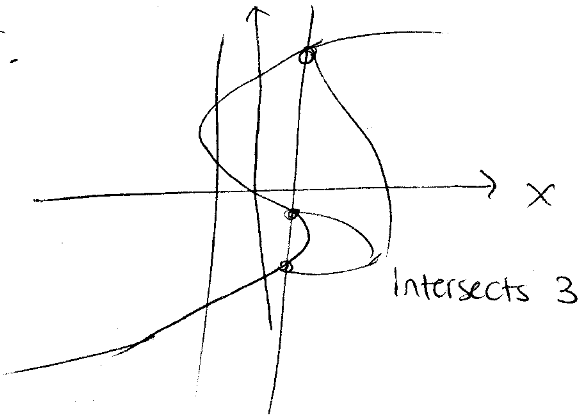
Solutions for HW #1

Section 1.1 — Four ways to represent a function.

5–8 : Determine whether the curve is the graph of a function of x, If it is, state the domain and range of the function.

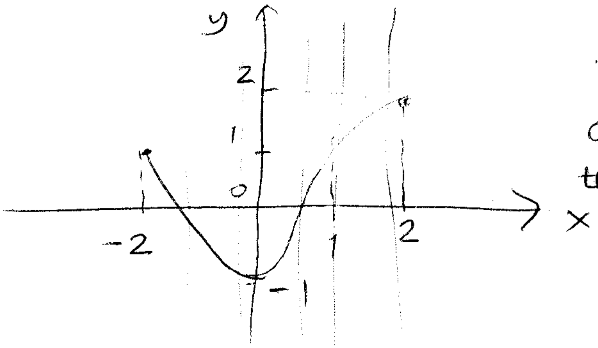
5.



Using the vertical line test, it can easily be observed that this curve doesn't represent a function.

Intersects 3 points on the curve.

6.

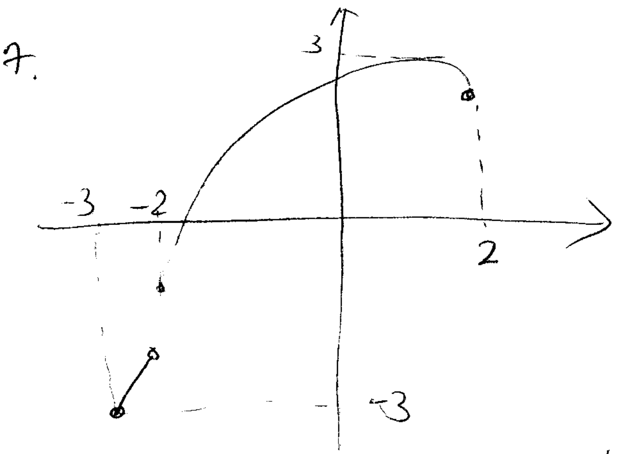


Vertical line test shows that any arbitrary line intersects only one point on the curve. So the curve represents a function.

Domain = $x \in [-2, 2]$

Range = $y \in [-1, 2]$

7.

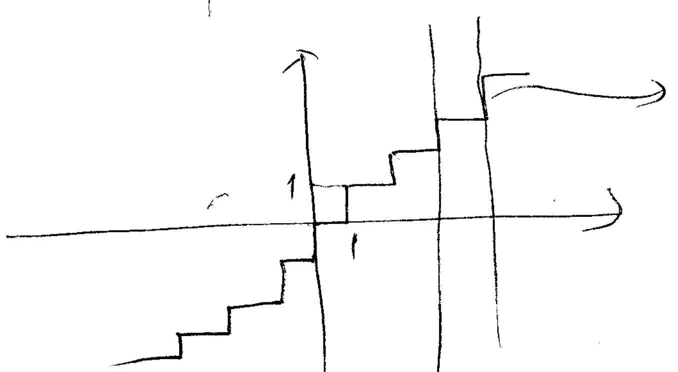


→ Vertical line test, function.

Domain = $x \in [-3, 2]$

Range = $y \in [-3, 3]$

8.



→ intersects more than one point

Not a function

29–33: Find the domain of the function

30) $f(x) = \dfrac{2x^3-5}{x^2+x-6}$ for $f(x)$ to be defined, $x^2+x-6 \neq 0$ must be satisfied. If,

$x^2+x-6 = (x+3)(x-2) = 0$ $x=-3$ OR $x=2$.

