Homework 10 – Solutions

Section 4.8

10. $g(x) = \dfrac{5 - 4x^3 + 2x^6}{x^6} = 5x^{-6} - 4x^{-3} + 2$ has domain $(-\infty, 0) \cup (0, \infty)$, so

$$G(x) = \begin{cases} 5\dfrac{x^{-5}}{-5} - 4\dfrac{x^{-2}}{-2} + 2x + C_1 = -\dfrac{1}{x^5} + \dfrac{2}{x^2} + 2x + C_1 & \text{if } x < 0 \\ -\dfrac{1}{x^5} + \dfrac{2}{x^2} + 2x + C_2 & \text{if } x > 0 \end{cases}$$

12. $f(x) = 3e^x + 7\sec^2 x \;\Rightarrow\; F(x) = 3e^x + 7\tan x + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

14. $f(x) = 2\sqrt{x} + 6\cos x = 2x^{1/2} + 6\cos x \;\Rightarrow\; F(x) = 2\left(\dfrac{x^{3/2}}{3/2}\right) + 6\sin x + C = \frac{4}{3}x^{3/2} + 6\sin x + C$

16. $f(x) = \dfrac{2 + x^2}{1 + x^2} = \dfrac{1 + (1 + x^2)}{1 + x^2} = \dfrac{1}{1 + x^2} + 1 \;\Rightarrow\; F(x) = \tan^{-1} x + x + C$

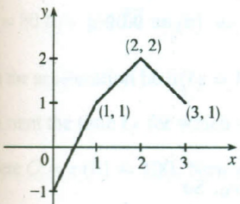
26. $f'(x) = 2x - 3/x^4 = 2x - 3x^{-4} \;\Rightarrow\; f(x) = x^2 + x^{-3} + C$ because we're given that $x > 0$.
$f(1) = 2 + C$ and $f(1) = 3 \;\Rightarrow\; C = 1$, so $f(x) = x^2 + 1/x^3 + 1$.

30. $f''(x) = 8x^3 + 5 \;\Rightarrow\; f'(x) = 2x^4 + 5x + C$. $f'(1) = 2 + 5 + C$ and $f'(1) = 8 \;\Rightarrow\; C = 1$, so $f'(x) = 2x^4 + 5x + 1$. $f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + D$. $f(1) = \frac{2}{5} + \frac{5}{2} + 1 + D = D + \frac{39}{10}$ and $f(1) = 0 \;\Rightarrow\; D = -\frac{39}{10}$, so $f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - \frac{39}{10}$.

32. $f''(t) = 3/\sqrt{t} = 3t^{-1/2} \;\Rightarrow\; f'(t) = 6t^{1/2} + C$. $f'(4) = 12 + C$ and $f'(4) = 7 \;\Rightarrow\; C = -5$, so $f'(t) = 6t^{1/2} - 5$ and hence, $f(t) = 4t^{3/2} - 5t + D$. $f(4) = 32 - 20 + D$ and $f(4) = 20 \;\Rightarrow\; D = 8$, so $f(t) = 4t^{3/2} - 5t + 8$.

38. $f'(x) = x^3 \;\Rightarrow\; f(x) = \frac{1}{4}x^4 + C$. $x + y = 0 \;\Rightarrow\; y = -x \;\Rightarrow\; m = -1$. Now $m = f'(x) \;\Rightarrow\; -1 = x^3 \;\Rightarrow\; x = -1 \;\Rightarrow\; y = 1$ (from the equation of the tangent line), so $(-1, 1)$ is a point on the graph of f. From f, $1 = \frac{1}{4}(-1)^4 + C \;\Rightarrow\; C = \frac{3}{4}$. Therefore, the function is $f(x) = \frac{1}{4}x^4 + \frac{3}{4}$.

39.



$$f'(x) = \begin{cases} 2 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 < x < 2 \\ -1 & \text{if } 2 < x \le 3 \end{cases} \;\Rightarrow\; f(x) = \begin{cases} 2x + C & \text{if } 0 \le x < 1 \\ x + D & \text{if } 1 < x < 2 \\ -x + E & \text{if } 2 < x \le 3 \end{cases}$$

$f(0) = -1 \;\Rightarrow\; 2(0) + C = -1 \;\Rightarrow\; C = -1$. Starting at the point $(0, -1)$ and moving to the right on a line with slope 2 gets us to the point $(1, 1)$. The slope for $1 < x < 2$ is 1, so we get to the point $(2, 2)$. Here we have used the fact that f is continuous. We can include the point $x = 1$ on either the first or the second part of f. The line connecting $(1, 1)$ to $(2, 2)$ is $y = x$, so $D = 0$. The slope for $2 < x \le 3$ is -1, so we get to $(3, 1)$. $f(3) = 1 \;\Rightarrow\; -3 + E = 1 \;\Rightarrow\; E = 4$. Thus

$$f(x) = \begin{cases} 2x - 1 & \text{if } 0 \le x \le 1 \\ x & \text{if } 1 < x < 2 \\ -x + 4 & \text{if } 2 \le x \le 3 \end{cases}$$

Note that $f'(x)$ does not exist at $x = 1$ or at $x = 2$.

