KOÇ UNIVERSITY

MATH 102

MIDTERM 2 April 24, 2012

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: KEY

Student ID no: ________________

Signature: ________________

(Check One):

(Selda Küçükçifçi – MWF 9:30-10:20) : __
(Selda Küçükçifçi – MWF 11:30-12:20) : __
(Şule Yazıcı – MWF 10:30-11:20) : __
(Ali Göktürk – MWF 12:30-13:20) : __
(Ali Göktürk – MWF 15:30-16:20) : __

PROBLEM	1	2	3	4	5	TOTAL
POINTS	25	34	15	15	15	104
SCORE						

Problem 1. (25 pts) Calculate the following limits.

(a) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right) = \lim_{x\to 0} \frac{x-\sin x}{x\sin x} \underset{\frac{0}{0}}{=} \lim_{x\to 0}\frac{1-\cos x}{\sin x + x\cos x}$ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{\sin x}{\cos x + \cos x - x\sin x} = \frac{0}{2} = 0.$

(b) $\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^x =$

$y = \left(1+\frac{5}{x}\right)^x$

$\ln y = x\ln\left(1+\frac{5}{x}\right)$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} x\ln\left(1+\frac{5}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(1+\frac{5}{x}\right)}{\frac{1}{x}} \underset{\frac{0}{0}}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{5}{x}}\cdot\frac{-1}{x^2}\cdot 5}{\frac{-1}{x^2}}$

$= \lim_{x\to\infty}\frac{5}{1+\frac{5}{x}} = 5 \qquad \lim_{x\to\infty}\ln y = 5 \Rightarrow \lim_{x\to\infty} y = e^5$

(c) $\lim_{x\to 0}\frac{10^x - e^x}{x-1} = \quad \frac{0}{-1} = 0$

Problem 2. Consider the function $f(x) = \ln(x^2 + 4)$.

(a) (2 pts) Find the domain of $f(x)$.

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(b) (2 pts) Find the x and y intercepts of the graph of f if they exist.

no x-intercept

y-intercept : $(0, \ln 4)$

(c) (2 pts) Find the horizontal and vertical asymptotes of the graph of f if they exist.

$\lim_{x \to \pm\infty} \ln(x^2+4) = \infty$ so no horizontal asymptote.

no vertical asymptote.

(d) (9 pts) Find the intervals on which the function f is increasing or decreasing and; determine the local extreme values of f.

$f'(x) = \dfrac{2x}{x^2+4}$ $\qquad f'(x) = 0 \Rightarrow x = 0$

		0	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

f is increasing on $(0, +\infty)$

f is decreasing on $(-\infty, 0)$

local minimum $(0, \ln 4)$

no local maximum

(e) (9 pts) Find the inflection points and determine the intervals where the graph of the function f is concave up and concave down.

$$f''(x) = \frac{2(x^2+4) - 2x \cdot 2x}{(x^2+4)^2} = \frac{8-2x^2}{(x^2+4)^2} = \frac{2(4-x^2)}{(x^2+4)^2}$$