Hence domain of this function is,

$$\boxed{x \in \mathbb{R} - \{-3, 2\}}$$

32) $g(t) = \sqrt{3-t} - \sqrt{2+t}$ for $g(t)$ to be defined,

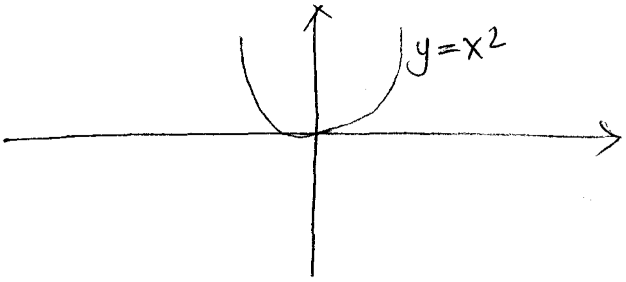
$3-t \geq 0$ and $2+t \geq 0$ must be satisfied.

$3-t \geq 0 \Rightarrow 3 \geq t$ and $2+t \geq 0 \Rightarrow t \geq -2$

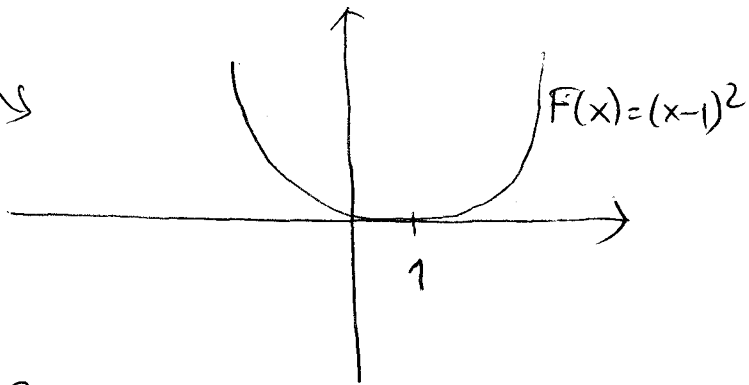
Hence domain of the function is, $\boxed{-2 \leq t \leq 3}$

35–46: Find the domain and sketch the graph of the function.

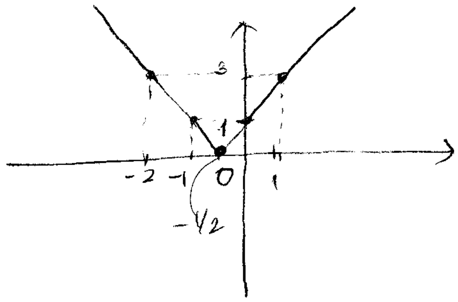
36) $F(x) = x^2-2x+1$, Domain : $x \in \mathbb{R}$.



$F(x) = x^2-2x+1 = (x-1)^2$



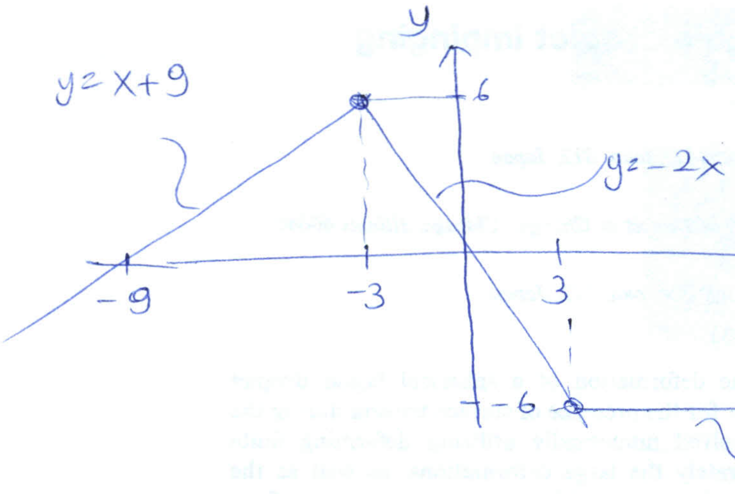
40) $F(x) = |2x+1|$ Domain : $x \in \mathbb{R}$



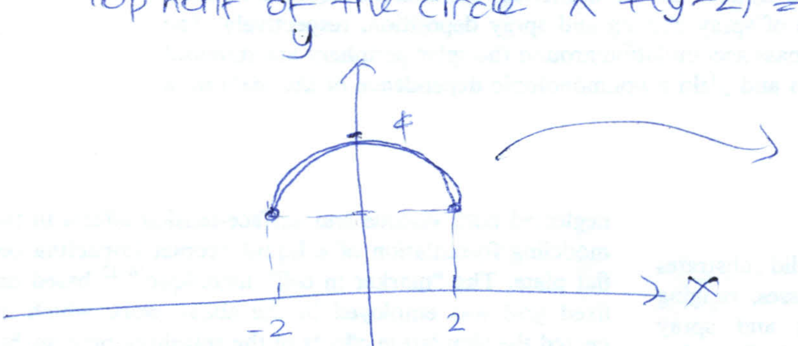
$y = |2x+1| \Rightarrow y \geq 0$.

