# KOÇ UNIVERSITY

# MATH 101 - FINITE MATHEMATICS

## Midterm I March 29, 2010

**Duration of Exam: 90 minutes**

**INSTRUCTIONS:** No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

**Name:** KEY

**Surname:**

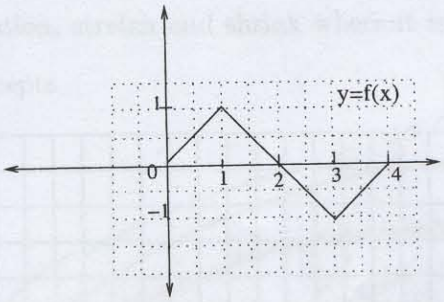
**Signature:**

**Section (Check One):**

Section 1: E. Şule Yazıcı Tu-Th(11:00) —
Section 2: E. Şule Yazıcı Tu-Th(14:00) —
Section 3: Mehmet Sarıdereli Tu-Th(15:30) —

| PROBLEM | POINTS | SCORE |
|---|---|---|
| 1 | 18 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 15 | |
| 5 | 35 | |
| 6 | 12 | |
| TOTAL | 105 | |

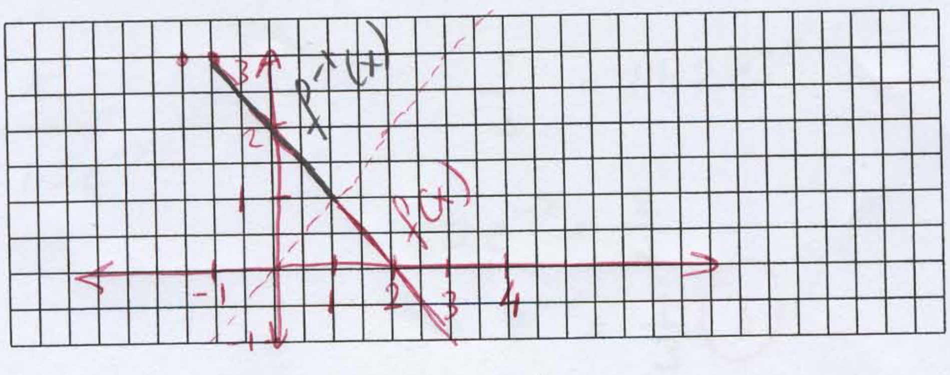
**1.** Given the graph of $y = f(x)$ below. Sketch the graph of the following functions.



