KOÇ UNIVERSITY

MATH 101 - FINITE MATHEMATICS

Midterm II May 3, 2010

Duration of Exam: 75 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: ____________________

Surname: ____________________ KEY

Signature: ____________________

Section (Check One):

Section 1: E. Şule Yazıcı Tu-Th(11:00) ___
Section 2: E. Şule Yazıcı Tu-Th(14:00) ___
Section 3: Mehmet Sarıdereli Tu-Th(15:30) ___

PROBLEM	POINTS	SCORE
1	15	
2	20	
3	15	
4	20	
5	15	
6	15	
TOTAL	**100**	

1. (15 points) Solve the following system of linear equation by using Gauss Jordan elimination.

$$\begin{cases} x_1 + 2x_2 - 2x_3 &= -1 \\ 2x_1 - x_2 + x_3 &= 3 \\ x_2 + x_3 &= 5 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 5 \end{array}\right] \quad -2R_1 + R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & -5 & 5 & 5 \\ 0 & 1 & 1 & 5 \end{array}\right] \quad -\frac{1}{5} R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & 5 \end{array}\right] \quad \begin{array}{l} -2R_2 + R_1 \to R_1 \\ -R_2 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 6 \end{array}\right] \quad \frac{1}{2} R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_2 + R_3 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \to \quad \begin{array}{l} x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{array}$$

$SS = \{(1, 2, 3)\}$

unique solution

consistent

independent system

2. Use Gauss Jordan elimination to bring the following matrices into their reduced form. Write the solution set for the corresponding systems. Find two particular solutions for the system if there exist more than one solution. Determine if the system is consistent, inconsistent, dependent or independent.

(a) (12 points) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 10 & 4 \end{array}\right]$ $-2R_2 + R_4 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 5$

$x_2 + x_3 + 5x_4 = 2$

let $x_3 = a$, $x_4 = b$

$SS = \{(5, -a-5b+2, a, b) \mid a, b \in \mathbb{R}\}$

Consistent
dependent system
infinitely many solutions

Example Solutions:
let $a=0$ $b=0$ $(5, 2, 0, 0)$
let $a=1$ $b=0$ $(5, 1, 1, 0)$

(b) (8 points) $\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 3 & -2 & 0 & 5 \\ 5 & -4 & -2 & 10 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 3 & -2 & 0 & 5 \\ 5 & -4 & -2 & 10 \end{array}\right]$$

$-3R_1 + R_2 \to R_2$
$-5R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 1 & 3 & 0 \end{array}\right]$$

$R_2 + R_1 \to R_1$
$-R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0 \neq 1$

inconsistent system

no solutions

$SS = \emptyset$

A list of formulas: $I = Prt$; $A = P(1+rt)$

$A = P(1+i)^n$; $APY = (1+\frac{r}{m})^m - 1$;

$A = Pe^{rt}$; $APY = e^r - 1$;

$FV = PMT\frac{[(1+i)^n - 1]}{i}$

$PV = PMT\frac{[1-(1+i)^{-n}]}{i}$, where $i = \frac{r}{m}$ and $n = mt$

3.(15 points) What annual rate compounded monthly has the same annual percentage yield as 8% compounded quarterly?

$APY_1 = APY_2$

$(1+\frac{0.08}{4})^4 - 1 = (1+\frac{r}{12})^{12} - 1$

$(1.02)^4 = (1+\frac{r}{12})^{12}$

$1+\frac{r}{12} = \sqrt[3]{1.02}$

$\frac{r}{12} = \sqrt[3]{1.02} - 1$

$r = (\sqrt[3]{1.02} - 1) \times 12$

$r = 0,0795$

$r = 7.95\%$

4. (20 points) Ali has been saving for his retirement since he was 20 years old by putting \$100 each month in an account that pays 7.5% interest compounded monthly. Now he is 50 years of age and wants to retire. If he wants to withdraw \$1300 each month from the account, find the number of withdrawals he can make.

$FV = PV$

$$100 \frac{(1+\frac{0,075}{12})^{360} - 1}{\frac{0,075}{12}} = 1300 \frac{1-(1+\frac{0,075}{12})^{-n}}{\frac{0,075}{12}}$$

$$\frac{(1,00625)^{360} - 1}{13} = 1 - (1,00625)^{-n}$$

$$(1,00625)^{-n} = 1 - \frac{(1,00625)^{360} - 1}{13}$$

$$-n \ln 1,00625 = \ln\left(1 - \frac{(1,00625)^{360} - 1}{13}\right)$$

$$n = -\frac{\ln\left(1 - \frac{(1,00625)^{360} - 1}{13}\right)}{\ln 1,00625} = n = \frac{\ln 0,352189699}{\ln 1,00625} =$$

$n = 167,49$

167 withdrawals

5. (15 points) A person purchased a \$200,000 home 20 years ago by paying 20% down (peşin) and signing a 30-year mortgage at 13.2% compounded monthly. Find the unpaid balance of the mortgage today.

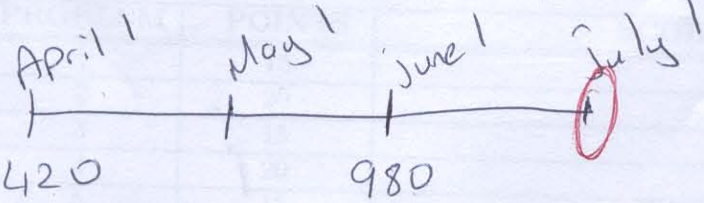
$$200{,}000 \times 0{,}8 = 160.000$$

$$PMT = \frac{160.000 \times \frac{0{,}132}{12}}{1-\left(1+\frac{0.132}{12}\right)^{-360}} = 1{,}794.97$$

$$\text{Unpaid balace} = 1{,}794.97 \, \frac{1-\left(1+\frac{0{,}132}{12}\right)^{-120}}{\frac{0{,}132}{12}}$$

$$= 119{,}272.89$$

6. (15 points) Ayşe's credit card company charges her interest at 2.3% per month. She has no debt on March 31, but buys a cell phone on April 1 for 420 TL, and a laptop for 980 TL on June 1. Show that she owes the credit card company 1452.19 TL on July 1 if she does not make any payments between April 1 and July 1.



$$420(1+0{,}023)^3 + 980(1+0{,}023) = 1452{,}19$$