46) $f(x) = \begin{cases} x+9, & \text{if } x<-3 \\ -2x, & \text{if } |x|\leq 3 \\ -6, & \text{if } x>3 \end{cases}$ Domain $x \in \mathbb{R}$

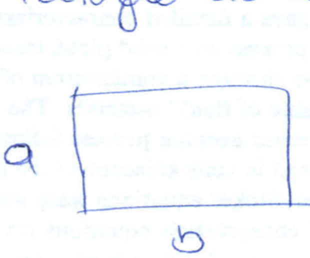
$|x| \leq 3 \Rightarrow -3 \leq x \leq 3$



50) " Top half of the circle $x^2+(y-2)^2=4$



54) A rectangle has area $16m^2$ Express the perimeter of the rectangle as a function of the length of one of its sides.



$ab = 16m^2 \Rightarrow a = 16/b$

Perimeter: $p(b) = 2b+2a = 2b+\frac{32}{b}$

70–72: Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

70) $f(x) = x|x|$

$f(-x) = -x|-x| = -x|x|$

$f(x) \neq f(-x)$

$f(-x) = -f(x) \Rightarrow$ ODD

72) $f(x) = 1 + 3x^3 - x^5$

$f(-x) = 1 + 3(-x)^3 - (-x)^5$

$= 1 - 3x^3 + x^5 \neq -f(x)$ OR

$f(x) \neq f(-x)$

not ODD OR EVEN.

Section 1.2 : mathematical Models: A catalog of Essential Functions

2) Classify each function as a power function, root function, polynomial - - -

(a) $y = \pi^x$: exponential function

(b) $y = x^\pi$: power function

(c) $y = x^2(2-x^3)$; polynomial degree 5

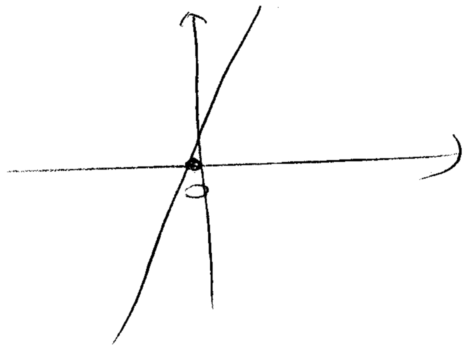
(d) $y = \tan t - \cos t$: trigonometric

(e) $y = \frac{s}{1+s}$: rational function.

