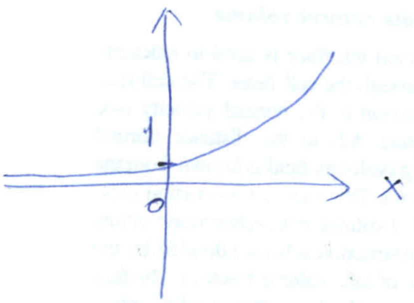
HW #2

Section 1.5: Exponential Functions.

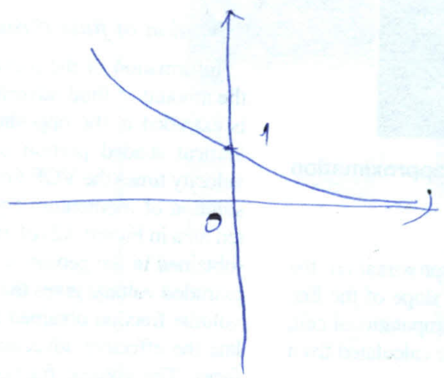
13-15: Sketch the graphs of the given functions.

13) $y=-2^{-x}$

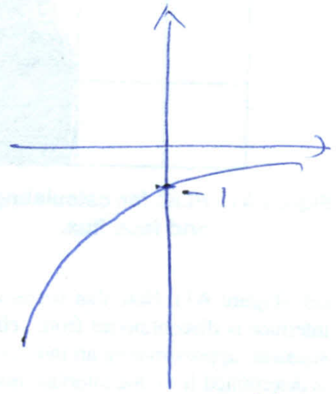
We start with the graph of $y=2^x$, reflect it about y-axis and about x-axis to obtain the graph of $y=-2^{-x}$.



$y=2^x$



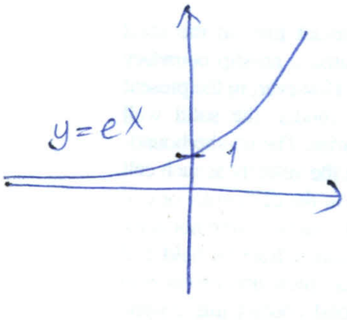
$y=2^{-x}$

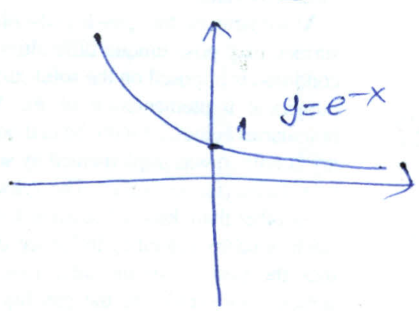


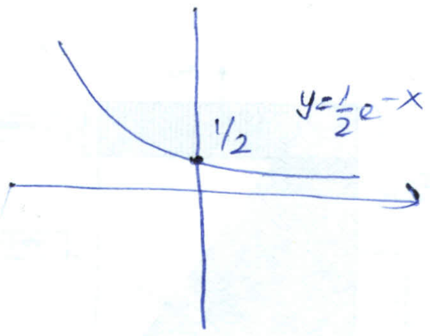
$y=-2^{-x}$.

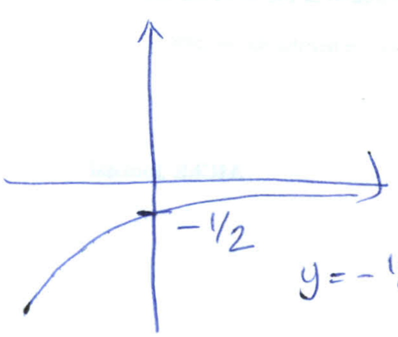
15) $y=1-\frac{1}{2}e^{-x}$

We start with the graph of $y=e^x$, reflect it about the y-axis, $(y=e^{-x})$. Then we compress the graph vertically by a factor of 2. $(y=\frac{1}{2}e^{-x})$ Then we reflect it about x-axis $(y=-\frac{1}{2}e^{-x})$ Finally, we shift the graph upword and get $y=1-\frac{1}{2}e^{-x}$

