

Math 101, Midterm 2: 120 minutes

Instructions: There are 5 questions ☺ in this exam. Please inspect the exam and make sure you have all 5 pages of questions. There are 20 extra points, so select the questions that seem easier to you. **Show your work** on the empty space below the questions and on the back pages.

Extremely Important:

You *MUST* write all answers in THE SPACE PROVIDED FOR ANSWERS.
Any answers not written in the space provided will NOT BE EVALUATED.
Currency has to be presented with two digits after the decimal point.

Remember: You *must* show your work to get proper credit.

Academic Honesty Code: Koç University Academic Honesty Code stipulates that “copying from others or providing answers or information, written or oral, to others is cheating.” By taking this exam, you are assuming full responsibility for observing the Academic Honesty Code.

NAME: _____ SECTION(Circle one): 1 2 3 4

Q.1:	/35
Q.2:	/25
Q.3:	/20
Q.4:	/25
Q.5:	/15
Total:	/100

Question 1:

Lütfi Evsiz wants to buy a house. He needs to borrow 285,000 TL. The best mortgage available is at 21.45% compounded monthly. (There is a bank that will loan him this amount at 21.45% compounded monthly)

a) How much does he have to pay per month if the loan is to be paid off in 20 years?

PMT = _____ TL

b) What is the total interest that he will pay? Total interest paid = _____ TL

c) By how much is the debt reduced with the 175th payment? By _____ TL
(unpaid balance reduction)

d) How much of the third payment will go to interest? _____ TL

e) Lütfi's wife starts depositing 280 TL a month into an account that pays 1.6% per month at the same time that Lütfi starts making payments for the house. After 5 years she gives him the money in this account. He gives this money together with the 60th payment he would normally make to the bank. What are the monthly payments he will now have to make to pay off the remaining debt in 15 years?

PMT = _____ TL

Question 2:

The augmented matrix of a system of equations is given below.

1	3	5	6	2	0
2	7	12	15	5	0
1	5	10	13	5	0
3	10	18	22	8	0
1	4	7	9	3	0

a) How many leading 1s does the reduced row echelon form of this system have? _____

b) How many free variables (parameters) are required to describe all solutions? _____

c) Give two solutions.

_____ and _____

Question 3:

A region is defined by the inequalities $2 \leq 4x - y$, $x + y \leq 8$, $0.5x + y \geq 2.5$, $y \geq 3x - 8$.
The objective function (cost function) is given by $P = ax + 3y$.

a) Which of the points (2,1), (3,5), and (1.5,5) are in the region?

b) Assuming $a = -2$, determine the maximum and minimum values of P subject to the given constraints.

$$P_{max} = \underline{\hspace{2cm}} \qquad P_{min} = \underline{\hspace{2cm}}$$

c) Give a value of a so that the optimum value of the objective function is at (4,4).

$$a = \underline{\hspace{2cm}}$$

d) For which value of a will the optimum value of the objective function be at all points between (1,2) and (3,1)?

$$a = \underline{\hspace{2cm}}$$

Question 4:

Use the **simplex method** to find the optimal value and optimal solution to the following standard maximization problem:

$$\max P = 4x + 5y, \quad x + 2y \leq 8, \quad 3x + y \leq 9, \quad x \geq 0, \quad y \geq 0$$

Write the equations involving the slack variables (initial system), and indicate the values of the basic variables and the non-basic variables for each step, starting with the initial simplex tableau.

Optimal Value: _____ Optimal Solution: _____

Question 5:

Find the solution for the matrix B in the following matrix equation. (Note: You must use the inverse of the 2x2 matrix on the left side, and you must find that inverse using Gauss-Jordan elimination)

$$B \cdot \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 & 6 \\ 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 \\ 1 & 0 \\ -2 & 3 \\ -1 & 0 \end{pmatrix}$$