(f) $y = \frac{\sqrt{x^3-1}}{1+\sqrt[3]{x}}$: algebraic function

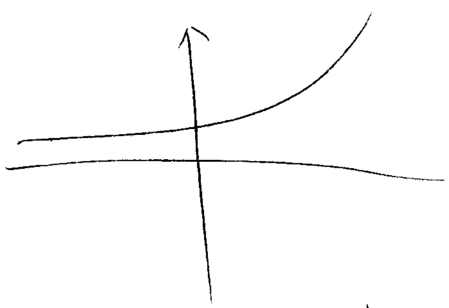
4) match each equation with its graph

(a) $y = 3x$: G



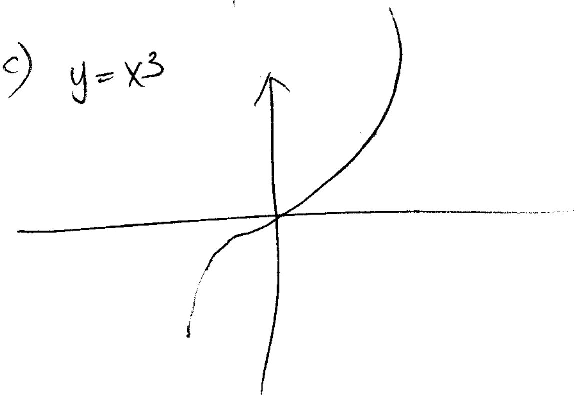
linear

(b) $y = 3^x$: f

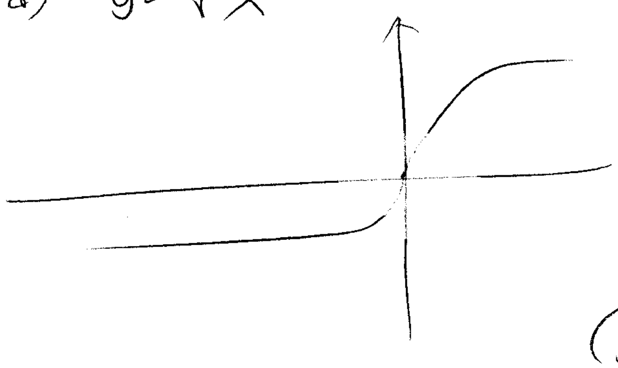


doesn't intersect "O"

(c) $y = x^3$



(d) $y = \sqrt[3]{x}$

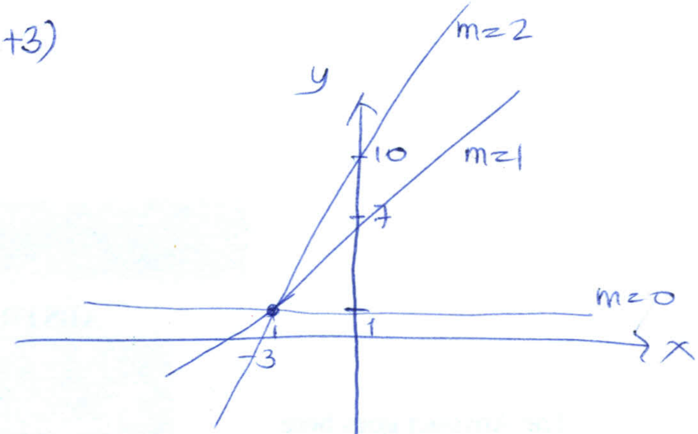


6) For $m \in \mathbb{R}$, $f(x) = 1 + m(x+3)$

If $m = 0 \Rightarrow f(x) = 1$

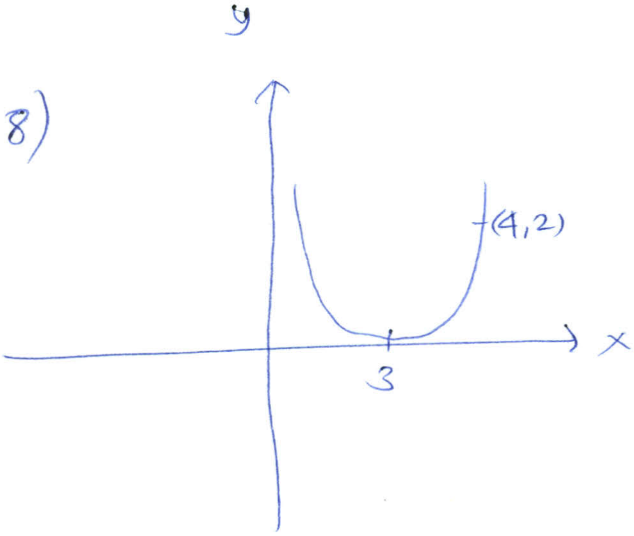
If $m = 1 \Rightarrow f(x) = x + 4$

If $m = 2 \Rightarrow f(x) = 2x + 7$



All $f(x)$ functions pass through $(-3,1)$ for $\forall m \in \mathbb{R}$.

8)



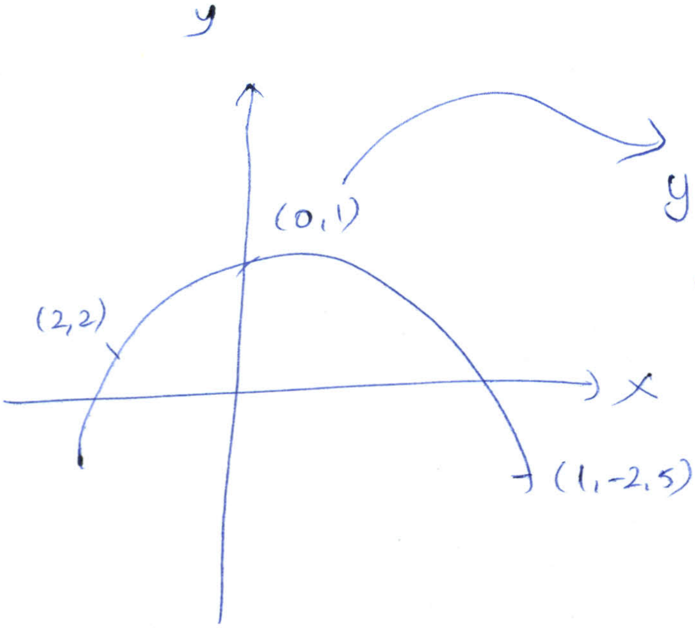
$y = a(x-3)^2 + 0$.