47. Marginal cost $= 1.92 - 0.002x = C'(x) \;\Rightarrow\; C(x) = 1.92x - 0.001x^2 + K$. But $C(1) = 1.92 - 0.001 + K = 562 \;\Rightarrow\; K = 560.081$. Therefore, $C(x) = 1.92x - 0.001x^2 + 560.081 \;\Rightarrow\; C(100) = 742.081$, so the cost of producing 100 items is \$742.08.

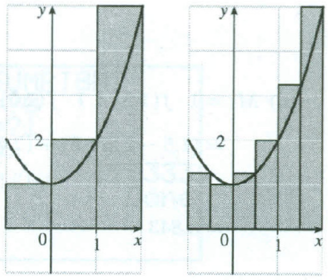
Seciton 5.1

5. (a) $f(x) = 1 + x^2$ and $\Delta x = \dfrac{2-(-1)}{3} = 1 \Rightarrow$

$R_3 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 5 = 8.$

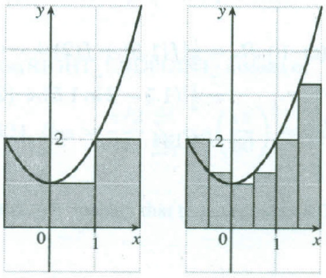
$\Delta x = \dfrac{2-(-1)}{6} = 0.5 \Rightarrow$

$$\begin{aligned} R_6 &= 0.5[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \\ &= 0.5(1.25 + 1 + 1.25 + 2 + 3.25 + 5) \\ &= 0.5(13.75) = 6.875 \end{aligned}$$



(b) $L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 5$

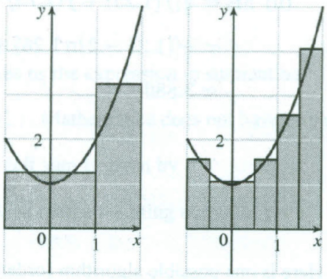
$$\begin{aligned} L_6 &= 0.5[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)] \\ &= 0.5(2 + 1.25 + 1 + 1.25 + 2 + 3.25) \\ &= 0.5(10.75) = 5.375 \end{aligned}$$



(c) $M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$

$= 1 \cdot 1.25 + 1 \cdot 1.25 + 1 \cdot 3.25 = 5.75$

$$\begin{aligned} M_6 &= 0.5[f(-0.75) + f(-0.25) + f(0.25) \\ &\quad + f(0.75) + f(1.25) + f(1.75)] \\ &= 0.5(1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625) \\ &= 0.5(11.875) = 5.9375 \end{aligned}$$



(d) M_6 appears to be the best estimate.

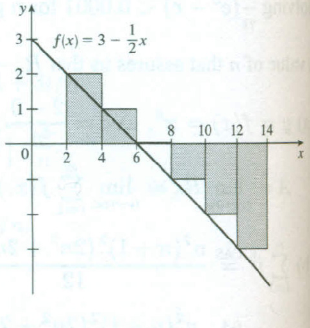
18. $f(x) = x^2 + \sqrt{1+2x}$, $4 \le x \le 7$. $\Delta x = (7-4)/n = 3/n$ and $x_i = 4 + i\Delta x = 4 + 3i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left[(4+3i/n)^2 + \sqrt{1 + 2(4+3i/n)}\right] \cdot \frac{3}{n}.$$

20. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5+x)^{10}$ on the interval $[0,2]$, since for $y = (5+x)^{10}$ on $[0,2]$ with $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$, $x_i = 0 + i\,\Delta x = \dfrac{2i}{n}$, and $x_i^* = x_i$, the expression for the area is $A = \displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(5 + \frac{2i}{n}\right)^{10} \frac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$ on $[5,7]$ or, in general, $y = ((5-n)+x)^{10}$ on $[n, n+2]$.

