KOÇ UNIVERSITY

MATH 101

Midterm 2 April 21, 2008

Duration of Exam: 90 minutes

INSTRUCTIONS: No books, no notes, no questions, and talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

Surname, Name: ANSWER KEY

Student ID no: ____________

Signature: ____________

Instructor's Name:HALUK ORAL

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

Problem 1 (20 pts) You want to make equal monthly deposits into an account which pays 10% compounded monthly for 20 years in order to then make equal monthly withdrawals of \$1000 for the next 15 years ,reducing the balance to zero.

(a)(10 pts) How much should be deposited each month for the first 20 years?

(b)(10 pts) What is the total interest earned during this 35 year process?

a)
$$PV = \frac{1000\left(1-\left(1+\frac{10}{100.12}\right)^{-15\times 12}\right)}{i} = \frac{PMT\left(1+\frac{10}{100.12}\right)^{20\times 12}-1}{i}$$

$PMT = 122{,}55$

b) Total interest = $\underbrace{180 \times 1000}_{\text{total withdrawals}} - \underbrace{240 \times 122{,}55}_{\text{Total deposits}} = 180000 - 29412$

$= 150588$

Problem 2 (20 pts) You want to invest your money in an account and you have two options. These are the accounts that pay

Account 1: 40% compounded monthly

Account 2: 50% compounded annually

(a)(10 pts) Which account will you to choose to invest your money in and Why?

(b)(10 pts) How long will it take your money to double if you choose to invest in **Account 1**?

a) $APY_{I} = \left(1 + \frac{40}{100 \cdot 12}\right)^{12} - 1 = 0.48$

$APY_{II} = \left(1 + \frac{50}{100}\right) - 1 = 0.50$

Since

$APY_{II} > APY_{I}$. I will choose Account 2.

b) $2\cancel{x} = \cancel{x}\left(1 + \frac{40}{100 \cdot 12}\right)^{n} \Rightarrow \ln 2 = n\left(\ln 31 - \ln 30\right)$

$n = \frac{\ln 2}{\ln 31 - \ln 30} = 21.14$ So, we need at least 22 moths to double our money

Problem 3 (20 pts) Let

$$x_1+3x_2=5$$
$$4x_1+ax_2=b$$

where a and b are constant numbers.

(a)(10 pts) Find the values of a and b so that this system has infinitely many solutions.

(b)(10 pts) Find the values of a and b so that this system has no solutions.

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 4 & a & b \end{array}\right] \xrightarrow{-4R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & a-12 & b-20 \end{array}\right]$$

a) $a-12 = b-20 = 0$ implies there exists infinitely many solutions. So, $a=12$, $b=20$ is the required answer.

b) $a-12=0$

$b-20 \neq 0$ $\qquad a=12 \quad b\neq 20$ is the required answer.

Problem 4 (20 pts) Solve the system

$$x_1+3x_2+2x_3=1$$
$$2x_1+10x_2+8x_3=10$$
$$3x_1+5x_2+7x_3=5$$

by using Gauss-Jordan elimination.

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 2 & 10 & 8 & 10 \\ 3 & 5 & 7 & 5 \end{array}\right] \begin{array}{l} -2R_1+R_2 \to R_2 \\ -3R_1+R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 4 & 4 & 8 \\ 0 & -4 & 1 & 2 \end{array}\right] \tfrac{1}{4} R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & -4 & 1 & 2 \end{array}\right] \begin{array}{l} -3R_2+R_1 \to R_1 \\ 4R_2+R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 5 & 10 \end{array}\right] \tfrac{1}{5} R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} -R_3+R_2 \to R_2 \\ R_1+R_3 \to R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

So $X_1 = -3$

$X_2 = 0$

$X_3 = 2$ is the solution.