$(4,2)$ is on the parabola,

$2 = a(4-3)^2 + 0$

$2 = a.1 + 0 \Rightarrow a = 2$

$f(x) = 2(x-3)^2$



$y = ax^2 + bx + 1$

$(-2,2), (1,-2.5)$ are on the curve, hence they must satisfy $f(x)$,

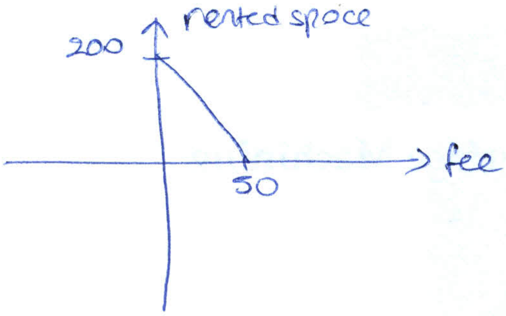
$$\left.\begin{aligned} 2 &= 4a - 2b + 1 \\ -2.5 &= a + b + 1 \end{aligned}\right\} \quad \begin{aligned} 4a - 2b &= 1 \\ a + b &= -3.5 \end{aligned}$$

$a = -1$, $b = -2.5$

$f(x) = -x^2 - 2.5x + 1$

12) $y = 200 - 4x$,

a)



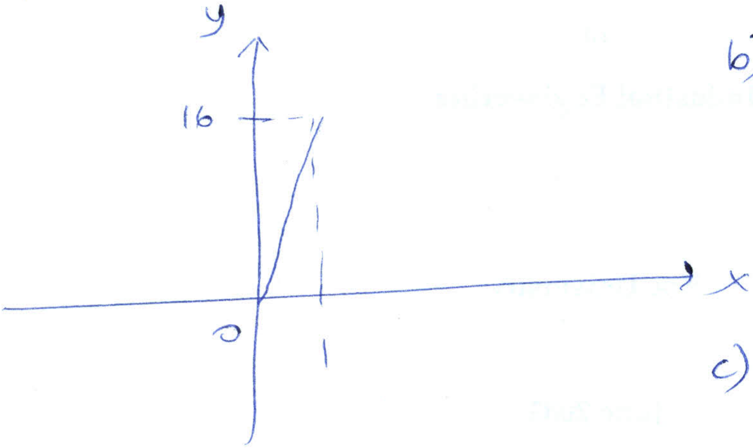
b) slope $= -4$; # of rented spaces decrease by 4 units as fee increase as 1 dollar,

x-Intercept: fee corresponding no rented spaces

y-Intercept: # of spaces corresponding no fee

16) \$2200 to manufacture 100 chairs, \$4800 " produce 300 " } Production cost, \$16 per chair.

a) $C(x) = 16x$



b) Slope of the graph, Cost Increases by \$16 as # of chairs Increase by 1.

c) cost corresponding no chairs produced, Section 1-3 — New Functions from Old Functions.