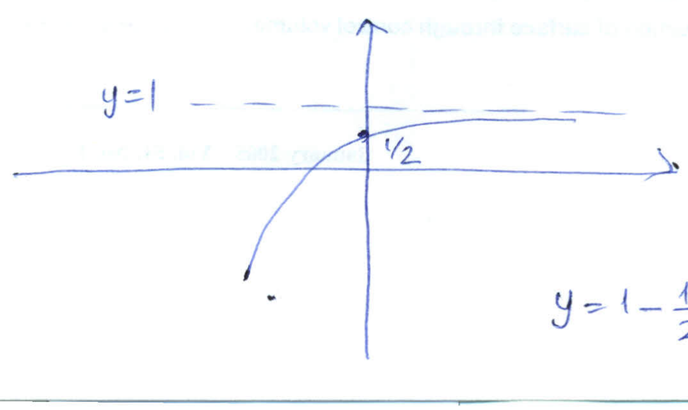








$y=-\frac{1}{2}e^{-x}$



$y=1-\frac{1}{2}e^{-x}$

19) Find the domain of the given functions.

(a) $f(x) = \dfrac{1-ex^2}{1-e^{1-x^2}}$ $\quad 1-e^{1-x^2} = 1 - \dfrac{e^1}{e^{x^2}} = \dfrac{e^{x^2}-e^1}{e^{x^2}}$

$$f(x) = \frac{(1-e^{x^2})\,e^{x^2}}{e^{x^2}-e^1} \Rightarrow e^{x^2}-e^1 \neq 0$$

$e^{x^2}-e^1 = e(e^{x^2-1}-1) = 0 \quad e \neq 0.$

Hence $e^{x^2-1} \neq 1 \Rightarrow x^2-1 \neq 0$

$x^2 \neq 1; \; x \neq \pm 1$

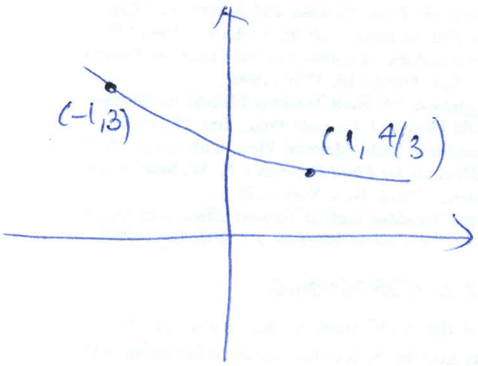
Domain; $x \in \mathbb{R} - \{-1, 1\}$

(b) $f(x) = \dfrac{1+x}{e^{\cos x}} \Rightarrow e^{\cos x} \neq 0 \qquad -1 \leq \cos x \leq 1$

For the given values of $\cos x$, $e^{\cos x}$ is always a finite nonzero number. Hence,

Domain : $x \in \mathbb{R}$

22) Find the exponential function. $f(x) = Ca^x$ whose graph is given



$f(x) = Ca^x$

$(-1,3)$: $3 = C.a^{-1} \Rightarrow C = 3a.$

$(1, 4/3)$: $\dfrac{4}{3} = Ca \Rightarrow Ca = 3a^2 = \dfrac{4}{3}$

$a^2 = \dfrac{4}{9} \Rightarrow a = \dfrac{\pm 2}{3}$

$C = \pm 2$

$f(x) = 2.\left(\dfrac{2}{3}\right)^x = 2.\left(\dfrac{3}{2}\right)^{-x}$

$\dfrac{2}{3} = \left(\dfrac{3}{2}\right)^{-1}$

30) A bacterial culture starts with 500 bacteria and doubles in size every half hour.

a) After 3 hours? 3 hours = 6 doubling periods. $500.2^6 = 32000$

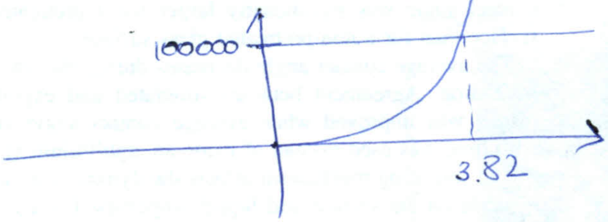
b) In t hours, there'll be 2t doubling periods. Population at the t

$y = 500.2^{2t}$

(c) After 40 minutes?

$$t = \frac{40}{60} = \frac{2}{3} \Rightarrow y = 500 \cdot 2^{2(2/3)} \approx 1260.$$