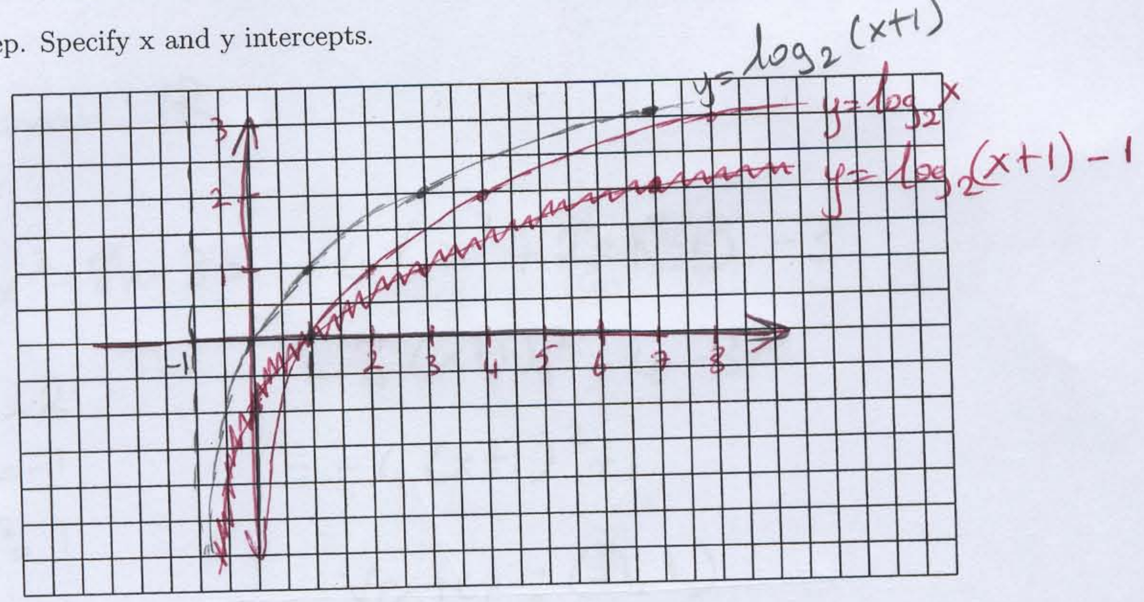

a) (6 points) $y = 2f(x)$


c) (12 points) Choose an interval for $x$ in $[0, 4]$ so that $f(x)$ is one-to-one in that interval and that the range of $f(x)$ in that interval is the same as the range of $f(x)$ when $x$ is in $[0, 4]$ (a restricted domain). Sketch the graph of $f^{-1}$ for this interval.

Restricted Domain [1,3]



**2.** (15 points) Sketch the graph of the function $f(x) = \log_2(x+1) - 1$ by using horizontal translation, vertical translation, stretch and shrink where it is appropriate. Indicate each step. Specify x and y intercepts.




**3.** a-) (5 points) Find $\ln(\frac{1}{\sqrt{ab^3}})$ =? if $\ln a = 2$ and $\ln b = 3$.

$$\ln\left(\frac{1}{\sqrt{ab^3}}\right) = \ln(ab^3)^{-1/2} = -\frac{1}{2}(\ln a + 3\ln b)$$

$$= -\frac{1}{2}(2 + 3.3) = \boxed{\frac{-11}{2}}$$

b-) (5 points) Solve $\frac{5}{1+2e^{-x}} = 3$ if $\ln 3 = 1.099$

$$\frac{5}{1+2e^{-x}} = 3 \qquad \frac{5}{3} = 1 + 2e^{-x}$$

$$2e^{-x} = \frac{2}{3}$$

$$e^{-x} = \frac{1}{3} \quad 3$$



$$\ln e^{-x} = \ln 1/3$$

$$-x = -\ln 3$$

$$x = \ln 3 = 1.099$$

**4.** (15 points) Find the domain, range, intercepts and the vertex of the parabola $f(x) = -4x^2 - 8x - 3$ using completing the square technique. Sketch the graph of the function.

Domain: $\mathbb{R}$

$-4x^2 - 8x - 3 = -4(x^2 + 2x + 1 - 1) - 3$

$-2h = 2 \qquad = -4(x+1)^2 + 4 - 3$

$h = -1 \qquad = -4(x+1)^2 + 1$

$h^2 = 1$

Vertex: $(-1, 1)$

Axis of symmetry: $x = -1$

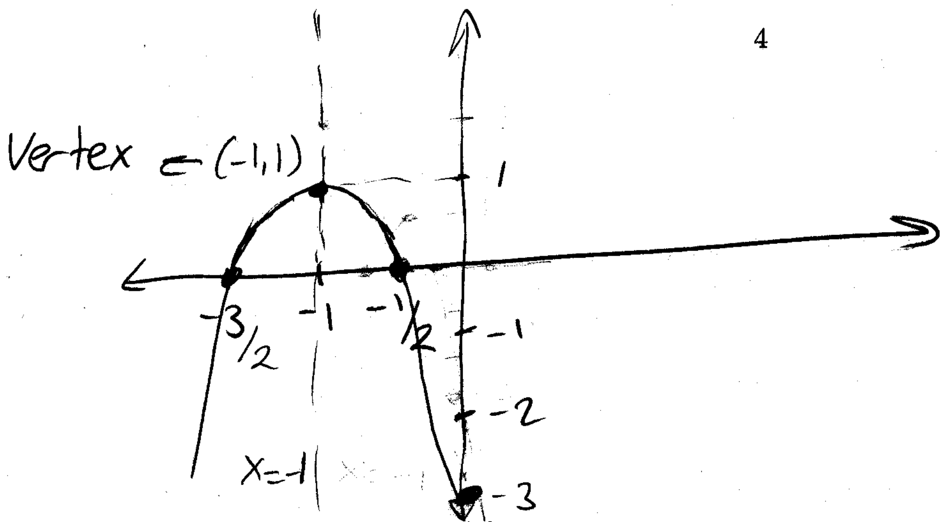
Range: $(-\infty, 1]$

$f(0) = -3 \qquad (0, -3) \to$ y-intercept

$-4x^2 - 8x - 3 = 0$

$$\frac{8 \mp \sqrt{64 - 4 \cdot -4 \cdot -3}}{-8} = \frac{8 \mp \sqrt{64 - 48}}{-8} = \frac{8 \mp \sqrt{16}}{-8}$$