2) Explain how each graph is obtained from the graph of $y=f(x)$

a) $y=f(x)+8$ the graph of $y=f(x)$ shifted 8 units upwards

b) $y=f(x+8)$ the graph of $y=f(x)$ shifted 8 units to left

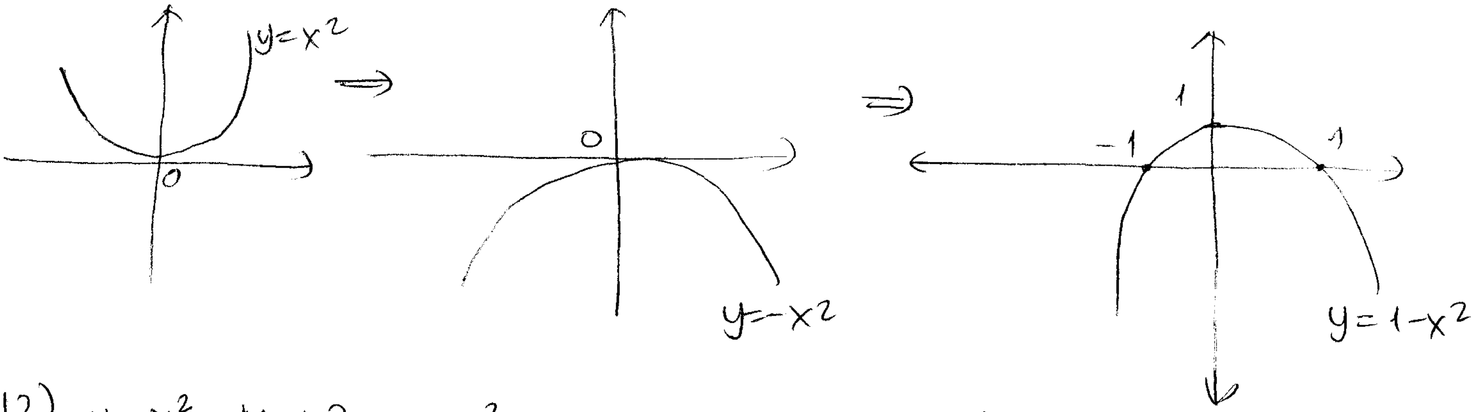
c) $y=8f(x)$ the graph of $y=f(x)$ stretched vertically by a factor of 8

d) $y=f(8x)$ " " " shrinked horizontally " 8

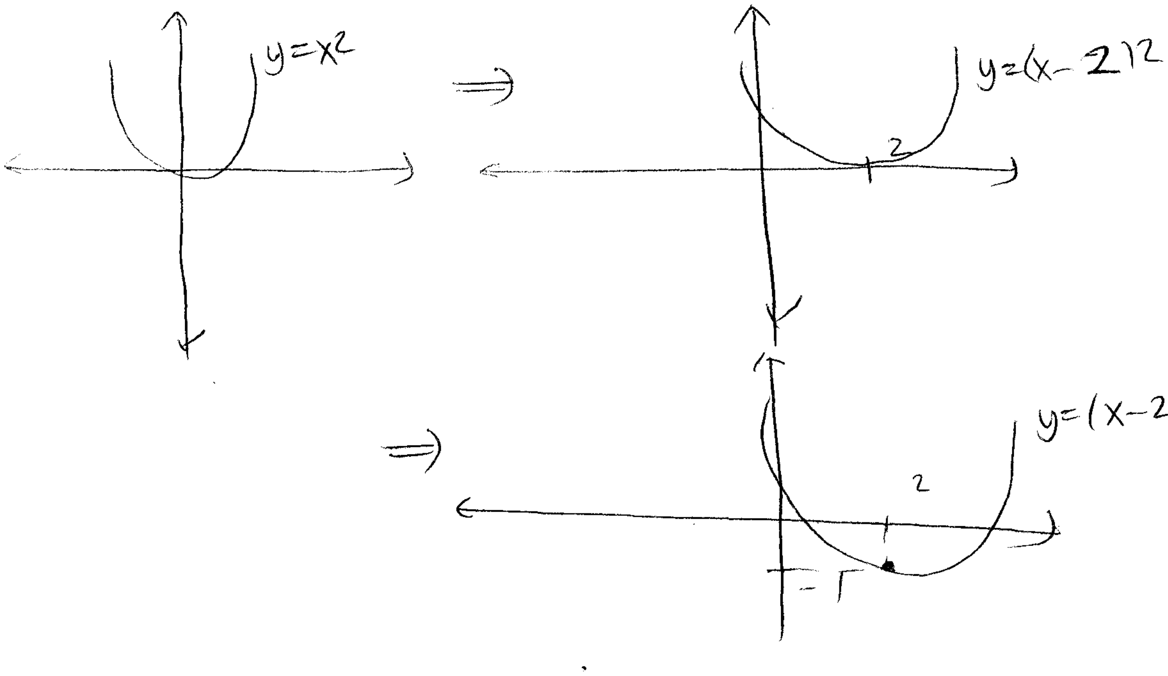
e) $y=-f(x)-1$ " " " reflected about x axis and shifted 1 unit down

f) $y=8f(\frac{1}{8}x)$ " " " stretched horizontally by a factor of 8 and vertically by a factor of 8.