(d) $y_1 = 500 \cdot 2^{2t}$, $y_2 = 100.000$, $t = 3.82$

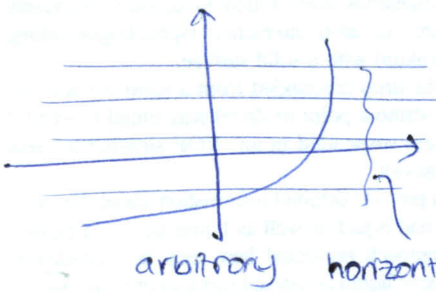


~~Exercises~~

Section 1.6: Inverse functions and logarithms.

6-10-11 : Is the given function one-to-one?

6)



No horizontal line intersects the graph more than once. It is one-to-one

10) $f(x) = 1 + 4x - x^2$

The graph of $f(x)$ is a parabola with axis of symmetry $x = -\frac{b}{2a} = \frac{-4}{2(-1)} = 2$.

$f(1) = 4$, $f(3) = 4$. So it is not one to one

11) $g(x) = |x| \Rightarrow g(-1) = g(1) = 1$, so g isn't one to one,

22-23-25-26: Find a formula for the inverse of the function.

22) $f(x) = \dfrac{4x-1}{2x+3} \Rightarrow y(2x+3) = 4x-1 \Rightarrow 2xy + 3y = 4x - 1$

$$3y + 1 = 4x - 2xy$$

$$3y + 1 = (4 - 2y)x \Rightarrow x = \frac{3y+1}{4-2y}$$

Interchange x and y; $y = \dfrac{3x+1}{4-2x} \Rightarrow f^{-1}(x) = \dfrac{3x+1}{4-2x}$

23) $f(x) = e^{2x-1}$ $\quad y = e^{2x-1} \Rightarrow \ln y = 2x-1$

$\Rightarrow 2x = \ln y + 1$

$x = \frac{\ln y + 1}{2}$ interchange x and y,

$y = \frac{\ln x + 1}{2}$ $\quad f^{-1}(x) = \frac{\ln x + 1}{2}$

25) $y = \ln(x+3) \Rightarrow x+3 = e^y \Rightarrow x = e^y - 3$ Interchange x and y,

$y = f^{-1}(x) = e^x - 3$

26) $y = \frac{e^x}{1+2e^x} \Rightarrow y + 2ye^x = e^x$

$\Rightarrow e^x.(1-2y) = y \Rightarrow e^x = \frac{y}{1-2y} \Rightarrow x = \ln\left(\frac{y}{1-2y}\right)$

Interchange x and y: $y = f^{-1}(x) = \ln\left(\frac{x}{1-2x}\right)$

36–38: Find the exact value of the expression.

36) a) $\ln(1/e) = \ln 1 - \ln e = 0 - 1 = -1$

b) $\log_{10}\sqrt{10} = \log_{10} 10^{1/2} = \frac{1}{2}\log_{10} 10 = 1/2$

38) a) $e^{-2\ln 5} = e^{\ln 5^{-2}} = e^{\ln(1/25)} = 1/25$

b) $\ln(\ln e^{e^{10}}) = \ln(e^{10}.\underbrace{\ln e}_{=1}) = \ln e^{10} = 10.\underbrace{\ln e}_{=1} = 10.$

41) Express the given quantity as a single logarithm

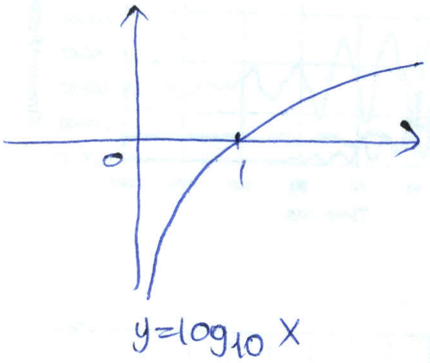
$\ln(1+x^2) + \frac{1}{2}\ln x - \ln\sin x = \ln(1+x^2) + \ln x^{1/2} - \ln$ sin x

=ln((1+x^2).sqrt(x)/sinx) $\quad = \ln\left\{\frac{(1+x^2).\cancel{\sqrt{x}}}{\cancel{x}\ \cancel{\sqrt{x}}}\right\} \cancel{-\ln}\left(\frac{\cancel{1+x^2}}{\sqrt{x}}\right)$

