KOÇ UNIVERSITY

MATH 102

SECOND MIDTERM DECEMBER 8, 2015

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________

Student ID no: KEY

Signature: ______________________

(Check One):
(Selda Küçükçifçi – MW 13:00-14:15) : ___
(Selda Küçükçifçi – MW 16:00-17:15) : ___

PROBLEM	1	2	3	4	5	TOTAL
POINTS	34	20	7	20	27	108
SCORE						

Problem 1 (34 pts) Let $f(x) = \dfrac{x^2}{x^2-1}$.

(a) (6 pts) Determine the asymptotes of f, if they exist.

$\lim\limits_{x\to\infty} \dfrac{x^2}{x^2-1} = 1$

$\lim\limits_{x\to-\infty} \dfrac{x^2}{x^2-1} = 1$

$y=1$ is the horizontal asymptote.

$\lim\limits_{x\to1^-} \dfrac{x^2}{x^2-1} = -\infty$ $\qquad$ $\lim\limits_{x\to-1^-} \dfrac{x^2}{x^2-1} = +\infty$

$\lim\limits_{x\to1^+} \dfrac{x^2}{x^2-1} = +\infty$ $\qquad$ $\lim\limits_{x\to-1^+} \dfrac{x^2}{x^2-1} = -\infty$

$x=1$ & $x=-1$ are vertical asymptotes.

(b) (16 pts) Find the intervals on which f is increasing/decreasing and the intervals on which f is concave up/down. Given that $f'(x) = \dfrac{-2x}{(x^2-1)^2}$ and $f''(x) = \dfrac{6x^2+2}{(x^2-1)^3}$.

		-1		0		1	
f'	$+$		$+$	0	$-$		$-$
f''	$+$		$-$		$-$		$+$
f	↗		↗		↘		↘

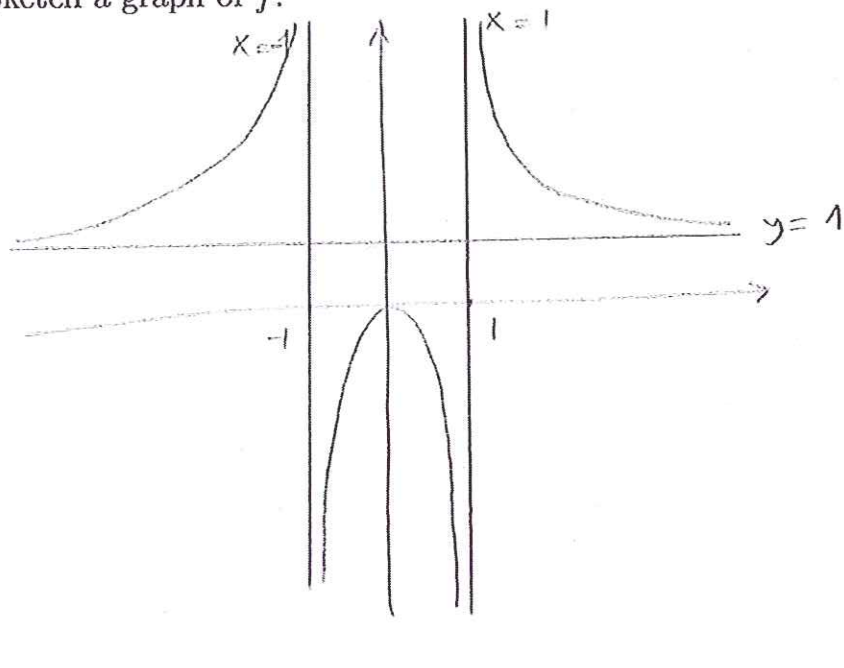
(c) (4 pts) Find the local maximum, local minimum and inflection points of f if they exist.

local maximum point: $(0,0)$

no local minimum

no inflection point

(d) (8 pts) Sketch a graph of f.



Problem 2 (20 pts) Find the absolute extremum (that is absolute maximum and absolute minimum values) of the function

$$f(x) = (x+1)^5 - 5x - 2 \text{ on } [-1, 1] \ .$$

$f'(x) = 5(x+1)^4 - 5 = 0$

$\Rightarrow (x+1)^4 = 1$

$\Rightarrow x+1 = 1$ or $x+1 = -1$

$x = 0$ $\quad$ $x = -2 \notin [-1,1]$.

$f(0) = 1 - 2 = -1 \leftarrow$ absolute minimum value.

$f(-1) = 5 - 2 = 3$

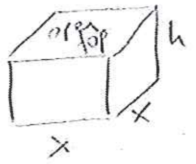
$f(1) = 32 - 5 - 2 = 25 \leftarrow$ absolute maximum value

Problem 3 (7 pts) Evaluate $\dfrac{d}{dx} \displaystyle\int_{x^2}^{1} \frac{\sin t}{t}\, dt.$

$$\frac{d}{dx} \int_{x^2}^{1} \frac{\sin t}{t}\, dt = -\frac{d}{dx} \int_{1}^{x^2} \frac{\sin t}{t}\, dt = -\frac{\sin(x^2)}{x^2} \cdot 2x$$

$$= -\frac{2\sin(x^2)}{x}.$$

Problem 4 (20 pts) A box with a square base and open top must have a volume of 4000 cm^3. Find the dimensions of the box that minimize the amount of material used.



$V = x^2 h = 4000 \Rightarrow h = \frac{4000}{x^2}$

$A = x^2 + 4xh$

minimize $\rightarrow A(x) = x^2 + 4x \cdot \frac{4000}{x^2} = x^2 + \frac{16000}{x}$

$A'(x) = 2x - \frac{16000}{x^2} = 0$

$\Rightarrow \quad 2x^3 = 16000 \Rightarrow x^3 = 8000 \Rightarrow x = 20$ cm.

$A''(x) = 2 - 16000(-2) \cdot \frac{1}{x^3}$

$A''(20) = 2 + \frac{32000}{(20)^3} > 0$ So $x = 20$ minimizes A.

then

$h = \frac{4000}{400} = 10$.

Problem 5 (a) Evaluate the integrals below.

(a) (7 pts) $\int \left(\frac{3}{x^3} + \sin x + \frac{1}{x}\right) dx$ $= 3\,\frac{x^{-2}}{-2} - \cos x + \ln|x| + C$

$= -\frac{3}{2x^2} - \cos x + \ln|x| + C$.

(b) (8 pts) $\int_0^1 (2x^3 + x)e^{x^4+x^2}\, dx$ $= \frac{1}{2}\int_0^2 e^u\, du = \frac{1}{2}\left[e^u\right]_0^2$

$u = x^4 + x^2$

$du = (4x^3 + 2x)\, dx$

$= \frac{1}{2}(e^2 - 1)$.

(c) (12 pts) $\int_1^2 x^3 \ln x\, dx$ $= \left[\frac{x^4}{4}\ln x\right]_1^2 - \frac{1}{4}\int_1^2 x^3\, dx$

$u = \ln x$ $\quad v = \frac{x^4}{4}$

$du = \frac{1}{x}\, dx$ $\quad dv = x^3\, dx$

$= \left[\frac{x^4}{4}\ln x\right]_1^2 - \frac{1}{4}\left[\frac{x^4}{4}\right]_1^2$

$= 4\ln 2 - \frac{1}{4}\left(4 - \frac{1}{4}\right)$

$= 4\ln 2 - 1 + \frac{1}{16} = 4\ln 2 - \frac{15}{16}$.