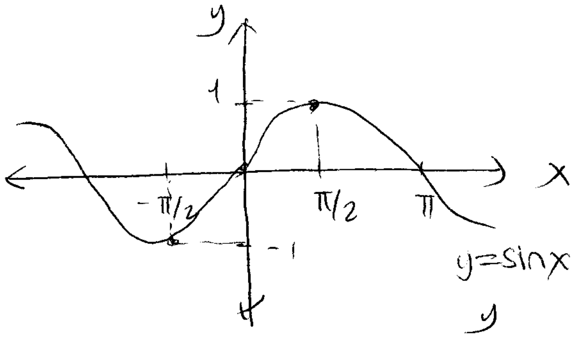
10) $y=1-x^2$, graph the function using standard function.



12) $y=x^2-4x+3 = x^2-4x+4-1 = (x-2)^2-1$



14) $y = 4\sin 3x$



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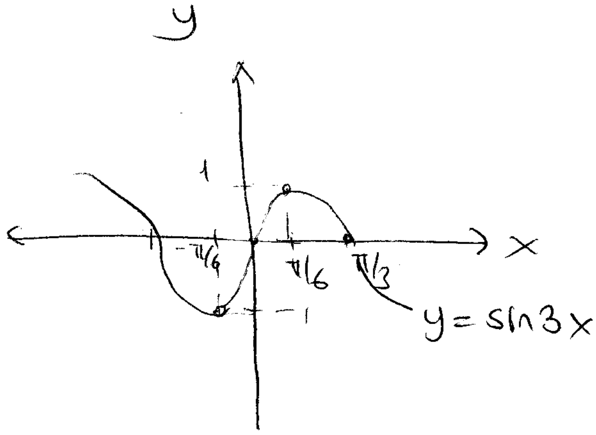

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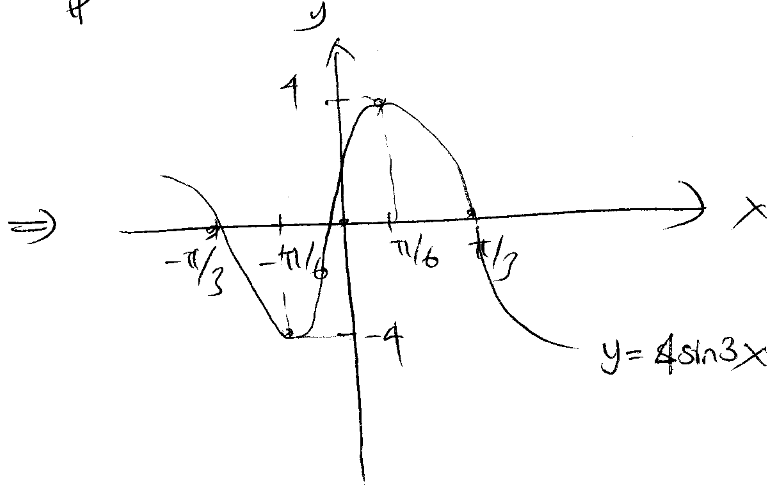
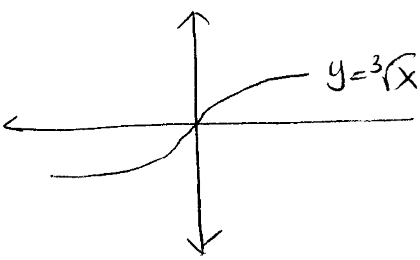


20) $y = 1 + \sqrt[3]{x-1}$



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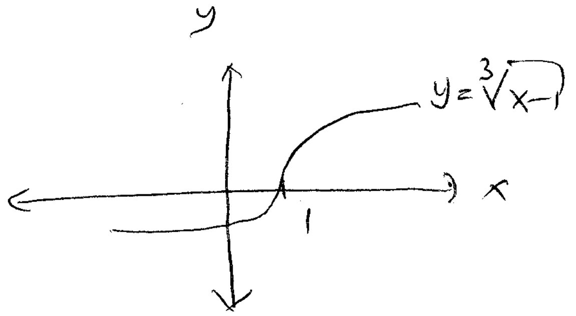

