) = 8$

x-intercept : $f(x) = 0 \Rightarrow x^2 - 6x + 8 = 0$

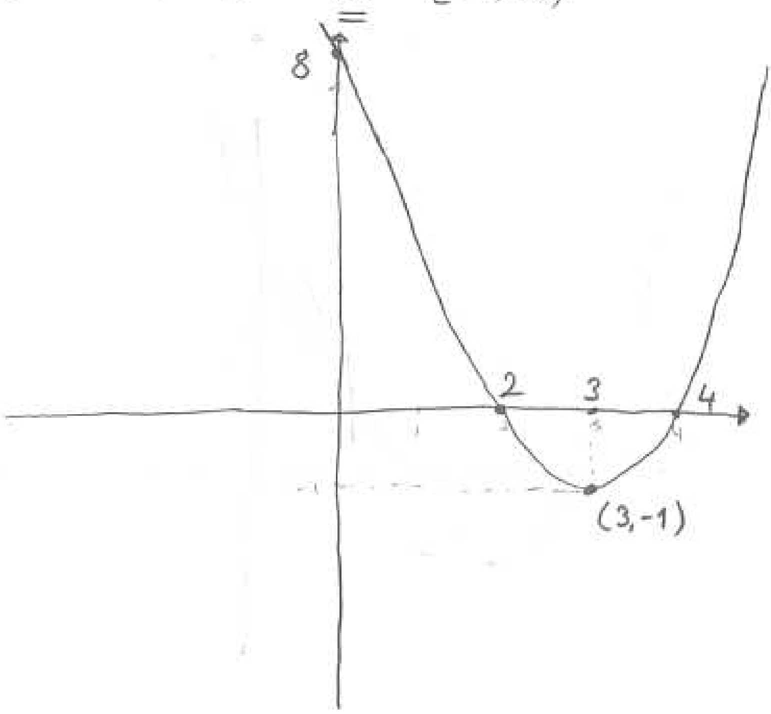
$\Rightarrow (x-4)(x-2) = 0$

$\Rightarrow x = 4,\ x = 2$ x-intercepts

c) Domain: All real numbers

Range : $x \geq -1$ or $[-1, \infty)$

d)



Problem 3 **(20 pts)** If $\log 2 = a$ and $\log 3 = b$,write the followings in terms of a and b.

(a)(4 pts) $\log 48$

(b)(5 pts) $\log 5$

(c)(4 pts) $\log 75$

(Hint:Where the logarithm is of base 10)

(d)(7 pts) Solve $\log_{20}(x)+\log_{20}(x+1)=1$

a) $\log 48 = \log 2^4 \cdot 3 = 4\log 2 + \log 3 = 4a + b$

b) $\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - a$

c) $\log 75 = \log 5^2 \cdot 3 = 2\log 5 + \log 3 = 2(1-a) + b$

$= 2 - 2a + b$

d) $\log_{20}(x) + \log_{20}(x+1) = 1$

$\log_{20} x(x+1) = 1 \Rightarrow x(x+1) = 20 \Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x+5)(x-4) = 0$

$x = -5$ or $x = 4$ But logarithm cannot take negative numbers

So $\boxed{x = 4}$

Problem 4 (25 pts) Find the inverse functions of

(a)(9 pts) y=3^{2x-1}+4

(b)(9 pts) y=4$\log_2(3x-2)$+5

(c)(7 pts) y=$\frac{x}{x+2}$

a) $y = 3^{2x-1} + 4 \Rightarrow y - 4 = 3^{2x-1} \Rightarrow 2x - 1 = \log_3(y-4)$

$$x = \frac{\log_3(y-4)+1}{2}$$

$$f^{-1}(x) = \frac{\log_3(x-4)+1}{2}$$

b) $y = 4 \cdot \log_2(3x-2) + 5 \Rightarrow \frac{y-5}{4} = \log_2(3x-2)$

$\Rightarrow 3x - 2 = 2^{\frac{y-5}{4}}$

$$x = \frac{2^{\frac{y-5}{4}} + 2}{3}$$

$$f^{-1}(x) = \frac{2^{\frac{x-5}{4}} + 2}{3}$$

c) $y = \frac{x}{x+2} \Rightarrow (x+2)y = x \Rightarrow xy + 2y = x$

$\Rightarrow xy - x = -2y$

$\Rightarrow x(y-1) = -2y$

$$x = \frac{-2y}{y-1} = \frac{2y}{1-y}$$

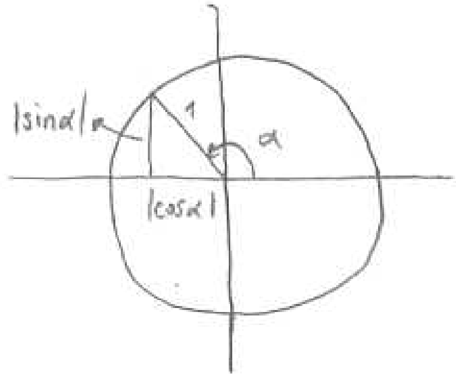
$$f^{-1}(x) = \frac{2x}{1-x}$$

Problem 5 (20 pts)

(a)(10 pts) Prove that $\sin^2\alpha + cos^2\alpha = 1$ for any α

(b)(10 pts) If $\frac{\pi}{2} \leq \alpha \leq \pi$ and $sin(\alpha) = \frac{5}{13}$,then find $\cos\alpha$,$\tan\alpha$,$\cot\alpha$,$\sec\alpha$,$\csc\alpha$.

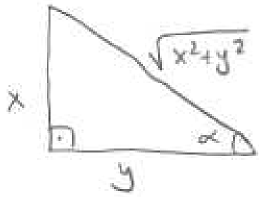
a)



for any α

$$\cos^2\alpha + \sin^2\alpha = 1$$

or



$$\sin\alpha = \frac{x}{\sqrt{x^2+y^2}}$$

$$\cos\alpha = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos^2\alpha + \sin^2\alpha = \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} = 1$$

or

$$\cos 0 = 1$$

$$\cos(\alpha - \alpha) = \cos\alpha \cdot \cos(-\alpha) - \sin\alpha \cdot \sin(-\alpha)$$
$$= \cos^2\alpha + \sin^2\alpha = 1$$

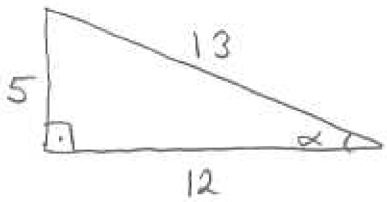
$$\boxed{\begin{array}{l} \cos -\alpha = \cos\alpha \\ \sin -\alpha = -\sin\alpha \end{array}}$$

b) $\frac{\pi}{2} \leq \alpha \leq \pi$

$\sin\alpha = \frac{5}{13}$

$\sin\alpha$ lies in second quadrant

$\cos\alpha$, $\tan\alpha$, $\cot\alpha$, $\sec\alpha$ should be negative



$\cos\alpha = \frac{-12}{13}$

$\tan\alpha = \frac{-5}{12}$

$\cot\alpha = \frac{-12}{5}$

$\sec\alpha = \frac{-13}{12}$

$\csc\alpha = 13/5$