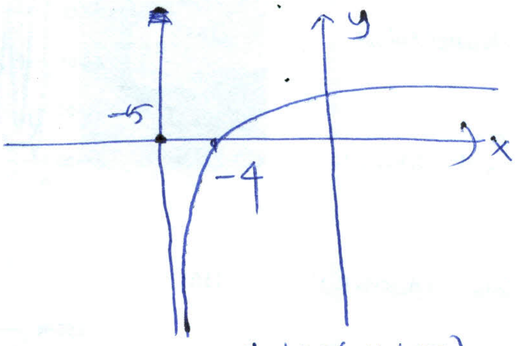
47) Sketch the given functions;

(a) $y=\log_{10}(x+5)$

Shift the graph of $y=\log_{10}x$ five units to the left to obtain the graph of $y=\log_{10}(x+5)$. (Vertical asymptote $x=-5$)

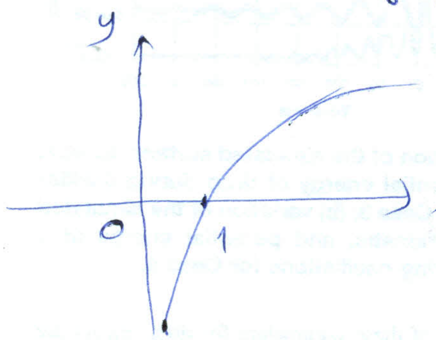


$y=\log_{10}x$

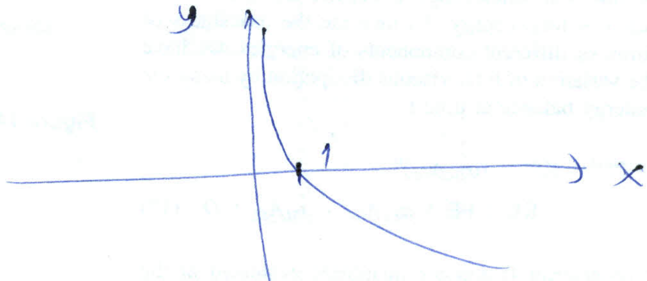


$y=\log_{10}(x+5)$

b) $y=-\ln x$ Reflect the graph of $y=\ln x$ about x-axis to obtain the graph of $y=-\ln x$.



$y=\ln x$



$y=-\ln x$.

50) Solve the given equations for x.

a) $\ln(x^2-1)=3 \Rightarrow x^2-1=e^3 \Rightarrow x^2=1+e^3$

$x=\pm\sqrt{1+e^3}$

b) $e^{2x}-3e^x+2=0$

$(e^x-2)(e^x-1)=0 \Rightarrow e^x=2$ OR $e^x=1$

Hence, $x=\ln 2$ OR $x=\ln 1=0$

54) Solve the inequalities for x.

a) $2 < \ln x < 9 \Rightarrow e^2 < x < e^9$.

b) $e^{2-3x} > 4 \Rightarrow \ln e^{2-3x} > \ln 4$

$2-3x > \ln 4$

$2 - \ln 4 > 3x \Rightarrow x < \frac{2-\ln 4}{3}$

56) Find the domain of f and f^{-1}.

$f(x) = \ln(2+\ln x)$

$2 + \ln x > 0. \Rightarrow \boxed{\ln x > -2}$ ~~Doman~~

Domain of f: $x > e^{-2}$

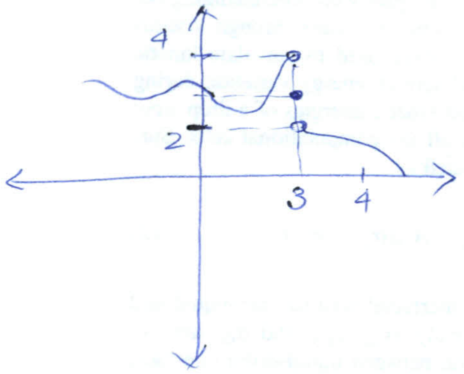
Let's find f^{-1}: $y = \ln(2+\ln x) \Rightarrow e^y = 2 + \ln x \Rightarrow \ln x = e^y - 2$

$\Rightarrow x = e^{e^y - 2}$ Interchange x and y,

$f^{-1} = e^{e^x - 2}$ Domain: $x \in \mathbb{R}$.