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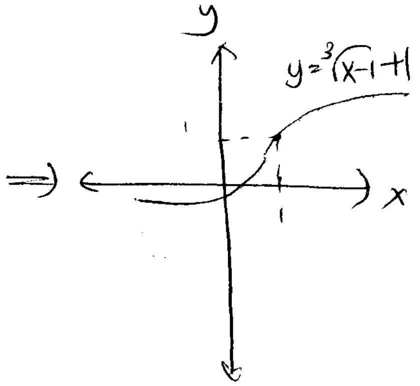
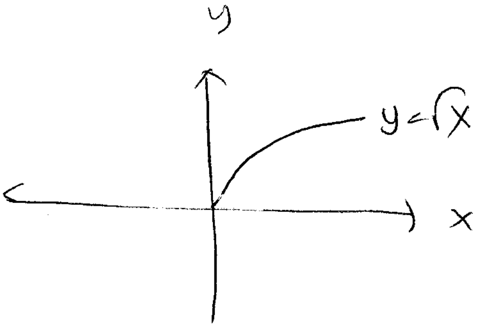


30) $f(x) = \sqrt{3-x}$



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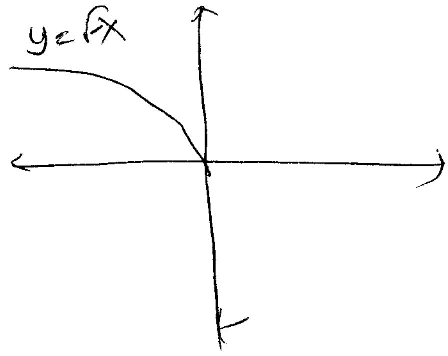

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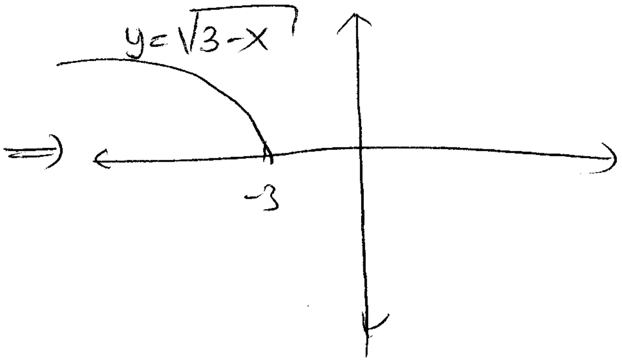

32) $f(x) = x-2$, $g(x) = x^2+3x+4$

a) $f \circ g = (x-2) \circ (x^2+3x+4) = x^2+3x+4-2 = x^2+3x+2$

b) $g \circ f = (x^2+3x+4) \circ (x-2) = (x-2)^2+3(x-2)+4$

$= x^2-4x+4+3x-6+4 = x^2-x+2$

c) $f \circ f = (x-2) \circ (x-2) = (x-2-2) = x-4$

d) $g \circ g = (x^2+3x+4) \circ (x^2+3x+4)$

$= (x^2+3x+4)^2+3(x^2+3x+4)+4$

$= x^4+9x^2+16+6x^3+8x^2+24x+3x^2+9x+12+4$

$= x^4+6x^3+20x^2+33x+32$

36) $f(x) = \dfrac{x}{1+x}$, $g(x) = \sin 2x$

a) $f \circ g = \left(\dfrac{x}{1+x}\right) \circ (\sin 2x) = \dfrac{\sin 2x}{1+\sin 2x}$

b) $g \circ f = (\sin 2x) \circ \left(\dfrac{x}{1+x}\right) = \sin\left(2\dfrac{x}{1+x}\right)$

c) $f \circ f = \left(\dfrac{x}{1+x}\right) \circ \left(\dfrac{x}{1+x}\right) = \dfrac{x/1+x}{1+\dfrac{x}{1+x}} = \dfrac{x}{\cancel{1+x}} \cdot \dfrac{\cancel{1+x}}{1+2x} = \dfrac{x}{1+2x}$

d) $g \circ g = (\sin 2x) \circ (\sin 2x) = \sin 2(\sin 2x)$

44) $G(x) = \sqrt[3]{\dfrac{x}{1+x}}$

If $f = \sqrt[3]{x}$, $g = \dfrac{x}{1+x}$

Then $G(x) = f \circ g$

64) $h = f \circ g = f(g(x))$

$g(x)$ is an odd functio.

Is h always an odd function? What if f is odd? What if f is even?

lets assume,

$f = x^2$ (EVEN), $g(x) = x^3$ (ODD) $\Rightarrow h(x) = f \circ g(x)$

$= f(g(x)) = (x^3)^2 = x^6$

then f(x) is EVEN

$f = x$ (ODD), $g(x) = x^3$ (ODD) $\Rightarrow h(x) = f(x^3) = x^3$ (ODD)

then f(x) is ODD.

Hence, f(x) isn't allways and odd function given that g(x) is odd; Depends on ~~g~~ selection of f(x).