$x_1 = \frac{8+4}{-8} = \frac{-3}{2} \qquad x_2 = \frac{8-4}{-8} = \frac{4}{-8} = \frac{-1}{2} \qquad x_1, x_2 \Rightarrow$ x intercepts



**5.**

a) (10 points) Find all values of $x$ in the interval $[-\pi, 2\pi]$ so that $\sqrt{3}\sin(x - \frac{\pi}{2}) = \frac{3}{2}$

$$\sqrt{3}\,\text{Sin}\,(x - \pi/2) = 3/2$$

$$\text{Sin}\,(x - \pi/2) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$x - \pi/2 = \frac{\pi}{3} + 2k\pi \qquad x - \pi/2 = \frac{2\pi}{3} + 2k\pi$$

$$x = \frac{5\pi}{6} + 2k\pi \qquad x = \frac{7\pi}{6} + 2k\pi$$

$$x \in \left\{ \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{-5\pi}{6} \right\}$$

Alternatively

$\text{Sin}\,(x - \pi/2) = \text{Sin}(-(\pi/2 - x)) =$

$-\text{Sin}\,(\pi/2 - x) = -\text{Cos}x = \frac{\sqrt{3}}{2}$

$$\boxed{\text{Cos}x = -\frac{\sqrt{3}}{2}}$$

$$x \in \{5\pi/6, 7\pi/6, -5\pi/6\}$$

b) (15 points) Calculate

i-) $\sin(\frac{\pi}{12}) = ?$

$$\text{Sin}\left(\frac{\pi}{12}\right) = \text{Sin}(\pi/3 - \pi/4) =$$

$$\text{Sin}\,\pi/3 \cdot \text{Cos}\,\pi/4 - \text{Sin}\,\pi/4 \cdot \text{Cos}\,\pi/3 = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

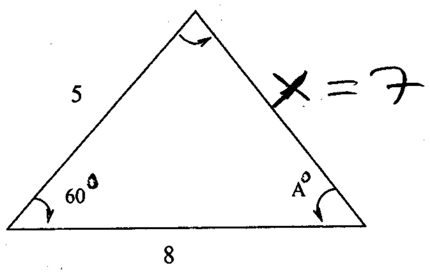
ii-) $\cos(\frac{7\pi}{12}) = ?$

$$\text{Cos}\left(\frac{7\pi}{12}\right) = \text{Cos}\left(\frac{\pi}{3} + \pi/4\right) = \text{Cos}\,\pi/3 \cdot \text{Cos}\,\pi/4 - \text{Sin}\,\pi/3 \cdot \text{Sin}\,\pi/4$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

iii-) $\tan^{-1}(-\sqrt{3}) = ?$ $\boxed{-\pi/3}$

c) (10 points) In the below triangle if $\sin A° = \frac{a}{b}\sqrt{c}$, where a,b,c are integers. Find a possible value for each a,b and c.




$$x^2 = 5^2 + 8^2 - 2.5.8.\cos 60°$$

$$25 + 64 - \cancel{2}.5.8.\frac{1}{\cancel{2}} = 49 \qquad x = 7$$

$$\frac{\sin A°}{5} = \frac{\sin 60°}{7}$$

$$\sin A° = \frac{5.\sqrt{3}}{2.7} = \frac{5\sqrt{3}}{14} \qquad \begin{array}{l} a = 5 \\ b = 14 \\ c = 3 \end{array}$$

**6.** (12 points) Find the equation of the line passing through the points $(1,3)$ and $(5,7)$ in the form $Ax + By = C$ and in the form $y = mx + b$ where $A, B, C, m$ and $b$ are real numbers.

$$m = \frac{7-3}{5-1} = \frac{4}{4} = 1$$

$$y = x + b$$

$$3 = 1 + b$$

$$b = 2$$

$$\boxed{y = x + 2}$$

$$\boxed{-x + y = 2}$$