Section 5.2

1. $f(x) = 3 - \frac{1}{2}x,\ 2 \le x \le 14.\ \Delta x = \dfrac{b-a}{n} = \dfrac{14-2}{6} = 2.$



Since we are using left endpoints, $x_i^* = x_{i-1}$.

$$L_6 = \sum_{i=1}^{6} f(x_{i-1})\,\Delta x$$
$$= (\Delta x)\,[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$$
$$= 2[f(2) + f(4) + f(6) + f(8) + f(10) + f(12)]$$
$$= 2[2 + 1 + 0 + (-1) + (-2) + (-3)] = 2(-3) = -6$$

The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the three rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

9. $\Delta x = (10-2)/4 = 2$, so the endpoints are 2, 4, 6, 8, and 10, and the midpoints are 3, 5, 7, and 9. The Midpoint Rule gives $\int_2^{10} \sqrt{x^3+1}\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = 2\left(\sqrt{3^3+1} + \sqrt{5^3+1} + \sqrt{7^3+1} + \sqrt{9^3+1}\right) \approx 124.1644.$

22. $\displaystyle\int_1^4 (x^2 + 2x - 5)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \qquad [\Delta x = 3/n \text{ and } x_i = 1 + 3i/n]$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1 + \frac{3i}{n}\right)^2 + 2\left(1 + \frac{3i}{n}\right) - 5\right]\left(\frac{3}{n}\right)$$
$$= \lim_{n\to\infty} \frac{3}{n}\left[\sum_{i=1}^{n}\left(1 + \frac{6i}{n} + \frac{9i^2}{n^2} + 2 + \frac{6i}{n} - 5\right)\right]$$
$$= \lim_{n\to\infty} \frac{3}{n}\left[\sum_{i=1}^{n}\left(\frac{9}{n^2}\cdot i^2 + \frac{12}{n}\cdot i - 2\right)\right] = \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}\sum_{i=1}^{n} i^2 + \frac{12}{n}\sum_{i=1}^{n} i - \sum_{i=1}^{n} 2\right]$$
$$= \lim_{n\to\infty}\left(\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{36}{n^2}\cdot\frac{n(n+1)}{2} - \frac{6}{n}\cdot n\right)$$
$$= \lim_{n\to\infty}\left(\frac{9}{2}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n} + 18\cdot\frac{n+1}{n} - 6\right)$$
$$= \lim_{n\to\infty}\left[\frac{9}{2}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 18\left(1 + \frac{1}{n}\right) - 6\right] = \tfrac{9}{2}\cdot 1\cdot 2 + 18\cdot 1 - 6 = 21$$

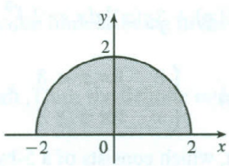
32. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

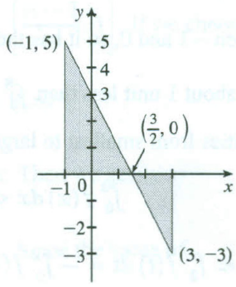
(c) $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ [area of a triangle]

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

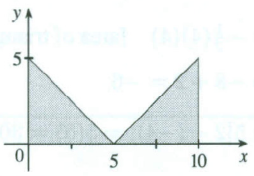
34. $\int_{-2}^{2} \sqrt{4 - x^2}\,dx$ can be interpreted as the area under the graph of $f(x) = \sqrt{4 - x^2}$ between $x = -2$ and $x = 2$. This is equal to half the area of the circle with radius 2, so $\int_{-2}^{2} \sqrt{4 - x^2}\,dx = \frac{1}{2}\pi\cdot 2^2 = 2\pi$.



36. $\int_{-1}^{3}(3-2x)\,dx$ can be interpreted as the area of the triangle above the x-axis minus the area of the triangle below the x-axis; that is, $\frac{1}{2}\left(\frac{5}{2}\right)(5)-\frac{1}{2}\left(\frac{3}{2}\right)(3)=\frac{25}{4}-\frac{9}{4}=4$.



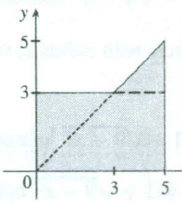
38. $\int_{0}^{10}|x-5|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2\left(\frac{1}{2}\right)(5)(5)=25$.



41. $\int_{-2}^{2}f(x)\,dx+\int_{2}^{5}f(x)\,dx-\int_{-2}^{-1}f(x)\,dx=\int_{-2}^{5}f(x)\,dx+\int_{-1}^{-2}f(x)\,dx$ [by Property 5 and reversing limits]

$=\int_{-1}^{5}f(x)\,dx$ [Property 5]

44. If $f(x)=\begin{cases}3 & \text{for } x<3\\ x & \text{for } x\geq 3\end{cases}$, then $\int_{0}^{5}f(x)\,dx$ can be interpreted as the area of the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus, $\int_{0}^{5}f(x)\,dx=5(3)+\frac{1}{2}(2)(2)=17$.



