KOÇ UNIVERSITY

MATH 102

EXAM 1 November 2, 2019

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________

Student ID no: KEY

Signature: ______________________

(Check One):

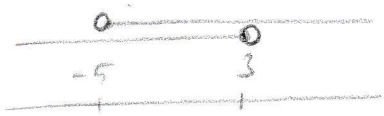
(Selda Küçükçifçi – TTh 8:30-9:45) : —
(Selda Küçükçifçi – TTh 13:00-14:15) : —
(E. Şule Yazıcı – TTh 16:00-17:15) : —

PROBLEM	POINTS	SCORE
1	20	
2	30	
3	30	
4	12	
5	10	
TOTAL	102	

Problem 1 (a) (10 points) Determine the domain of the function $f(x) = \dfrac{\log(3-x)}{\sqrt{x+5}}$.

$3 - x > 0$ and $x + 5 > 0$

$x < 3$ and $x > -5$



So the domain of f is $(-5, 3)$.

(b) (10 points) At what point is the tangent line to the curve $y = \sqrt{1+2x}$ parallel to the line $3y - x = 18$?

$3y - x = 18 \Rightarrow y = \dfrac{x+18}{3}$ So $m = \dfrac{1}{3}$.

$y' = \dfrac{1}{2}(1+2x)^{-1/2} \cdot 2 = \dfrac{1}{\sqrt{1+2x}}$

$\dfrac{1}{3} = \dfrac{1}{\sqrt{1+2x}} \Rightarrow 1 + 2x = 9 \Rightarrow x = 4.$

$P(4, 3)$.

Problem 2 (30 points) Find the following limits, if they exist. (Do not use l'Hospital's rule.)

(a) $\lim_{x\to-\infty} \frac{3x^{10}+x^5+1}{1-x^5}$ $= \lim_{x\to-\infty} \frac{3x^5+1+\frac{1}{x^5}}{\frac{1}{x^5}-1} = +\infty$

(b) $\lim_{x\to 1} \frac{|x-1|}{x^2-1}$

$$\lim_{x\to 1^+} \frac{|x-1|}{x^2-1} = \lim_{x\to 1^+} \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \lim_{x\to 1^+} \frac{1}{x+1} = \frac{1}{2}$$

$$\lim_{x\to 1^-} \frac{|x-1|}{x^2-1} = \lim_{x\to 1^-} \frac{-(\cancel{x-1})}{(\cancel{x-1})(x+1)} = \lim_{x\to 1^-} \frac{-1}{x+1} = \frac{-1}{2}$$

So $\lim_{x\to 1} \frac{|x-1|}{x^2-1}$ does not exist.

(c) $\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}$ $= \lim_{x\to 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{x(16-x)(4+\sqrt{x})} = \lim_{x\to 16} \frac{16-x}{x(16-x)(4+\sqrt{x})}$

$$= \lim_{x\to 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16\times 8} = \frac{1}{128}$$

Problem 3 (30 points) Find the derivative of the following functions. Simplify your answer, if it is possible.

(a) $f(x) = x^2 \cos x + 4 \tan x$

$$f'(x) = 2x\cos x - x^2 \sin x + 4\sec^2 x$$

(b) $f(x) = 2^{x^2+1}$

$$f'(x) = 2^{x^2+1} \cdot \ln 2 \cdot 2x$$

(c) $f(x) = \left(\dfrac{x-1}{3x^2+x}\right)^{10}$.

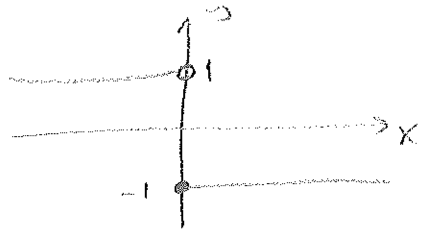
$$f'(x) = 10 \left(\frac{x-1}{3x^2+x}\right)^9 \frac{(3x^2+x) - (x-1)(6x+1)}{(3x^2+x)^2}$$

$$= \frac{10(x-1)^9 (3x^2+x-6x^2+6x-x+1)}{(3x^2+x)^{11}}$$

$$= \frac{10(x-1)^9(-3x^2+6x+1)}{(3x^2+x)^{11}}$$

Problem 4 (12 points) Find a function f defined on $[-1,1]$ such that $f(-1) = 1 > 0$ and $f(1) = -1 < 0$ but there is no $c \in [-1,1]$ such that $f(c) = 0$.

$$f(x) = \begin{cases} -1 & x \geq 0 \\ 1 & x < 0 \end{cases}$$



there is no $c \in [-1,1]$ such that $f(c)=0$.

Problem 5 (10 points) Let $F(x) = \sin(g(x))$. If $F'(3) = 10, g(3) = \pi$, find $g'(3)$.

$$F'(x) = \cos(g(x)) \cdot g'(x)$$

$$10 = \underbrace{\cos \pi}_{-1} \times g'(3)$$

So $g'(3) = -10$.