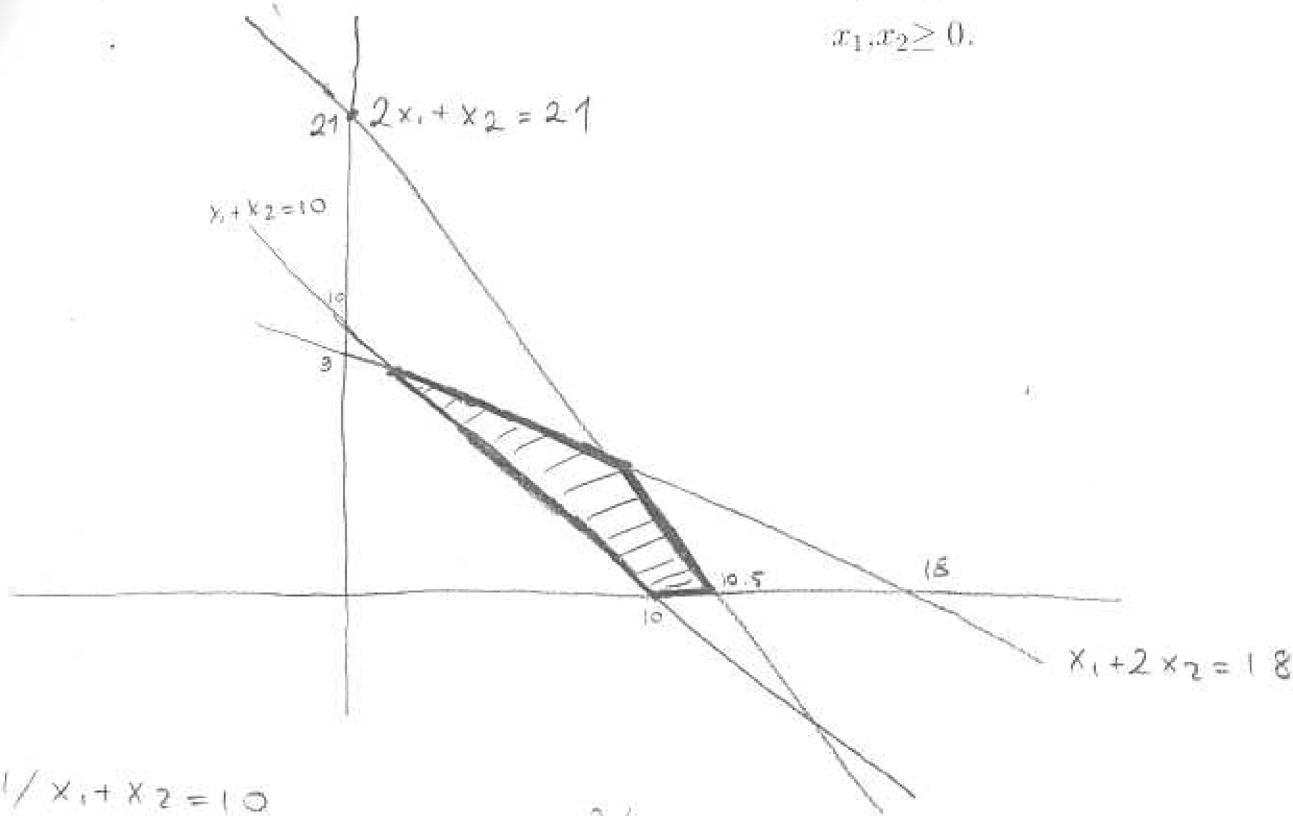
Problem 5 (20 pts) Maximize $P=2x_1+5x_2$

subject to $x_1+2x_2\leq 18$

$2x_1+x_2\leq 21$

$x_1+x_2\geq 10$

$x_1,x_2\geq 0.$



$-1/\ x_1+x_2=10$

$x_1+2x_2=18$

$x_2=8 \quad x_1=2$

$-2/\ x_1+2x_2=18$

$2x_1+x_2=21$

$-3x_2=-15$

$x_2=5 \quad x_1=8$

So, the corner points are $(10,0)$, $(\frac{21}{2},0)$, $(2,8)$, $(8,5)$

$(10,0) \Rightarrow P=2\cdot 10+5\cdot 0=20$

$(\frac{21}{2},0) \Rightarrow P=2\cdot\frac{21}{2}+5\cdot 0=21$

$(2,8) \Rightarrow P=2\cdot 2+5\cdot 8=44$

$(8,5) \Rightarrow P=2\cdot 8+5\cdot 5=41$

$P=44$ is maximum at $x_1=2$ & $x_2=8$