52. If $0\leq x\leq 2$, then $1\leq 1+x^2\leq 5$ and $\dfrac{1}{5}\leq\dfrac{1}{1+x^2}\leq 1$, so $\dfrac{1}{5}(2-0)\leq\displaystyle\int_0^2\frac{1}{1+x^2}\,dx\leq 1(2-0)$; that is, $\dfrac{2}{5}\leq\displaystyle\int_0^2\frac{1}{1+x^2}\,dx\leq 2$.

Section 5.3

6.

$\int_1^8\sqrt[3]{x}\,dx=\int_1^8 x^{1/3}\,dx=\left[\frac{3}{4}x^{4/3}\right]_1^8=\frac{3}{4}(8^{4/3}-1^{4/3})=\frac{3}{4}(2^4-1)=\frac{3}{4}(16-1)=\frac{3}{4}(15)=\frac{45}{4}$

8. $\int_{-5}^{5}e\,dx=[ex]_{-5}^{5}=5e-(-5e)=10e$

14. $\int_0^{\pi/4}\sec\theta\tan\theta\,d\theta=[\sec\theta]_0^{\pi/4}=\sec\frac{\pi}{4}-\sec 0=\sqrt{2}-1$

18. $\int_0^5(2e^x+4\cos x)\,dx=\left[2e^x+4\sin x\right]_0^5=(2e^5+4\sin 5)-(2e^0+4\sin 0)=2e^5+4\sin 5-2\approx 290.99$

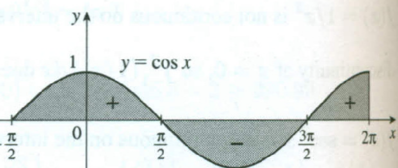
26. $$\int_1^2\frac{(x-1)^3}{x^2}\,dx=\int_1^2\frac{x^3-3x^2+3x-1}{x^2}\,dx=\int_1^2\left(x-3+\frac{3}{x}-\frac{1}{x^2}\right)dx=\left[\tfrac{1}{2}x^2-3x+3\ln|x|+\frac{1}{x}\right]_1^2$$

$$=\left(2-6+3\ln 2+\tfrac{1}{2}\right)-\left(\tfrac{1}{2}-3+0+1\right)=3\ln 2-2$$

30. $\int_0^{3\pi/2}|\sin x|\,dx=\int_0^{\pi}\sin x\,dx+\int_{\pi}^{3\pi/2}(-\sin x)\,dx=\left[-\cos x\right]_0^{\pi}+\left[\cos x\right]_{\pi}^{3\pi/2}=[1-(-1)]+[0-(-1)]=2+1=3$

38. $\int_{-\pi/2}^{2\pi} \cos x \, dx = \Big[\sin x\Big]_{-\pi/2}^{2\pi} = \sin 2\pi - \sin(-\pi/2)$

$= 0 - (-1) = 1$



44. $\int v(v^2+2)^2\,dv = \int v(v^4+4v^2+4)\,dv = \int (v^5+4v^3+4v)\,dv = \frac{v^6}{6} + 4\frac{v^4}{4} + 4\frac{v^2}{2} + C = \frac{1}{6}v^6 + v^4 + 2v^2 + C$

48. $\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\sin x \cos x}{\sin x}\,dx = \int 2\cos x\,dx = 2\sin x + C$

49. $A = \int_0^2 (2y - y^2)\,dy = \left[y^2 - \frac{1}{3}y^3\right]_0^2 = \left(4 - \frac{8}{3}\right) - 0 = \frac{4}{3}$

50. $y = \sqrt[4]{x} \quad \Rightarrow \quad x = y^4$, so $A = \int_0^1 y^4\,dy = \left[\frac{1}{5}y^5\right]_0^1 = \frac{1}{5}$.

72. (a) For $0 \le x \le 1$, we have $x^2 \le x$. Since $f(x) = \cos x$ is a decreasing function on $[0, 1]$, $\cos(x^2) \ge \cos x$.

(b) $\pi/6 < 1$, so by part (a), $\cos(x^2) \ge \cos x$ on $[0, \pi/6]$. Thus,

$\int_0^{\pi/6} \cos(x^2)\,dx \ge \int_0^{\pi/6} \cos x\,dx = \Big[\sin x\Big]_0^{\pi/6} = \sin(\pi/6) - \sin 0 = \frac{1}{2} - 0 = \frac{1}{2}$.