KOÇ UNIVERSITY

MATH 102

EXAM 2 December 7, 2018

Duration of Exam: 90 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________

Student ID no: KEY

Signature: ______________________

(Check One):

(Selda Küçükçifçi – MW 8:30-19:45) : ___
(Selda Küçükçifçi – MW 11:30-12:45) : ___
(Hasan İnci – MW 16:00-17:15) : ___

PROBLEM	POINTS	SCORE
1	12	
2	15	
3	30	
4	18	
5	28	
TOTAL	103	

Problem 1 (12 points) Using implicit differentiation determine y', where

$$2x(y+1)^2 + 10\ln(x-2) = 0.$$

$$2(y+1)^2 + 2x\,2(y+1)\,y' + 10\,\frac{1}{x-2} = 0$$

$$4x(y+1)\,y' = -2(y+1)^2 - \frac{10}{x-2}$$

$$y' = \frac{-2(y+1)^2 - \frac{10}{x-2}}{4x(y+1)} = \frac{-2(y+1)^2(x-2) - 10}{4x(x-2)(y+1)}$$

Problem 2 (15 points) Determine $\lim_{x\to\infty} \underbrace{\left(\cos\frac{1}{x}\right)^x}_{y}$.

$$y = \left(\cos\frac{1}{x}\right)^x$$

$$\ln y = x\ln\left(\cos\frac{1}{x}\right)$$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\ln\left(\cos\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(\cos\frac{1}{x}\right)}{\frac{1}{x}} \qquad \frac{0}{0}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{\cos\frac{1}{x}}\cdot\left(-\sin\frac{1}{x}\right)\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = 1\cdot 0 = 0.$$

So $\lim_{x\to\infty} y = e^0 = 1$

Problem 3 Let $f(x) = 2 - \frac{3}{x} - \frac{3}{x^2}$. $= \frac{2x^2-3x-3}{x^2}$

(a) (6 points) Determine the asymptotes of f, if they exist.

$\lim_{x \to 0^+} \frac{2x^2-3x-3}{x^2} = -\infty$ $\qquad$ $\lim_{x \to \pm\infty} \frac{2x^2-3x-3}{x^2} = 2$

$\lim_{x \to 0^-} \frac{2x^2-3x-3}{x^2} = -\infty$ $\qquad$ So $y = 2$ is the horizontal asymptote.

So $x=0$ is the vertical asymptote.

(b) (12 points) Find the intervals on which f is increasing/decreasing and the intervals on which f is concave up/down.

$f'(x) = \frac{(-3)(-1)}{x^2} - \frac{3(-2)}{x^3} = \frac{3}{x^2} + \frac{6}{x^3}$

$= \frac{3x+6}{x^3}$

$f''(x) = \frac{3(-2)}{x^3} + \frac{6(-3)}{x^4} = \frac{-6x-18}{x^4}$

$f'(x)=0 \Rightarrow x=-2$ $\qquad$ $f''(x)=0 \Rightarrow x=-3$

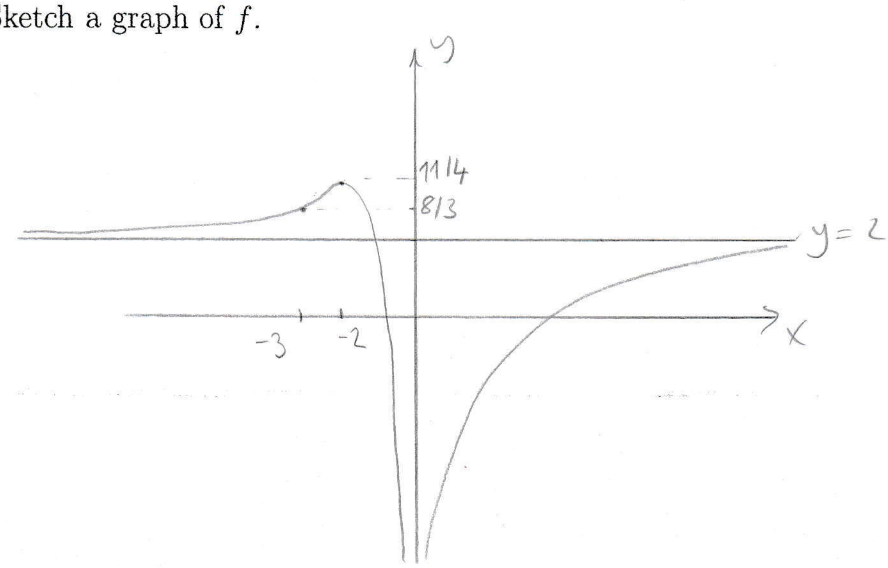
		-3		-2		0	
f'	$+$		$+$	0	$-$		$+$
f''	$+$	0	$-$		$-$		$-$
f	$\nearrow$		$\nearrow$		$\searrow$		$\nearrow$

(c) (4 points) Find the local maximum, local minimum and inflection points of f if they exist.

local maximum: $(-2, f(-2)) = (-2, 11/4)$

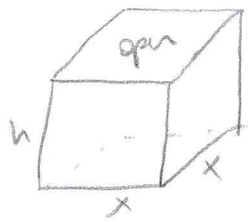
inflection point: $(-3, f(-3)) = (-3, 8/3)$.

(d) (8 points) Sketch a graph of f.



Problem 4 (18 points) If 2700 cm^2 of material is available to make a box with a square base and open top, find the largest possible volume of the box.

(The volume of the square box is $V = x^2h$, where x is the length of the side of the square base and h is the height of the box.)



$V = x^2 h$

$V(x) = x^2\left(\frac{675}{x} - \frac{1}{4}x\right)$

$V(x) = 675x - \frac{1}{4}x^3$

$x^2 + 4hx = 2700$

$h = \frac{2700 - x^2}{4x}$

$V'(x) = 0 \Rightarrow 675 - \frac{3}{4}x^2 = 0$

$h = \frac{675}{x} - \frac{1}{4}x$

$x^2 = \frac{4}{3}675 = 900$

$x = 30$.

$V''(x) = -\frac{6}{4}x$

$V''(30) = -\frac{3}{2} \times 30 < 0$

So $x = 30$ maximizes the volume.

then $h = \frac{675}{30} - \frac{1}{4}30$

$= \frac{45}{2} - \frac{15}{2} = 15$

So $V = 900 \times 15 = 13\,500$ cm^3.

Problem 5 Evaluate the followings.

(a) (8 points) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ $= \int 2e^u\, du = 2e^u + C = 2e^{\sqrt{x}} + C.$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\, dx$

(b) (8 points) $\int_0^1 (\sqrt[4]{x}+1)^2\, dx$ $= \int_0^1 (\sqrt{x} + 1 + 2\sqrt[4]{x})\, dx = \left[\frac{x^{3/2}}{3/2} + x + 2\frac{x^{5/4}}{5/4} \right]_0^1$

$= \frac{2}{3} + 1 + \frac{8}{5} = \frac{10+15+24}{15} = \frac{49}{15}$

(c) (12 points) $\frac{d}{dx}\int_{x^2}^3 \sqrt{t+e^t}\, dt$ $= -\frac{d}{dx}\int_3^{x^2} \sqrt{t+e^t}\, dt$

$= -\sqrt{x^2+e^{x^2}} \cdot 2x$