Section 2.2

4) For the function f whose graph is given, state the value of each quantity, if it exists.



a) $\lim_{x\to 0} f(x) = 3$

b) $\lim_{x\to 3^-} f(x) = 4$

c) $\lim_{x\to 3^+} f(x) = 2$

d) $\lim_{x\to 3} f(x)$ doesn't exist because (b) and (c) aren't equal

e) $f(3) = 3$.

6)

a) $\lim_{x \to -3^-} h(x) = 4$

b) $\lim_{x \to -3^+} h(x) = 4$

c) $\lim_{x \to -3} h(x) = 4$

d) $h(-3)$ isn't defined.

e) $\lim_{x \to 0^-} h(x) = 1$

f) $\lim_{x \to 0^+} h(x) = -1$

g) $\lim_{x \to 0} h(x)$ doesn't exist.

h) $h(0) = 1$

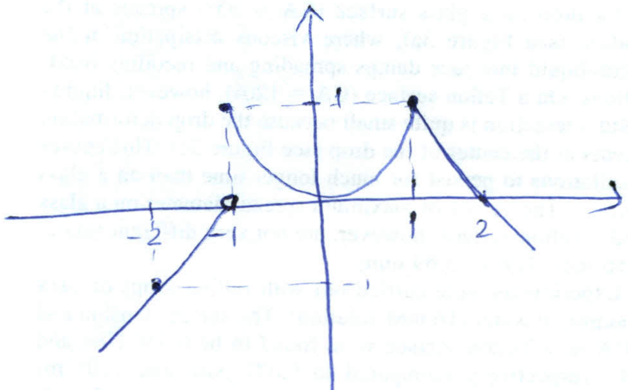
i) $\lim_{x \to 2} h(x) = 2$

j) $h(2)$ isn't defined

k) $\lim_{x \to 5^+} h(x) = 3$

l) $\lim_{x \to 5^-} h(x) = 3$ limit doesn't exist

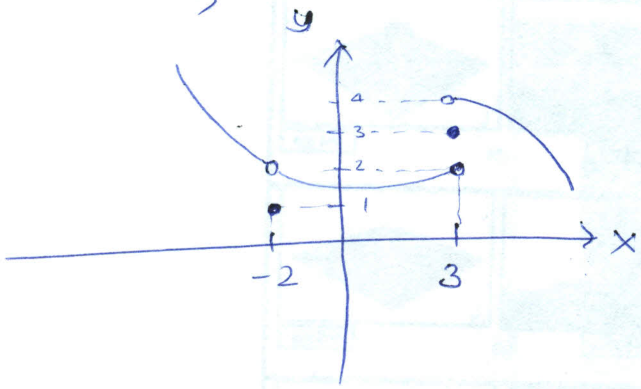
7) $f(x) = \begin{cases} 1 + x, & x < -1 \\ x^2, & -1 \leq x \leq 1 \\ 2 - x, & x \geq 1 \end{cases}$



at $a = 1$, $\lim_{x \to 1} f(x) = 1$ (exists).

(except $(a = -1)$, limit exists at all points and equal to the value of the function.

15) Sketch the graph of an example of a function f with the given conditions,



21) $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$

x	f(x)
1	0.236068
0.5	0.242641
0.1	0.248457
0.05	0.249224
0.01	0.249844

x	f(x)
-1	0.267949
-0.5	0.258343
-0.1	0.251582
-0.05	0.250786
-0.01	0.250156

$\Rightarrow \lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} = \frac{1}{4}$

23) $f(x) = \frac{x^6 - 1}{x^{10} - 1}$, $\lim_{x \to 1} \frac{x^6 - 1}{x^{10} - 1}$

x	f(x)
0.5	0.985337
0.9	0.719397
0.95	0.660186
0.99	0.612018
0.999	0.601200

x	f(x)
1.5	0.183369
1.1	0.484119
1.05	0.540783
1.01	0.588022
1.001	0.5988000

$\Rightarrow \lim_{x \to 1} f(x) = \frac{3}{5} = 0.6$