$f''(0) = 0 \Rightarrow \quad x = 2 \ \& \ x = -2$

		-2		2	
f''	$-$	0	$+$	0	$-$
f	concave down		concave up		concave down

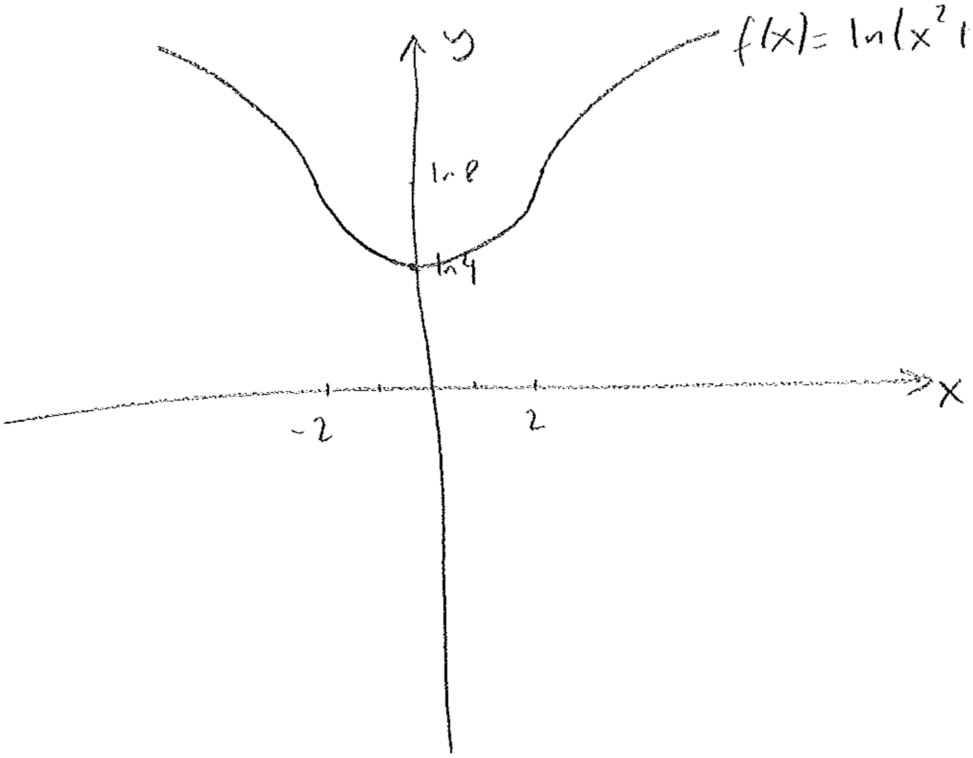
f is concave up on $(-2, 2)$

f is concave down on $(-\infty, -2) \cup (2, +\infty)$

inflection points: $(-2, \ln 8)$ & $(2, \ln 8)$.

(f) (10 pts) Sketch the graph of the function f.

		-2		0		2	
f'	$-$		$-$		$+$		$+$
f''	$-$	0	$+$		$+$	0	$-$
f	⌒ decreasing		◡ decreasing		◡ increasing		⌒ increasing



$f(x) = \ln(x^2+4)$.

Problem 3. (15 pts) Find the absolute extremum of the function $f(x) = 2x^3 - 9x^2 + 1$ on [-1,1].

$f'(x) = 6x^2 - 18x = 0 \Rightarrow 6x(x-3) = 0$

$\Rightarrow x = 0$ & $x = 3$

$3 \notin [-1, 1]$

$f(0) = 1$ ← absolute maximum

$f(-1) = -2 - 9 + 1 = -10$ ← absolute minimum

$f(1) = 2 - 9 + 1 = -6$

Problem 4. (15 pts) The total cost of producing x garbage disposals per day is given by the function

$$C(x) = 4000 + 10x + 0.1x^2$$

Find the minimum average cost.

$A(x) = \frac{C(x)}{x} = \frac{4000}{x} + 10 + 0.1x$

$A'(x) = -\frac{4000}{x^2} + 0.1 = 0 \Rightarrow 0.1 = \frac{4000}{x^2}$

$\Rightarrow x^2 = 40000 \Rightarrow x = \pm 200$

$x > 0$ So $x = 200$

$A''(x) = -4000(-2)x^{-3} = \frac{8000}{x^3}$

$A''(200) = \frac{8000}{(200)^3} > 0$ So $x = 200$ minimizes the average cost.

minimum average cost $A(200) = \frac{4000}{200} + 10 + 0.1(200) = 20 + 10 + 20 = 50$.

Problem 5. (15 pts) Find the function $f(x)$, where $f''(x) = 6x^2 + \dfrac{x^{-3/2}}{4}$, $f'(1) = 1$ and $f(1) = 0$.

$$f'(x) = 2x^3 + \frac{1}{4}\frac{x^{-1/2}}{-1/2} + C$$

$$f'(x) = 2x^3 - \frac{1}{2}x^{-1/2} + C$$

$$f'(1) = 2 - \frac{1}{2} + C = 1 \Rightarrow C = 1 - \frac{3}{2} = -\frac{1}{2}$$

$$f'(x) = 2x^3 - \frac{1}{2}x^{-1/2} - \frac{1}{2}$$

$$f(x) = \frac{2}{4}x^4 - \frac{1}{2}\frac{x^{1/2}}{1/2} - \frac{1}{2}x + D$$

$$f(x) = \frac{1}{2}x^4 - \sqrt{x} - \frac{1}{2}x + D$$

$$f(1) = \frac{1}{2} - 1 - \frac{1}{2} + D = 0$$

$$D = 1$$

So $f(x) = \frac{1}{2}x^4 - \sqrt{x} - \frac{1}{2}x + 1$.